A Physical Sine-to-Square Converter Noise Model

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***Abstract*—While sinusoid signal sources are used whenever low phase noise is required, conversion to a square wave-form is necessary when interfacing with digital circuits. Although have been analyzed a few times in various context, to the best knowledge of the author, there is no complete treatment and explanation of all noise sources within a sine-to-square converter. We attempt to give a quantitative, predictive and physically based noise model of sine-to-square converters without fitting parameters other than those imposed by the circuit itself.**

I. Introduction

In a lot of settings, a sinusoidal signal is given as an input, be it from a precision frequency source or measurement equipment like a dual mixer time difference system [1], but is required to be used in a digital or quasi digital environment. But for the use in digital electronics, the sinusoidal signal has to be converted into a square wave signal, either explicitly using a sine-to-square converter or implicitly, when the signal enters the digital logic. Although such circuits have been employed for a long time and been analyzed a few times, a complete noise model is still missing. Especially, there are very few attempts on a physically based noise model that which can predict the output noise based on circuit parameters and does not need fitting parameters.

II. Related Work

One of the first analysis of noise in sine-to-square converters was done by Collins [2]. Collins analyzed the jitter of multi-stage converters due to white noise with respect to the input slew-rate and noise bandwidth. Although giving insight on how to design multi-stage converters, Collins did not give any insight on the sources of noise and their behavior under different conditions.

In [3] Sepke et al. analyzed the noise contribution of comparators in analog-to-digital converters. While the circuit model is, on a first glance, similar, the use of the comparator after a sample-and-hold circuit changes the analysis considerably and thus its applicability to sine-to-square converters is limited.

In [4] Calosso and Rubiola measured the noise in an Field-Programmable Gate Array (FPGA) used as a sine-to-square converter. Even though they gave scaling laws for various types of noise, these were not rigorously analyzed and thus could not be related to noise parameters of the circuit directly.

III. Circuit Model

A sine-to-square converter can be modeled by a comparator (or a linear amplifier) followed by a number of amplifier stages. For simplicity, we assume here, that the converter consists of an ideal, noiseless and zero-delay comparator with a hysteresis $\pm H$ followed by a single noiseless amplifier with a delay t_{amp} (see figure 1). We use this split also to separate noise contributions due to different processes: Any noise that is related directly or indirectly to the input signal is folded into the comparator's input noise which we further combine with the input-offset voltage for simplicity. All noise related to delays within the circuit are folded into the amplifier.

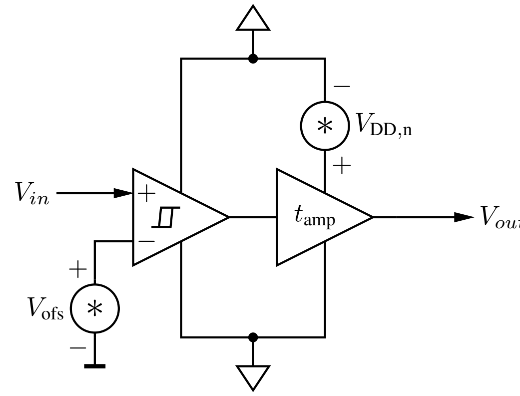

Figure 1. The circuit model of a sine-to-square converter is simplified to a noiseless comparator input stage with some hysteresis $\pm H$ and all input related noise being lumped together into the offset voltage V_{ofs}. Noise due to power supply variation $V_{\text{DD,n}}$ is modeled using an amplification stage with a delay of t_{amp} that only depends on the supply voltage.

The input signal

$$V_{\text{i}}(t) = (V_0 + V_{\text{i,AM}}(t))\sin(\omega_0 t + \varphi_i(t)) \quad (1)$$

with the two noise parts, the amplitude noise $V_{\text{i,AM}}(t)$ and the phase noise $\varphi_i(t)$ enters the comparator, which has a hysteresis of $\pm H(t)$ and an input offset voltage of $V_{\text{ofs}}(t)$. The output of the comparator gets further amplified and delayed by time $\Delta t_{\text{amp}}(t)$ by the following amplifier. We further assume that fluctuations and noise on the power supply $V_{\text{DD,n}}$ do not affect the comparator (e.g. it being an ideally symmetrical differential pair) and model the effect of $V_{\text{DD,n}}$ as variations in the amplifier delay t_{amp}.

