A Fresh Look at the Design of Low Jitter Hard Limiters

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***Abstract*—Hard limiters or zero-crossing detectors have received very little attention in the last two decades. Yet our understanding of how noise propagates through non-linear circuits has improved quite considerably. We have here a fresh look at hard limiters and their analysis using the tool of the Impulse Sensitivity Function as introduced by Hajimiri and Lee.**

***Index Terms*—hard limiter, zero crossing detector, noise, flicker noise**

I. Introduction

Accurate and stable zero-crossing detection is one of the corner stones of the Dual Mixer Time Difference (DMTD) method of precisely measuring phase and frequency differences[2]. The simplest way to build a zero-crossing detector is by use of limiting amplifiers, i.e. by transforming the (most often sinusoidal) signal into a square wave signal. Collins described in [3] how to design the slope-gain in multi-stage limiting amplifiers such that the jitter due to the amplification and squaring-up is minimal. Even though, DMTD is not the only application for zero-crossing detectors, their noise properties have received very little attention over the years since Collins' paper.

In the meantime, noise in sustaining amplifiers of oscillators have received a great deal of attention and new models to describe the noise generation and amplification have been formulated. Most notably are the models by Hajimiri and Lee[1] and Demir et al.[4]. Especially the approach of Hajimiri and Lee using an Impulse Sensitivity Function (ISF) can be easily adapted to settings outside of oscillators, given they are driven by periodic signals.

In this paper, we will have a fresh look at the noise transfer function and optimal gain settings of multi-stage liming amplifiers. In particular, we will review the transform and aliasing in the amplifier stages and the effect of bandwidth limiting on both the slope gain and noise gain. Using the ISF formalism, the noise analysis of Collins can be extended from a jitter-only description as in [3] to one that distinguishes between white and flicker noise. The difference in the transfer functions of white and flicker noise is especially important with low frequency input signals as are common with DMTD systems.

II. Squaring Up

Zero-crossing detection is usually performed by amplifying and limiting the input signal. The amplification enhances the slope at the zero-crossing, while the limiting ensures that the signal amplitude stays within reasonable limits such that the electronics does not require any high-voltage components (where high-voltage can start as low as $5\,\mathrm{V}$, depending on the components and technology used). Stability and noise considerations generally limit the maximum achievable amplification in a single stage. Thus, to achieve the high gain required in zero-crossing detectors (thousands to millions) the use of multiple stages is almost a given for practical implementations, unless the overall system level design allows to take short-cuts. This of course raises the question, how the gain should be split between the different stages in order to get the lowest possible noise and jitter at the output.

III. Related Work

To the best knowledge of the author, the first mention of multi-stage zero-crossing detectors and their associated noise was by Dick et al. in [5]. Unfortunately, Dick et al. did not do an analysis of the noise propagation and amplification within the zero-crossing detector. One of the first analysis of noise in zero-crossing detectors has been done by Collins [3]. Collins analyzed the jitter of multi-stage detectors due to white noise with respect to the input slew-rate and noise bandwidth using a first order filter.. The analysis has been done in the time domain which simplified some of the aspects quite considerably. Unfortunately, it is quite difficult to extend Collins work to filters of higher order. In [6] Calosso and Rubiola measured the noise in an Field-Programmable Gate Array (FPGA) used as a sine-to-square converter. Even though they gave scaling laws for various types of noise, these were not rigorously analyzed and thus could not be related to noise parameters of the circuit directly. In [7] the author has given an in-depth analysis of noise propagation in zero-crossing detectors using no filters between the stages, for both white noise and flicker noise. This paper is an extension of this work.

As in [7], this paper relies on an adaption of the the Impulse Sensitivity Function (ISF) formalism introduced by Hajimiri and Lee in [1] for analysis of noise of oscillators. The ISF formalism is based on the insight that the noise

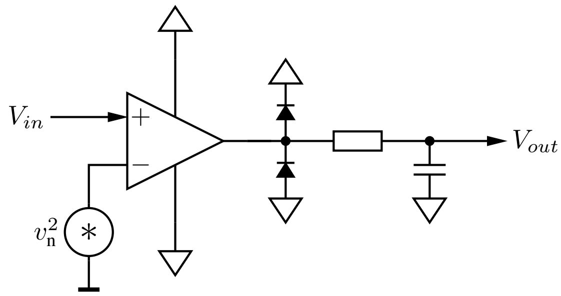

Fig. 1. The circuit model, adapted from [3] of a single stage zero-crossing detector is simplified to a noiseless amplifier input stage, with all input referred noise being lumped together into the noise voltage v_{n}^2. After the amplifier there is a (noise-less) limiter and a first order filter.

