# Assigning Papers to Referees Objectives, Algorithms, Open Problems

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# Overview

- Motivation
- Informal Problem Definition
- Formal Definition
- Algorithms and Hardness
- Truthfulness

Slides and paper are available at my home page

# Motivation

I was program chair of ESA 2008.

After submission closes and before reviewing starts, the PC chair assigns the papers to the PC members (called reviewers in the sequel).

What constitutes a good assignment?

# Informal Problem Definition I

- $n$ reviewers, $r$ indexes reviewers
- $m$ papers, $p$ indexes papers
- $v_{rp}$, the value of paper $p$ for reviewer $r$
  the interest of reviewer $r$ in paper $p$
  the qualification of reviewer $r$ for paper $p$
  the rank of paper $p$ for reviewer $r$
- valuations can be determined in many different ways:
  - the PC chair invents them
  - papers and reviewers provide key words, $v_{rp}$ is a function of the number of common key words
  - reviewers provide values in $\{$ NO, LOW, MEDIUM, HIGH $\}$
  - a combination of the above (our recommendation)
  - EasyChair (Andrei Voronkov), the system used for ESA 2008, asks the reviewers for bids

# Informal Problem Definition II

- $n$ reviewers, $r$ indexes reviewers  $m$ papers, $p$ indexes papers
- edge-labelled bipartite graph $G = (\text{papers} \cup \text{reviewers}, E)$
- $(r, p) \notin E$ means that $r$ cannot review $p$  conflict of interest
- for $e = (r, p) \in E$, $v_{rp} \in \{1, \ldots, \Delta\}$ is the rank of $r$ for $p$
- an assignment $M$ is a subset of the edges

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- Objectives
  - Coverage: each paper is reviewed (at least) $k$ times
  - Load-Balance: load is shared evenly among reviewers;
    every rev. reviews $h = \lceil mk/n \rceil$ or $h-1$ papers; today: $mk/n \in \mathbb{N}$

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  - Quality: papers are reviewed by qualified reviewers and reviewers get the papers that they are interested in
  - Fairness: papers are treated fairly, reviewers are treated fairly

# Quality w.r.t. a Reviewer (Paper)

- balanced assignment: $k$ reviews per paper, $h$ reviews per reviewer
- signature of reviewer $r$: $s^r = (s^r_\Delta, \ldots, s^r_1)$

  $s^r_\ell =$ number of papers with valuation $\ell$ assigned to $r$
- order signatures either
  - lexicographically or
  - by weight

$$w(s^r) = \sum_{1 \le \ell \le \Delta} w_\ell s^r_\ell$$

  where $w_\ell = \ell$ or $w_\ell = 2^\ell$ or ...
- reviewers prefer assignments that give them a high signature (selfish view)

# EasyChair's Solution

- convert the $v_{rp}$'s to numbers (LOW = 1, MEDIUM = 2, HIGH = 3)
- compute an maximum weight balanced assignment

  EasyChair computes an approximation

- value of assignment = sum of the values of the reviewers

$$\sum_r w(s^r)$$

- LEDA running time: $0.1$ sec for ESA instance
- maximum weight assignments are not necessarily "fair"