As we are only interested in the phase noise contribution of the amplifier, we will ignore the input phase noise $\varphi_i(t)$ for the further analysis. The amplitude noise $V_{\text{i,AM}}(t)$ is included to determine its effect on the output phase noise, due to AM-PM conversion through the hysteresis of the comparator.

Multi-stage converters can easily be modeled by series connection of the elementary stage in figure 1. In longer chains

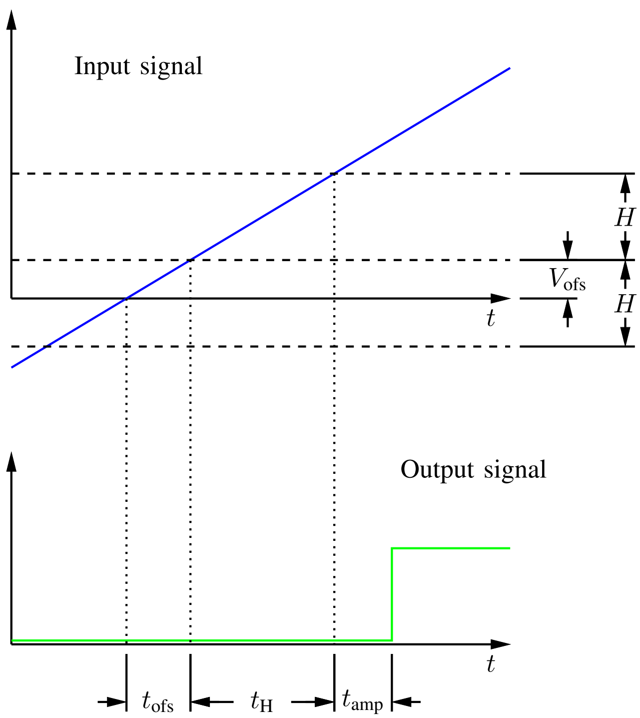

Figure 2. The zero crossing of the input signal gets delayed by the offset voltage V_{ofs} and the hysteresis $\pm H$ and the delay t_{amp} through the amplifier stage. The offset voltage and hysteresis related delays t_{ofs} and t_{H} depend not only on the values of V_{ofs} and H respectively, but also on the slew-rate of the input signal. The amplifier delay t_{amp} only depends on the supply voltage $\mathrm{V_{DD}}$.

or with sufficiently large input amplitude, there will be a slew-rate saturation. This can be used to simplify all following stages to single amplifier stages, without the comparator, and fold the input noise into the variation of the amplifier delay t_{amp}. If filters are used between stage, like in the case of Collins style sine-to-square converters [2] care has to be taken to account for the change in noise properties in each stage.

IV. Noise Sources

Assuming the hysteresis H of the comparator is symmetric around the zero point with offset voltage V_{ofs}, i.e., $V_{\mathrm{ofs}} \pm H$, and both are small enough such that the small angle sine approximation can be used around the zero-crossing point (see figure 2), then the propagation delay through the comparator is

$$t_{\mathrm{delay}}(t) = t_{\mathrm{ofs}}(t) + t_{\mathrm{H}}(t) + t_{\mathrm{amp}}(t) \tag{2}$$

$$= \frac{V_{\mathrm{ofs}}(t) + H(t)}{\omega_0(V_0 + V_{\mathrm{i,AM}}(t))} + t_{\mathrm{amp}}(t). \tag{3}$$

As we are interested in the variation of the delay, we will be looking at $\Delta t_{\mathrm{delay}}$. Assuming noise is small one can split the contributions:

$$\Delta t_{\mathrm{delay}}(t) = \Delta t_{\mathrm{ofs}}(t) + \Delta t_{\mathrm{H}}(t) + \Delta t_{\mathrm{AM}}(t) + \Delta t_{\mathrm{amp}}(t) \tag{4}$$

with:

$$\Delta t_{\mathrm{ofs}}(t) = \frac{\partial \Delta t_{\mathrm{delay}}}{\partial V_{\mathrm{ofs}}} \Delta V_{\mathrm{ofs}}(t) = \frac{1}{\omega_0 V_0} \Delta V_{\mathrm{ofs}}(t) \tag{5}$$