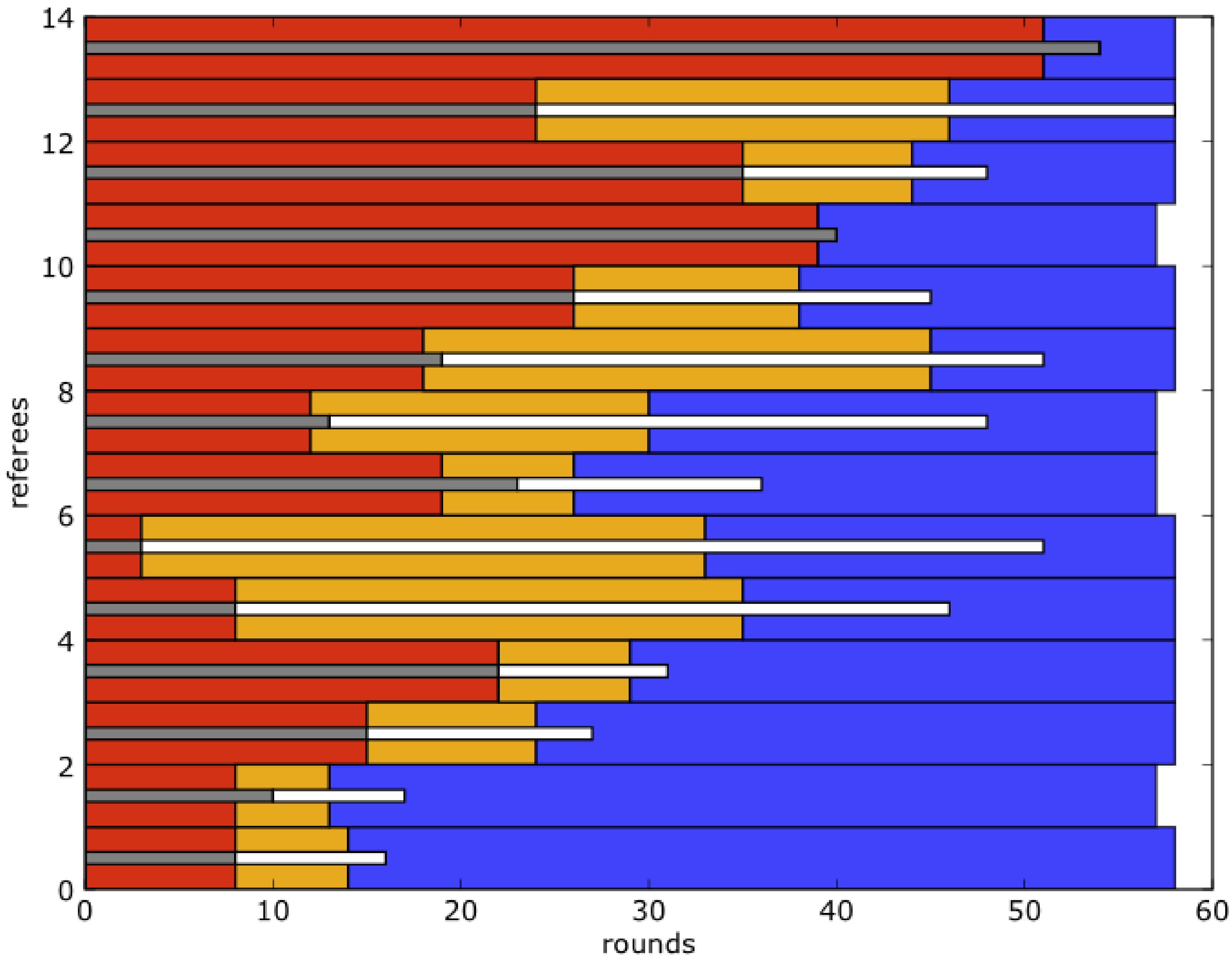
# Fairness

- four papers, two reviewers, each paper needs to be reviewed once
- reviewers agree in their valuation: two papers are H, two papers are L
- consider

| | | |
|---|---|---|
| Assignment A: | reviewer 1: L L | reviewer 2: H H |
| Assignment B: | reviewer 1: L H | reviewer 2: L H |

- both assignments have weight $2w(H) + 2w(L)$, but Assignment B is more fair than Assignment A
- whenever valuation $v_{rp}$ depends only on $p$, all assignments have the same weight

# Max Weight Assignment



# Formalization of Fairness

- PC work is a group effort; therefore special attention should be given to the reviewer that is least satisfied by an assignment
- recall $s^r(M) =$ signature of reviewer $r$ in assignment $M$
- signatures are ordered (lexicographically or by weight)
- for an assignment $M$

$$\min_r \; s^r(M)$$

  is the worst signature of any reviewer $r$
- we want the balanced assignment that maximizes the minimum signature

$$\max_M \min_r \; s^r(M)$$

- and among these assignments?
- the one that maximizes the second smallest signature, and among these, the one ... leximax solution

# Results

$\Delta \geq 3$: problem is NP-complete

all $\Delta$: approx. such that every reviewer looses at most $\Delta$ wrt optimum

$\Delta = 2$: efficient algorithm

experiments: good solutions for ESA data

# Signatures are Ordered by Weight

- inspired by allocation of indivisible goods (Santa Claus problem)
- sources
  - Bezakova, Dani: ACM SIGecom 2005
  - Lenstra, Schmoys, Tardos: Math Program. 1990
- the values $v_{rp}$ are numbers and it makes sense to add them
- binary variables $x_{rp}$ with $x_{rp} = 1$ iff paper $p$ is assigned to reviewer $r$
- load and coverage constraints:
  - $\sum_p x_{rp} = h$ for every reviewer $r$
  - $\sum_r x_{rp} = k$ for every (real) paper $p$

$S_r := \sum_p v_{rp} x_{rp}$ is value (utility) for reviewer $r$

# A Hardness Result

- goal: maximize the smallest signature

- It is NP-hard to compute a balanced assignment approximating the minimum signature within less than $\Delta/2$ for all $\Delta \geq 3$

# An Approximation Result

- fractional balanced assignments
  - every fractional balanced assignment gives rise to a vector $(t_1, t_2, \ldots, t_n)$, where $t_i =$ utility for reviewer $i$
  - let $(t_1^*, t_2^*, \ldots, t_n^*)$ be an optimal fractional assignment, i.e., it maximizes $sort(t_1^*, \ldots, t_n^*)$ (sort in increasing order)
  - $(t_1^*, t_2^*, \ldots, t_n^*)$ is unique and efficiently computable
  - fractional assignment: we may assign papers fractionally, e.g., $0.3$ to reviewer 1, $0.5$ to reviewer 2, $0.2$ to reviewer 3.

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  - $(t_1^*, t_2^*, \ldots, t_n^*)$ is unique and efficiently computable
- in polynomial time on can compute an integral assignment $M$ such that

$$S_r > t_r^* - \Delta \qquad \text{for all } r$$

i.e., each reviewer is within $\Delta$ of its utility in optimal fractional assignment

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MAX-PLANCK-GESELLSCHAFT

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- approach: compute optimal fractional assignment and round

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- remark: ESA instance ($\Delta = 3$): $h = 58$ and $58 \le S_r \le 174$, but
  $k = 4$ and $4 \le S_p \le 12$

# Finding the Optimum Fractional Solution

- proceed in rounds: in $j$-th round we compute $j$-th entry of $(s_1^*, \ldots, s_n^*) := sort(t_1^*, \ldots, t_n^*)$
- assume that we know the first $j-1$ entries of $s^*$ and the reviewers $r_1$ to $r_{j-1}$ defining them
- consider the following LP: maximize $q$ subj. to
  - $x$ guarantees coverage and load balance
  - $\sum_p v_{r_i p} x_{r_i p} = s_i^*$ for $1 \leq i < j$
  - $\sum_p v_{rp} x_{rp} \geq q$ for the remaining $r$
- let $q^*$ be the optimal value.
- find the reviewer(s) that cannot do better than $q^*$

  change one of the $\geq q$ into a $> q$ and check feasibility
- set $s_j^*$ to $q^*$ and $r_j$ to this reviewer

# Rounding Fractional Solutions

- let $x(e)$, $e \in E$ be any fractional solution satisfying the load and coverage constraints
- let $s_r(x) := \sum_p v_{rp} x_{rp}$, value for reviewer $r$
- let $s_p(x) := \sum_r v_{rp} x_{rp}$, value for paper $p$

- in polynomial time, we can find an integer assignment $y(e)$, $e \in E$, such that
  - $y$ satisfies the load and coverage constraints
  - $s_r(y) > s_r(x) - \Delta$ for all reviewers $r$
  - $s_p(y) > s_p(x) - \Delta$ for all papers $p$.
- observe that we have a guarantee for reviewers and papers

# The Rounding Scheme

- given a fractional assignment $x(e)$, $e \in E$ round to $y(e) \in \{0, 1\}$

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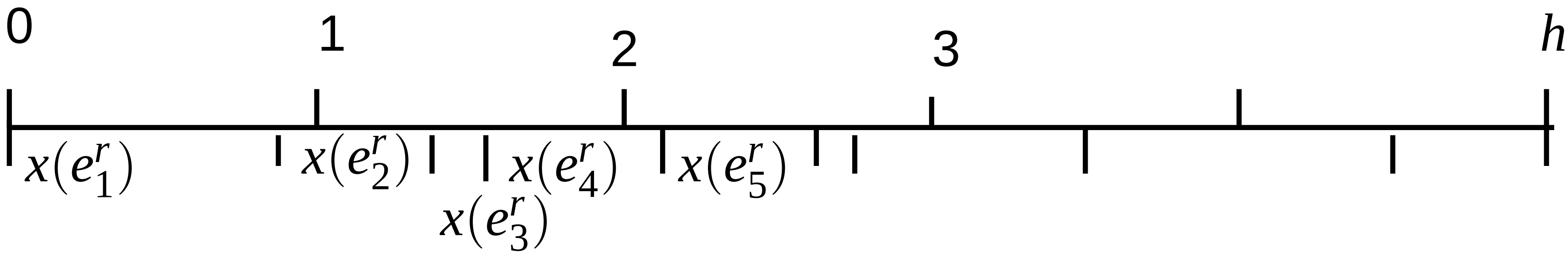
- given a fractional assignment $x(e)$, $e \in E$ round to $y(e) \in \{0,1\}$
- consider a fixed reviewer $r$, order the incident edges in order of decreasing weight, say $w(e_1^r) \geq w(e_2^r) \geq \ldots$.
- visualize the values $x(e_1^r)$, $x(e_2^r)$, $\ldots$



# The Rounding Scheme

- given a fractional assignment $x(e)$, $e \in E$ round to $y(e) \in \{0,1\}$
- consider a fixed reviewer $r$, order the incident edges in order of decreasing weight, say $w(e^r_1) \geq w(e^r_2) \geq \ldots$.
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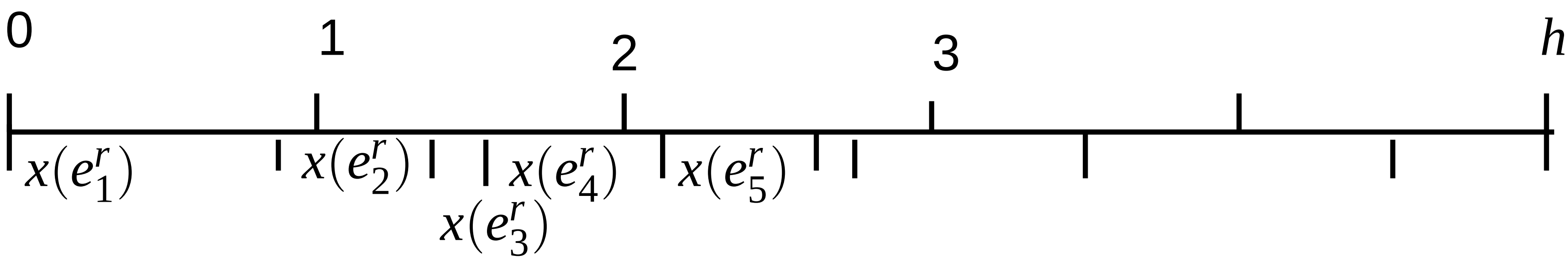
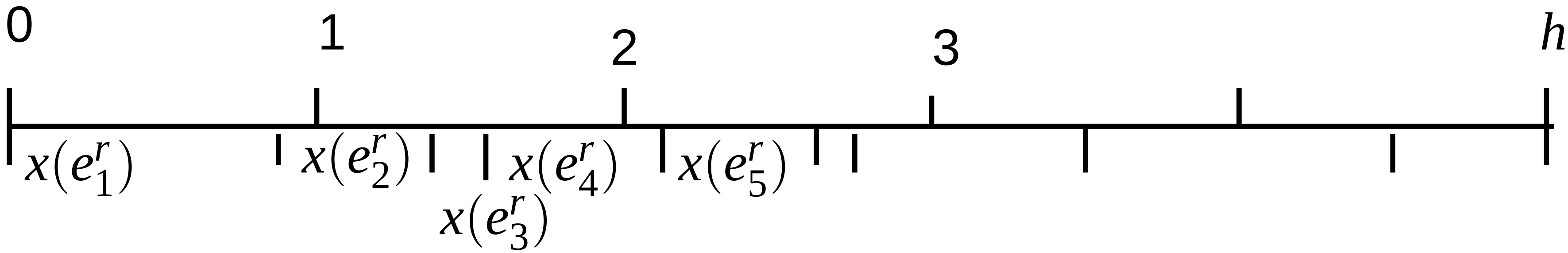