$$\Delta t_{\mathrm{H}}(t) = \frac{\partial \Delta t_{\mathrm{delay}}}{\partial H} \Delta H(t) = \frac{1}{\omega_0 V_0} \Delta H(t) \tag{6}$$

$$\Delta t_{\mathrm{AM}}(t) = \frac{\partial \Delta t_{\mathrm{delay}}}{\partial V_{\mathrm{i,AM}}} \Delta V_{\mathrm{i,AM}}(t) \approx \frac{H}{\omega_0 V_0^2} \Delta V_{\mathrm{i,AM}}(t) \tag{7}$$

The delay through the amplifier depends on the supply voltage $\mathrm{V_{DD}}$, thus any noise on the supply voltage will modulate the delay. As the delay depends on many factors like architecture, process, temperature etc., we simplify the relationship to

$$\begin{aligned}\Delta t_{\mathrm{amp}}(t) &= \frac{\partial \Delta t_{\mathrm{amp}}}{\partial \mathrm{V_{DD}}} V_{\mathrm{DD,n}}(t) + \mathcal{O}(V_{\mathrm{DD,n}}^2) \\ &= c V_{\mathrm{DD,n}}(t) + \mathcal{O}(V_{\mathrm{DD,n}}^2) \\ &\approx c V_{\mathrm{DD,n}}(t),\end{aligned} \tag{8}$$

with the circuit dependent parameter c. This, of course, removes time dependent delay variations due to e.g., aging or temperature, which can be significant at long time scales. But these contributions are easy to add later in the analysis and are left out, at the moment, for simplicity.

V. Noise Translation and Scaling

The phase noise is defined by

$$S_\varphi(f) = \varphi_{\mathrm{rms}}^2(f) \tag{9}$$

with the phase fluctuation φ measured over a bandwidth of 1 Hz [5]. As the phase relates to delay with $\varphi = \omega_0 t$ we can write

$$S_\varphi(f) = \omega_0^2 \left\langle \Delta t^2 \right\rangle_f \tag{10}$$

with $\langle x \rangle_f$ denoting, informally speaking, the average over all $x(t)$ with a measurement interval $1/f$. Or more formally, the absolute value of the Fourier coefficient at frequency f of the signal $x(t)$:

$$\langle x \rangle_f = \left| \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-2\pi \mathrm{j} f t} \, \mathrm{d}t \right| \tag{11}$$

For reasons of being concise, we ignore here the mathematical details of integrating over time series of random signals, which might potentially be non-continuous and assume all random signals are of finite bandwidth and thus integrable. We also assume all integrals go over finite time (measurement) intervals in order for them to be defined in case of $1/f^\alpha$ noises, which otherwise would lead to infinite signal power. For a discussion of integration over random signals see e.g., [6] and [7].

As all discussed noise sources are assumed to be independent, we can write

$$\begin{aligned}S_\varphi(f) &= \omega_0^2 \left\langle \Delta t_{\mathrm{delay}}^2 \right\rangle_f \\ &= \frac{1}{V_0^2} \left\langle \Delta V_{\mathrm{ofs}}^2 \right\rangle_f + \frac{1}{V_0^2} \left\langle \Delta H^2 \right\rangle_f \\ &\quad + \frac{H^2}{V_0^4} \left\langle \Delta V_{\mathrm{i,AM}}^2 \right\rangle_f + \omega_0^2 c^2 \left\langle V_{\mathrm{DD,n}}^2 \right\rangle_f\end{aligned} \tag{12}$$

Different frequency scaling for different noise sources becomes already evident. The input related noise processes do not scale with ω_0 while the delay related noise does scale with ω_0^2. These are the φ-type and x-type noises, respectively, as discussed in [4].