- goal: at least one of $y(e^r_1)$, $y(e^r_2)$ is one,
  at least two of $y_{(}e^r_1)$, ..., $y(e^r_4)$ are one, ...
- more generally: $x(e^r_1) + \ldots + x(e^r_\ell) \geq j \quad \Rightarrow \quad y(e^r_1) + \ldots + y(e^r_\ell) \geq j$
- such an integral solution exists and it yields the desired approximation

# The Approximation Quality

- given a fractional solution $x(e)$, $e \in E$, round to $y(e) \in \{0,1\}$
- reviewer $r$, order incident edges by weight $w(e_1^r) \geq w(e_2^r) \geq \ldots$.
- assume: $x(e_1^r) + \ldots + x(e_\ell^r) \geq j \quad \Rightarrow \quad y(e_1^r) + \ldots + y(e_\ell^r) \geq j$

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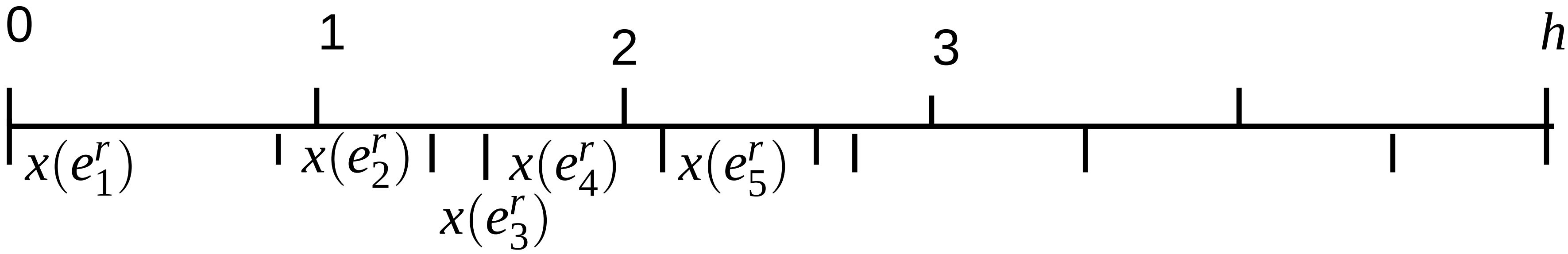


- how much can we loose by rounding? No more than

$$(w(e_1^r) - w(e_2^r)) + (w(e_2^r) - w(e_4^r)) + \ldots$$

since fractional value of $[1,2]$ at most $w(e_2^r)$ and integral value at least $w(e_4^r)$

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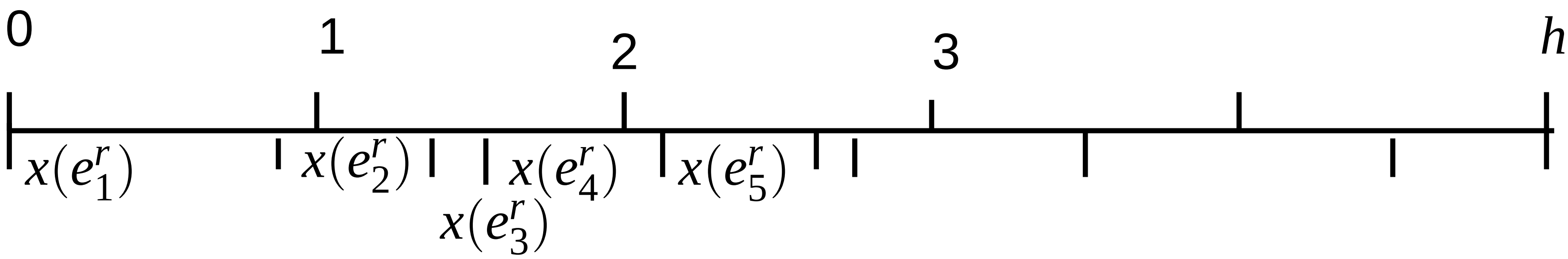


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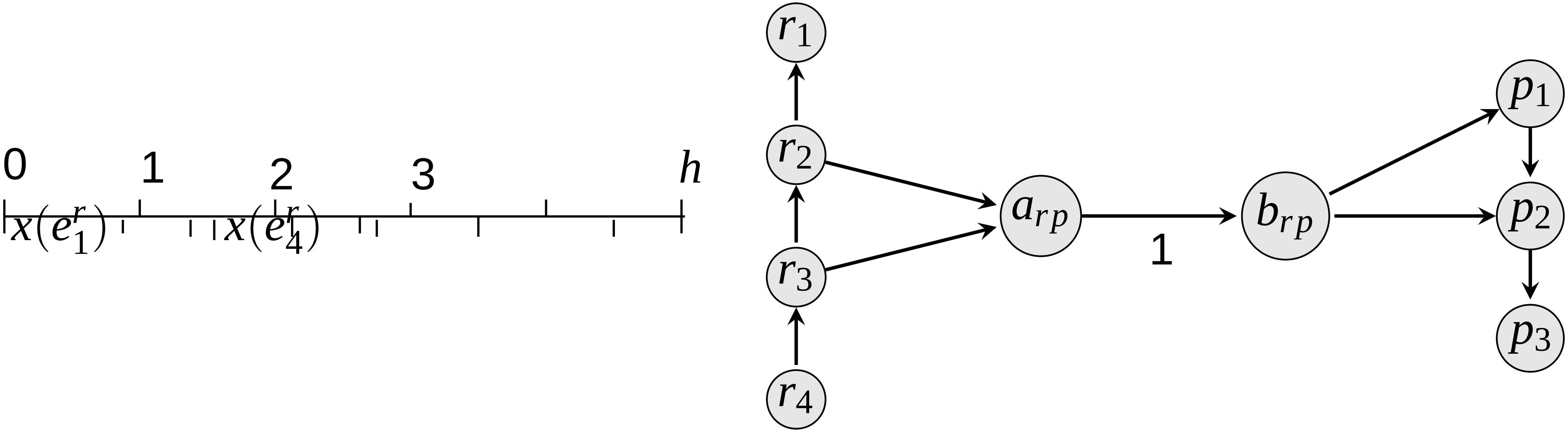
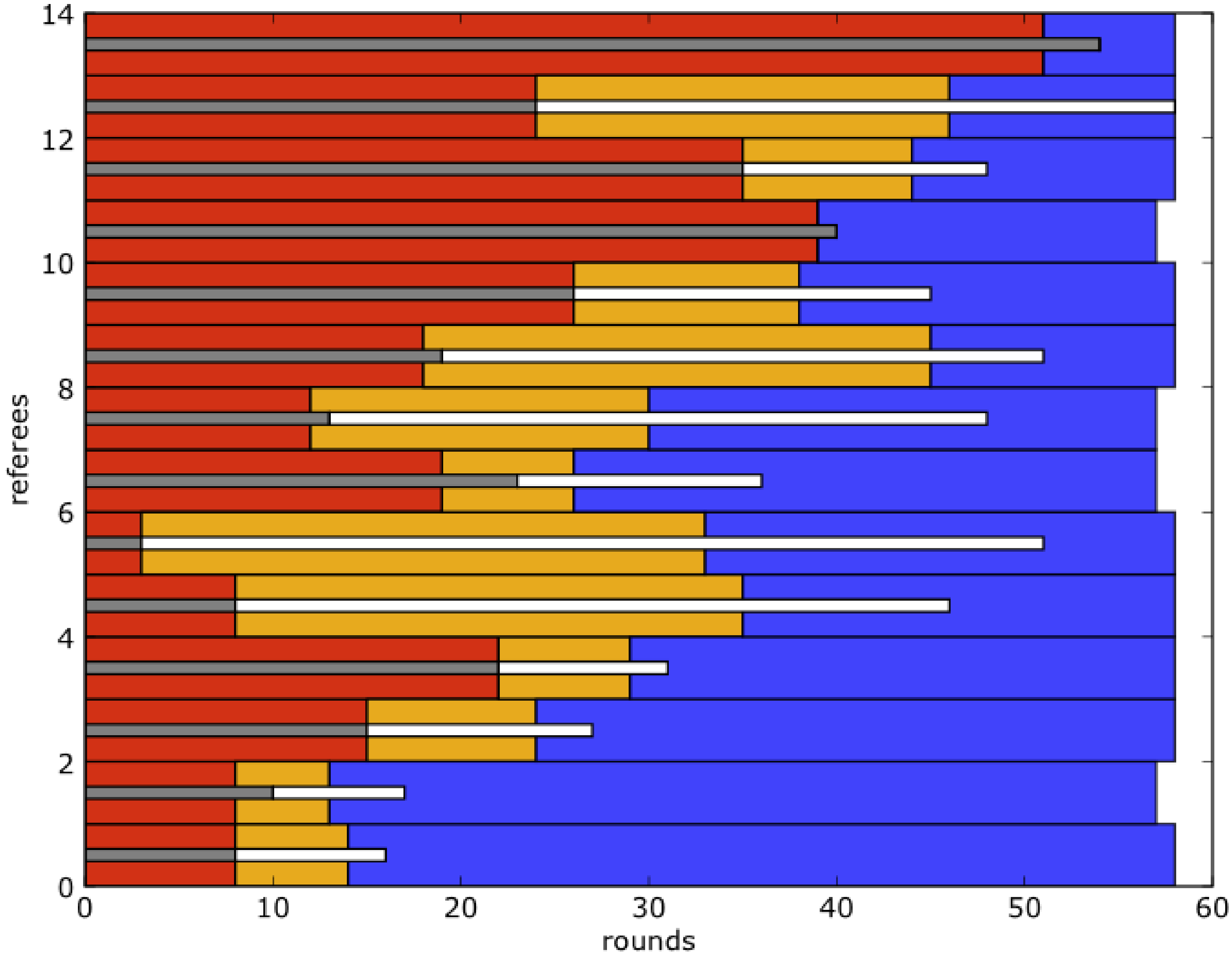
- this telescopes to no more than $\Delta$