A. Impulse Sensitivity Function (ISF)

In [8] Egan noted that white phase noise gets aliased due to sampling. Formally, this can be described by using the ISF as introduced by Hajimiri and Lee in [9]. We slightly modify it to adapt it for the more general setting of sine-to-square converters:

$$\Delta\varphi(t) = \int_{-\infty}^{t} \Gamma(\tau) n(\tau)\, \mathrm{d}\tau \tag{13}$$

with $\Gamma(t)$ being the ISF and $n(t)$ being the effecting noise. Please note that $\Gamma(t)$ is implicitly also a function of the circuit and its parameters, which also include the input signal. In other words, if the shape or amplitude of the input signal changes, this will potentially result in a change of the shape of $\Gamma(t)$. The ISF for a sine-to-square converter can be approximated by a comb of alternating positive and negative rectangular pulses:

$$\Gamma(t) = \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t}{\tau_w} - nT_0\right) - \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t}{\tau_w} - nT_0 - \tau_\mathrm{d}\right) \tag{14}$$

with a period $T_0 = 2\pi/\omega_0$ and a pulse width of τ_w. τ_d denotes the phase shift between the positive and the negative pulses and is related to the duty cycle of the output signal and depends, in our circuit model, on the input signal amplitude V_0 and the offset voltage V_ofs. In a first order approximation, For a $50\,\%$ duty cycle $\tau_\mathrm{d} = T_0/2$. For simplicity, we assume that the positive and negative pulse, which relate to the positive and negative zero crossing respectively, are of the same magnitude and width, which is not necessarily the case. It should be noted, that τ_w depends on the output slew-rate of the converter, thus is proportional to T_0. Hence, for slew-rate limited converters, τ_w becomes independent of T_0 (in first order). The Fourier series of the function $\Gamma(t)$ can be expressed as

$$\begin{aligned}\Gamma(t) =& \frac{\tau_w}{T_0} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{n\omega_0\tau_w}{2}\right) \mathrm{e}^{-\frac{1}{2}\mathrm{j}n\omega_0\tau_w} \mathrm{e}^{-\mathrm{j}n\omega_0 t} \\ &+ \frac{\tau_w}{T_0} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{n\omega_0\tau_w}{2}\right) \mathrm{e}^{-\frac{1}{2}\mathrm{j}n\omega_0\tau_w} \mathrm{e}^{-\mathrm{j}n\omega_0 t} \mathrm{e}^{-\mathrm{j}n\omega_0\tau_\mathrm{d}}\end{aligned} \tag{15}$$

Looking at the Fourier series directly explains two phenomena reported in [4]: The $1/\omega_0$ scaling of white noise and the $1/\omega_0^2$ and $1/\omega_0$ scaling of flicker noise:

B. Scaling of White Noise

Under the assumption that $\tau_\mathrm{d} = T_0/2$ the Fourier transform of $\Gamma(t)$ in equation (15) becomes a Dirac comb like structure with Dirac pulses at odd multiples of ω_0 due to its periodic nature and because the even harmonics cancel out. These Dirac pulses $\delta(f - (2n+1)\omega_0)$ have approximately constant amplitude $a_n$up to the frequency $1/(\pi\tau_\mathrm{d})$ from which on they decay with $1/f$ or $20\,\mathrm{dB/dec}$. The multiplication with noise in equation (13) results in a mixing process (c.f. [7]) that converts all noise in distance ω_0 to one of the Dirac pulses $a_n\delta(f - (2n+1)\omega_0)$ down into the signal passband around ω_0 with an amplitude that is proportional to the amplitude of the Dirac pulse. Because the noise in each down-converted frequency region is independent, the total down converted noise becomes a geometric sum

$$\begin{aligned} S_{\varphi,white,total} &\propto \sum_n a_n^2 && (16)\\ &= \sum_{n \le \frac{2}{\tau_\mathrm{d}\omega_0}} a_n^2 + \sum_{n > \frac{2}{\tau_\mathrm{d}\omega_0}} a_n^2 && (17)\\ &\approx \frac{2}{\tau_\mathrm{d}\omega_0} a_1^2 + a_1^2 \sum_{k=1}^{\infty} \frac{1}{k} && (18)\\ &= a_1^2 \left(\frac{2}{\tau_\mathrm{d}\omega_0} + \sum_{k=1}^{\infty}\frac{1}{k}\right) && (19)\end{aligned}$$