# Existence: A Flowproblem

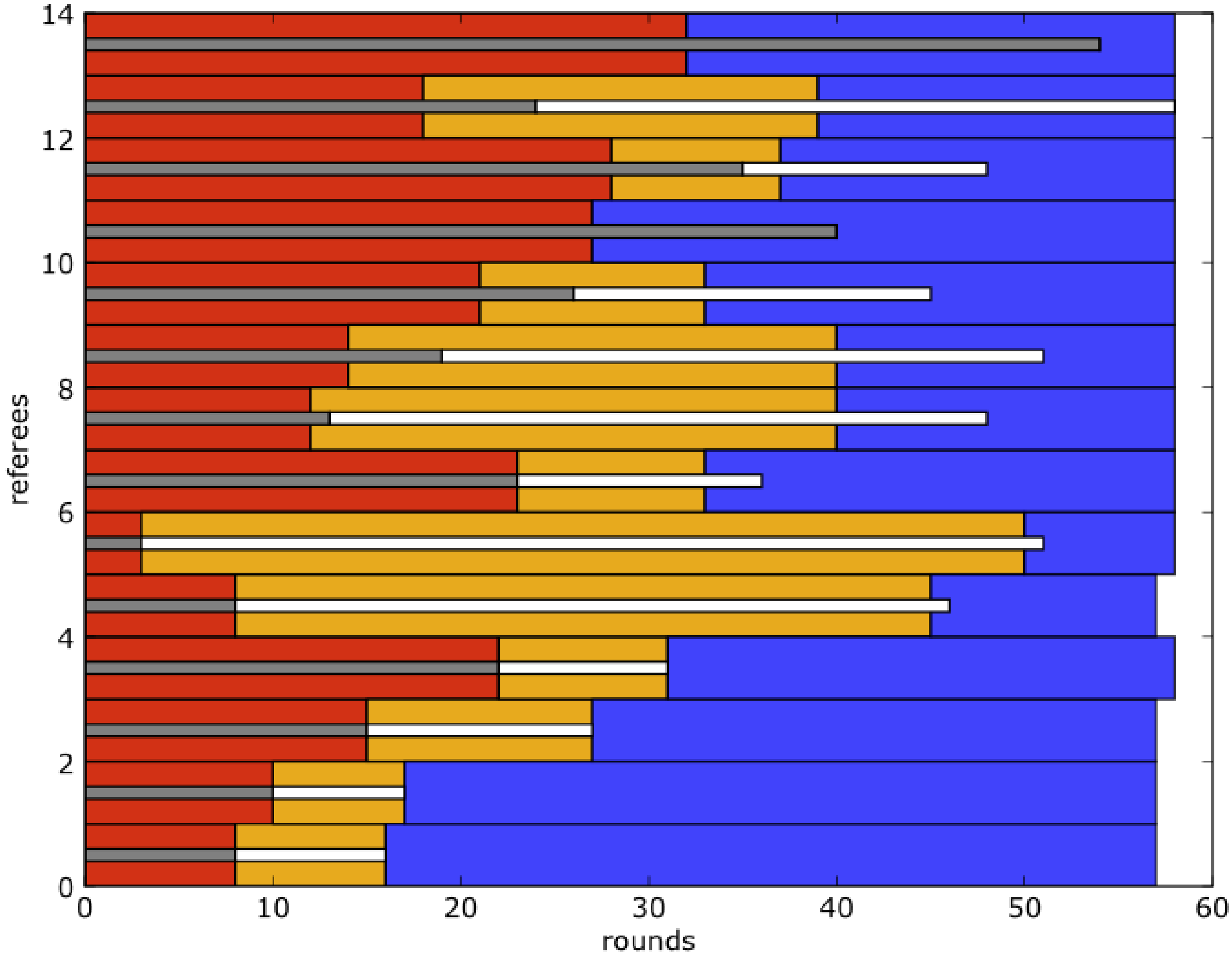



- have $h$ nodes for each reviewer (supply one) and $k$ nodes for each paper (demand one)
- $e_4^r = rp$ belongs to second and third group with respect to $r$ and first and second group with respect to $p$.
- fractional flow is feasible
- all capacities are integral $\Rightarrow$ integral flow exists
- flow out of $\{r_1, \ldots, r_i\}$ is at least $i$, flow into $\{p_1, \ldots, p_j\}$ is at least $j$.

# Max Weight Assignment



# Our Assignment (leximax)



# Two Ranks

- ordering signatures by weight or lexicographically is the same
- consider (reviewers are ordered by signature)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| H | H | H | H | H | H | H | H | L | L | L |
| H | H | H | H | H | H | L | L | L | L | L |
| H | H | H | H | H | H | L | L | L | L | L |
| H | H | H | H | H | L | L | L | L | L | L |
| H | H | L | L | L | L | L | L | L | L | L |

- we want the assignment for which the H — L – staircase is as far to the right as possible
- this is the same as saying that the H — L – staircase is as far down as possible.
- we will next see a polynomial time alg for the case of two ranks

# A Polynomial Time Algorithm for Two Ranks

- the following alg computes an assignment for any value of $\Delta$

  for $\Delta = 2$, it computes leximax solution

  for $\Delta \geq 3$, it also seems to work well

- ranks are in $\{1, .., \Delta\}$, large ranks are better than small ranks

- we view the assignment as proceeding in rounds:

| revs | papers | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 7 | 4 | 9 | 1 |
| 2 | 5 | 4 | 2 | 3 | 7 |
| 3 | 3 | 1 | 4 | 7 | 9 |

| revs | ranks (sorted) | | | | |
|---|---|---|---|---|---|
| 1 | 5 | 5 | 3 | 1 | 1 |
| 2 | 5 | 4 | 2 | 2 | 2 |
| 3 | 3 | 1 | 1 | 1 | 1 |

- signature of a round:

  (# of rank $\Delta$ papers, # of rank $\Delta - 1$ papers, . . . , # of rank 1 papers)

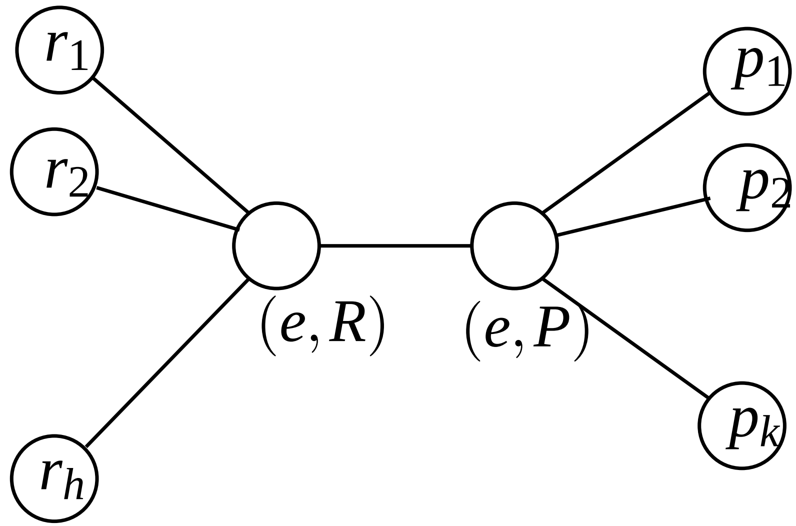
# Rank-Maximality

- we view the assignment as proceeding in rounds:
- signature of a round:

  (# of rank $\Delta$ papers, # of rank $\Delta - 1$ papers, ..., # of rank 1 papers)
- objective:
  - maximize signature of first round and subject to this signature of second round and ...
  - for two ranks: objective yields lex-max solution
  - for more than two ranks: ????
- polynomial time algorithm via
  - weighted bipartite matching problem with exponentially large weights
  - running time, $1$ sec for ESA instance

# The Weighted Bipartite Matching Problem

- vertex $r_\ell$ represents reviewer $r$ in round $\ell$, $1 \le \ell \le h$
- vertex $p_c$ represents copy $c$ of paper $p$, $1 \le c \le k$
- for an edge $e = (r, p)$ of rank $d$, we have vertices $(e, R)$ and $(e, P)$ and the edges shown




- If $p$ is assigned to $r$ in round $\ell$, $(r_\ell, (e,R))$ and $((e,P), p_c)$ are in $M$.
- If $p$ is not assigned to $r$ in any round, $((e,R),(e,P)) \in M$.

- the edges from nodes $r_\ell$ to the nodes $(e,R)$ are weighted

# The Weights

- if $e = (r, p)$ has rank $d$, we give the edge connecting $r_\ell$ and $(e, R)$ weight

$$(n+1)^d W^{k-\ell+1} \qquad \text{where } W = (n+1)^{r+1}$$

- weights for a single round:
  - a paper of rank $d$ contributes weight $(n+1)^d$ to the weight of a round; because then
  - $n$ rank $d-1$ assignments cannot make up for one rank $d$ assignment
  - maximum weight of a round: $n(n+1)^r$, set $W = (n+1)^{r+1}$
- total weight of assignment = $w_1 W^k + w_2 W^{k-1} + \ldots + w_k W^0$

  $w_\ell =$ weight of round $\ell$ and $k$ is the number of rounds

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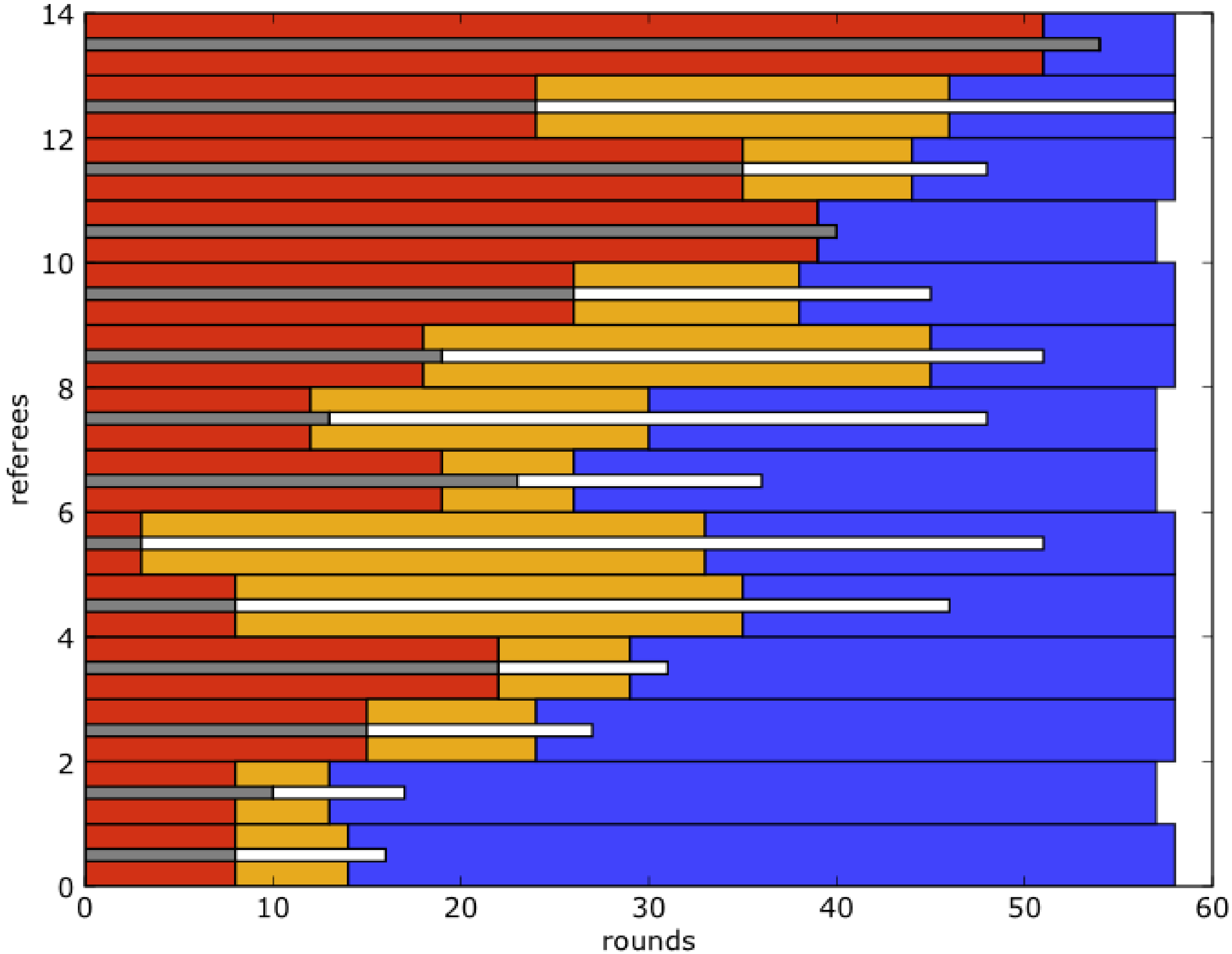