There harmonic series in equation (19) grows slowly and can be approximated by $H_n = \sum_{k=1}^{n} \frac{1}{k} = \ln n + \gamma + \mathcal{O}(1/n)$, with γ being the Euler-Mascheroni constant (c.f. [10, section 1.2.7]). Even though $H_\infty = \infty$ and thus $S_{\varphi,white,total} = \infty$, the sum is in reality limited. One reason is that the ISF edges have a finite steepness, which adds a second sinc term to $\Gamma(t)$ and thus a second corner frequency after which it decays with $40\,\mathrm{dB/dec}$ or $1/f^2$. Another is the limited bandwidth of the circuit, which acts similarly by adding a cut off frequency, after which the noise (and signal) decay with an additional $20\,\mathrm{dB/dec}$. The sum $H_\infty^{(r)} = \sum_{k=1}^{\infty} \frac{1}{k^r}, r > 1$ is bounded by a small constant (e.g., $H_\infty^{(2)} = \pi^2/6$) [10], thus we can express the total white phase noise as:

$$\begin{aligned} S_{\varphi,white,total} &\propto a_1^2 \left(\frac{2}{\tau_\mathrm{d}\omega_0} + \frac{c_\mathrm{BW}}{\omega_0}\right) && (20)\\ &\propto a_1^2 \frac{c'_\mathrm{BW}}{\omega_0} && (21)\end{aligned}$$

with c_BW and c'_BW being (noise) bandwidth depending constants of the circuit. We conclude that the total white noise of the sine-to-square converter gets an additional scaling with a factor of $1/\omega_0$ due to aliasing induced by the periodicity of the ISF. Thus we end up with:

$$\begin{aligned} S_{\varphi,\mathrm{white}}(f) \propto\ & \frac{1}{\omega_0 V_0^2}\left\langle \Delta V_\mathrm{ofs}^2 \right\rangle_f + \frac{1}{\omega_0 V_0^2}\left\langle \Delta H^2 \right\rangle_f \\ &+ \frac{H^2}{\omega_0 V_0^4}\left\langle \Delta V_\mathrm{i,AM}^2 \right\rangle_f + \omega_0 c^2 \left\langle V_\mathrm{DD,n}^2 \right\rangle_f \end{aligned} \tag{22}$$

The proportionality factor of equation (22) depends on the equivalent noise bandwidth, respectively how many harmonics contribute to aliasing, and on the ratio τ_w/T_0 from $\Gamma(t)$.

In case $\tau_\mathrm{d} \neq T_0/2$, then $\Gamma(t)$ will also have even harmonics. For white noise, the even harmonics will act the same way as the odd harmonics and add to the proportionality factor of equation (22). For most systems, one can safely assume that the duty cycle will be close to $50\,\%$ and thus the even harmonics will be small. Hence it is possible to ignore the effects of even harmonics in a first order approximation.

C. Scaling of Flicker Noise

Flicker noise is, initially, only present around DC, thus the first harmonic up converts the flicker noise into the signal band. Hence, the flicker component derives from equation (12) directly as:

$$S_{\varphi,\text{flicker}}(f) \propto \frac{1}{V_0^2}\left\langle \Delta V_{\text{ofs}}^2 \right\rangle_f + \frac{1}{V_0^2}\left\langle \Delta H^2 \right\rangle_f + \frac{H^2}{V_0^4}\left\langle \Delta V_{\text{i,AM}}^2 \right\rangle_f + \omega_0^2 c^2 \left\langle V_{\text{DD,n}}^2 \right\rangle_f \tag{23}$$

D. Scaling in a Multi-Stage Sine-to-Square Converter

If multiple gain stages are used in a sine-to-square converter, then each stage acts upon the noise and thus the harmonics of the ISF of each stage alias noise noise into the signal band. Even if all stages are the same, each stage will have a different $\Gamma(t)$ as τ_w will change with the slew rate of the input signal of each stage. Thus a simple multiplication of S_φ with the number of stages will, in general, not lead to an accurate result. Nevertheless one can derive scaling rules quite easily:

For white noise, the harmonics of the additional stages each up convert noise, which is then down converted into the signal band by the following stage. As the equivalent noise bandwidth of each stage individually stays constant, the scaling rules in section V-B remain unchanged and thus equation (22) is still valid with only a larger proportionality factor.