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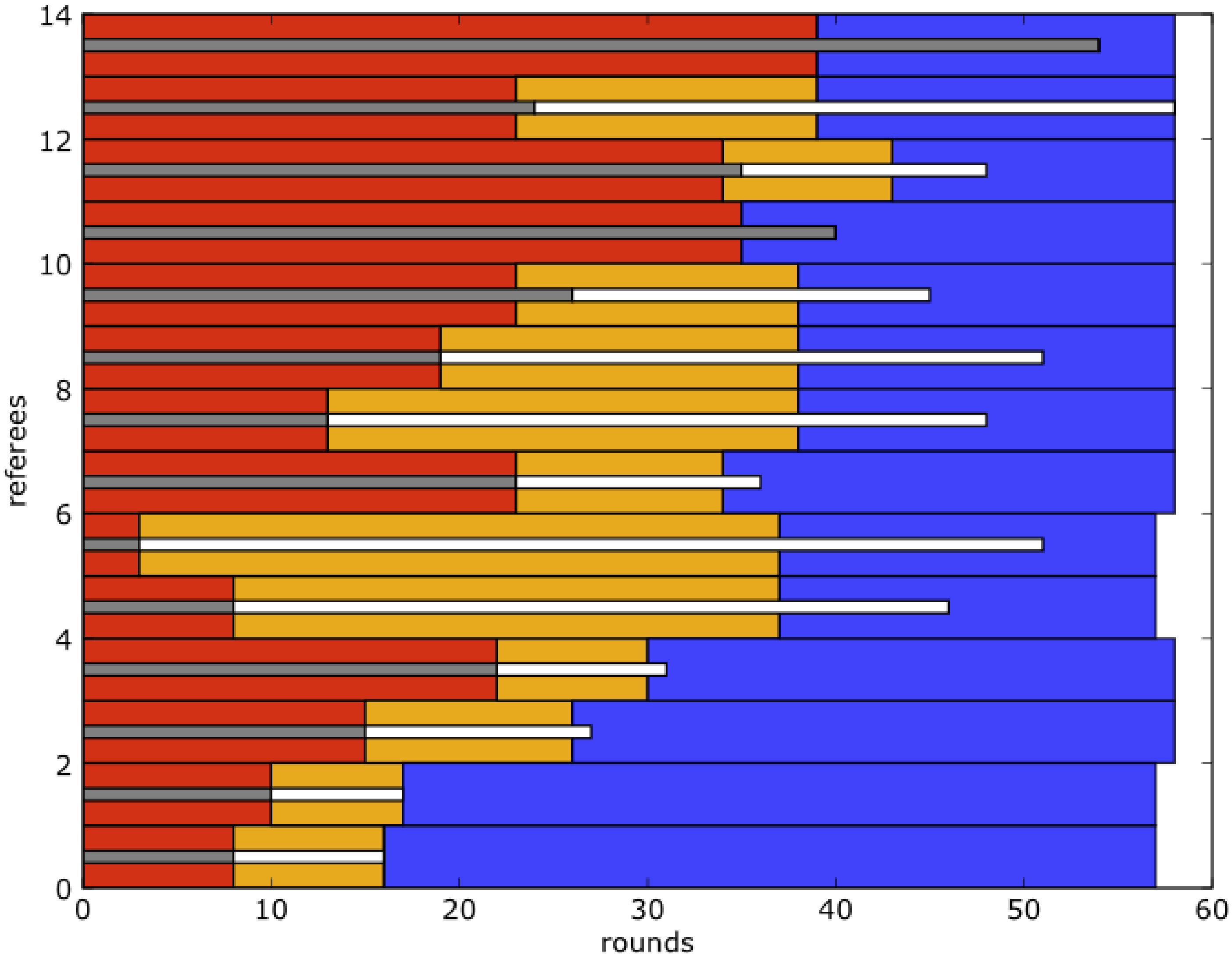


first 22 rounds are perfect

# Truthfulness

- the goal of a reviewer is to maximize his signature

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- NO
  - assume we have three reviewers, three papers and each paper needs to be reviewed twice.
  - the reviewers have equal valuations: they rate papers 1 and 2 high and paper 3 medium.
  - assume reviewers 2 and 3 tell the truth; then
  - reviewer 1 should lie about paper 3 and state a low rating.
  - he will get papers 1 and 2.

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- more extreme: a reviewer declares a conflict for all but $h$ papers

# What Next?

- are these the right objectives; alternative objectives?
- more algorithms (exact and approximate)
- a deeper investigation of truthfulness
- a better way to determine valuations?

  bids + keywords + wisdom of PC chair
- more experiments in collaboration with Andrei Voronkov (EasyChair)
- incorporation into EasyChair