For flicker noise, the up conversion and the following down conversion of consecutive stages change the behavior slightly. If the duty cycle is exactly $50\,\%$, then only odd harmonics will exist, and hence none of the present harmonics will see any flicker noise in a distance of ω_0. Thus only the first harmonic of each stage will up convert flicker noise into the signal band and, as with white noise, equation (23) is still valid with a slightly larger proportionality factor. But, due to the presence of V_{ofs}, the duty cycle will deviate from $50\,\%$ and give rise to even harmonics. Please note, that not only the DC component of V_{ofs} will lead to even harmonics, but also its higher frequency noise component. Hence the scaling of flicker noise will change its properties depending on the frequency spectrum of V_{ofs}.

Because now there are harmonics at a spacing of ω_0, the previously up converted flicker noise is seen by a harmonic at a distance of ω_0 and is thus down converted again into the signal band. Unlike the aliasing of white noise, the aliased flicker noise ultimately has the same origin, thus all down converted flicker noise components are correlated. Thus for S_φ, this leads to a scaling proportional to ω_0^2

From equation (14) we see that the power of the even harmonics relates to the power of the odd harmonics with $|1+\exp(\mathrm{j}\omega_0\tau_\text{d})| = |\sin(\omega_o\tau_\text{d})|$. Assuming $\tau_\text{d} \approx T_0/2$ we can replace τ_d by it's deviation (noise value) from $T_0/2$:

$$\tau_{\text{d,n}} = \tau_\text{d} - T_0/2 = \frac{1}{\omega_0 V_0} V_{\text{ofs}}(t) \tag{24}$$

leading to

$$|\sin(\omega_o\tau_\text{d})| = |\sin(\omega_o\tau_{\text{d,n}})| = \left|\sin\left(\frac{1}{V_0}V_{\text{ofs}}(t)\right)\right| \approx \left|\frac{1}{V_0}V_{\text{ofs}}(t)\right| \tag{25}$$

To evaluate the effects of equation (25) on the noise spectrum, we have to take into account, that $\tau_{\text{d,n}}$ represents a jitter value due to V_{ofs}. As such, it is subject to same aliasing and thus scaling laws as S_φ. If the amplification of the first converter stage is large, we can safely assume that $\tau_{\text{d,n}}$ is dominated by the first stage. If we also assume that the noise equivalent bandwidth is large and thus (the jitter) $\tau_{\text{d,n}}$ is dominated by white noise. We then can ignore the contribution and scaling due to flicker noise. Due to aliasing of white noise, we get an additional scaling term of $1/\omega_0$, as we have already seen with equation (20). Thus the power of the even harmonics will scale approximately with $1/(\omega_0 V_0)\left\langle V_{\text{ofs}}\right\rangle_f$.

The scaling due to aliasing will act differently on different types of noise. While all input related noise sources will see the full effect of aliasing, the V_{DD} related noise component will not. As the V_{DD} related noise acts as a delay in each stage of the multi stage converter, it will only see part of the flicker noise aliasing, depending on which stage was the source of the noise. Thus V_{DD} related noise will see an additional scaling factor between 1 and ω_0 depending on the exact structure of the sine-to-square converter and which stages contribute how much to the output noise.

Putting the arguments above together, we can conclude that $S_{\varphi,\text{flicker}}$ of a multistage sine-to-square converter gets an additional ω_0^2 term due to aliasing of correlated noise and an $1/\omega_0$ term due to the power scaling of the even harmonics:

$$S_{\varphi,\text{flicker,multi}}(f) \propto \omega_0 S_{\varphi,\text{flicker}}(f) \propto \frac{\omega_0}{V_0^2}\left\langle \Delta V_{\text{ofs}}^2 \right\rangle_f + \frac{\omega_0}{V_0^2}\left\langle \Delta H^2 \right\rangle_f + \frac{\omega_0 H^2}{V_0^4}\left\langle \Delta V_{\text{i,AM}}^2 \right\rangle_f + \omega_0^\alpha c^2 \left\langle V_{\text{DD,n}}^2 \right\rangle_f \tag{26}$$

with α, the scaling factor of the V_{DD} related noise, being between 2 and 3.

VI. Example Analysis of Measurements

Calosso and Rubiola presented measurements of a Cyclone III FPGA in [4]. Using the analysis in the previous sections, we will review the noise analysis of Calosso and Rubiola.

For convenience, we have reproduced the plot of the noise measurement in figure 4. For high offset frequencies, the $1/\omega_0$ scaling of white φ-type noise is nicely visible. Similarly, for low offset frequencies and high signal frequencies, the ω_0^2 scaling of flicker x-type noise shows up.

But for small offset frequencies and small signal frequencies the behavior changes. To fully understand the noise in this

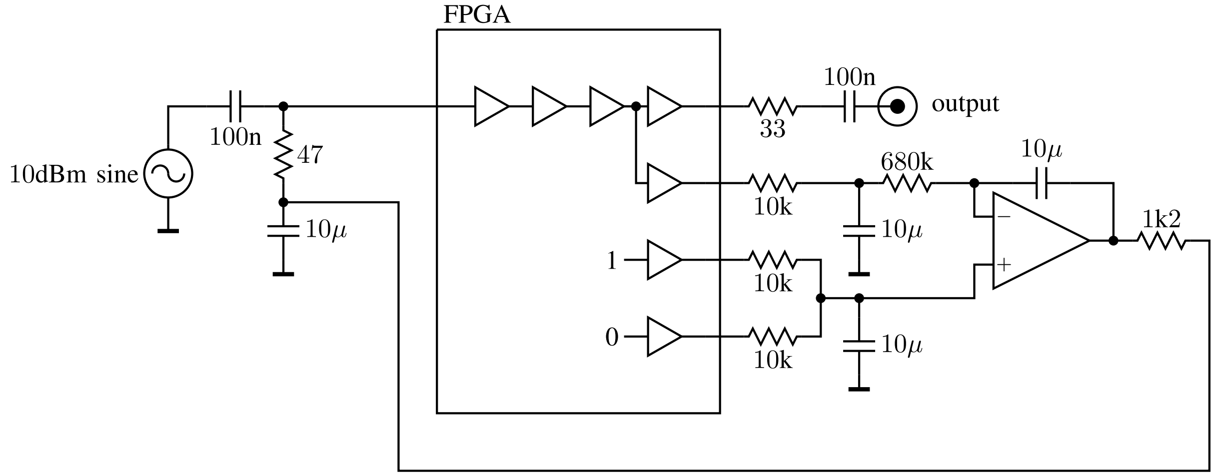

Figure 3. The circuit used in [4]. The FPGA acts as as a sine-to-square converter and buffer. The operation amplifier forms an integrator to stabilize the duty cycle of the output and steer the input offset voltage to that effect. Courtesy of Claudio Calosso and Enrico Rubiola.

area we first have to look at the circuit that was used in [4], which is shown in figure 3. The FPGA acts as a sine-to-square converter with multiple stages. There are two output buffers used, one for the output, which is fed to the noise measurement equipment and one that is fed, through a low pass filter, into an integrator. The integrator adjusts the offset voltage of the input signal such that the duty cycle is kept at $50\,\%$. Analysis of the transfer characteristics of this stabilization circuit reveals a $3\,\mathrm{dB}$ frequency at approximately $28\,\mathrm{mHz}$.

Armed with this information understanding figure 4 becomes easy. For offset frequencies below $28\,\mathrm{mHz}$ the integrator in figure 3 compensates any deviation from $50\,\%$ duty cycle, thus eliminating the second order harmonics and aliasing of flicker noise. Hence only (unaliased) x-type noise is visible.

In the range between approximately $28\,\mathrm{mHz}$ and 1–$1000\,\mathrm{kHz}$ when white noise becomes dominant, we see aliased φ-type noise, due to the multi-stage nature of the FPGA for lower signal frequencies. For higher signal frequencies, flicker noise scales with ω_0^2, thus α of equation (26) must be close to 2, which in turn would suggest, that most of the V_{DD} related noise originates from the output stage or the last few stages.

VII. Conclusion

Starting from a simple circuit model, we have analyzed how input and power supply noise affect the sine-to-square conversion. The scaling of the additive noise in dependence of the input frequency is has been shown for different noise contributors and under different settings. These scaling laws then have been used to explain the noise measurements of [4].

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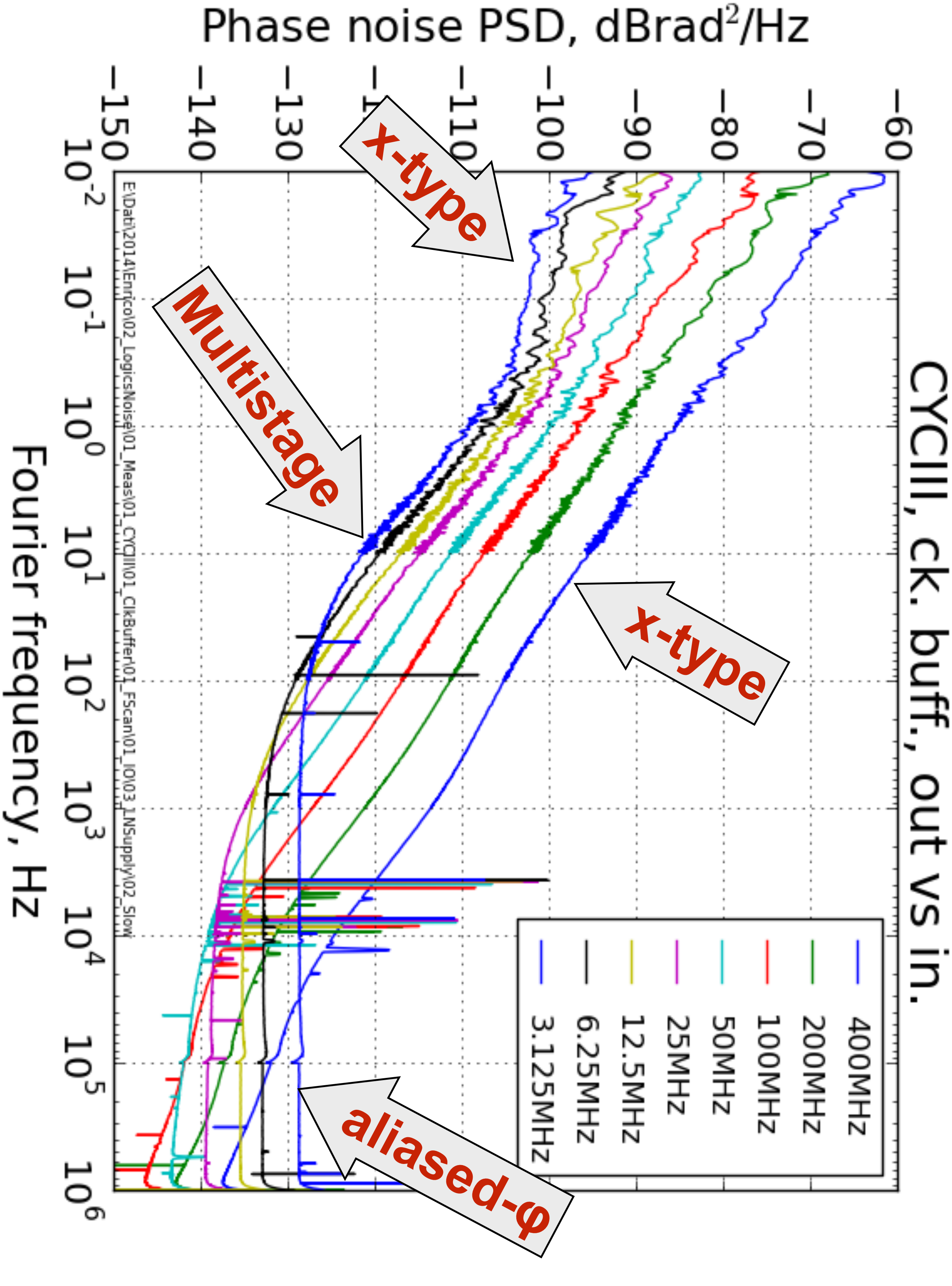

Figure 4. The noise measurement of [4], slightly adapted. The white noise region shows the $1/\omega_0$ scaling of aliased φ-type noise. The flicker noise region shows both the unaliased x-type noise at high ω_0 and aliased flicker noise at low ω_0. At low offset frequencies, the aliasing of flicker noise goes away due to duty cycle stabilization in figure 3. Courtesy of Claudio Calosso and Enrico Rubiola.