transfer through the circuit is modulated by the signal and thus has to be regarded as a linear time-variant function. Several authors later provided ways to efficiently calculate the ISF for a given circuit. The approaches can be grouped into circuit analysis in state space (e.g. [8]), SPICE based simulation and ISF extraction (e.g. [9]) or on analysis of the circuit in the frequency domain (e.g. [10]).

IV. Circuit Model

The circuit model used in this paper is a slightly adapted version of what Collins used in [3]. A single stage consists of a noiseless amplifier, where all its noise is lumped together into a single noise source at its input v_{n}^2. For brevity of notation, we also include the signal's noise in v_{n}^2 and assume an otherwise noiseless signal at each stage. Please note, that we assume here that the amplifier has no offset voltage and the noise voltage v_{n}^2 has zero mean. This slightly artificial assumption is justified by the need to precisely control the duty cycle to $50\,\%$ and, by extension, the offset voltage in order to eliminate amplifying flicker noise due to the otherwise present even order harmonics. See [7] for an in-depth analysis of flicker noise amplification in multi-stage sine-to-square converters. For sake of brevity, it is further assumed the circuit does not contain any variable time delay term (e.g. due to power supply noise). The effects of any such delay noise can be analyzed in a similar manner as the input referred noise effects. See [6] and [7] for details.

The switch in Collins circuit model has been replaced by the more natural clamping diodes. One can still assume these diodes to be noise free in a very good approximation of the real circuit performance. On one hand, only the clamping diodes of the last stage will contribute to the output noise. On the other hand, the noise contribution of the diodes can be neglected compared to the input referred noise that has been amplified through even a single amplifier stage, even if said input noise would be as low as the noise of a single diode.

The filter is here modeled by a (noiseless) RC-filter as a stand in of any, more general low pass filter that could be used.

V. Circuit Analysis

The assumption that there is no offset voltage and thus no even harmonic components in the signal and ISF together with the assumption that there is no variable (noise dependent) delay result in the flicker noise being dominated by the input referred noise (c.f. [6] and [7]). I.e. the output flicker noise is dominated by the flicker noise of the first stage (c.f. Friis formula [11]) and the up-conversion of the flicker noise is due to the fundamental of the ISF at each stage. This leaves us the analysis of the white noise propagation through the circuit.

Following Collins, we want to optimize the variance of the jitter $J^2 = N_{\mathrm{out}}^2/\rho_{\mathrm{out}}^2$ with N_{out}^2 being the output noise power density and ρ_{out} being the output slew rate.

A. Slew Rate

The output slew rate is obviously the slew rate of the output signal $V_{\mathrm{out}}(t)$ at the zero crossing. Assuming a purely sinusoidal input signal of frequency ν_0 with amplitude 1, total gain $G_n = \prod_{i=1}^{n} g_i$, with g_i being the individual stage gains[1] and clamping back to 1 again, the output signal becomes $V_{\mathrm{out}}(t) = \min\{\max\{G_n\cos(2\pi\nu_0 t), -1\}, 1\}$. This is a periodic function with the Fourier series coefficients (frequency ν_0 normalized to 1) being

$$\widehat{V}_{\mathrm{out}}[k] = \frac{1}{\pi}\left(\left(\frac{2}{k} - \frac{2k}{k^2-1}\right)\sin(k\tau) + \frac{2G_n}{k^2-1}\sin(\tau)\cos(k\tau)\right) \tag{1}$$

with $\tau = \arccos(1/G_n)$ being half of the rise/fall time of the output signal. The slew rate can then easily calculated as

$$\rho_{\mathrm{out}} = \sum_{k=1}^{\infty} k\widehat{V}_{\mathrm{out}}[k] \tag{2}$$

Following Collins we can approximate V_{out} by a trapezoidal function for large G_n, which leads to $\widehat{V}_{\mathrm{out}}[k]$ decaying with $1/k$ up to the frequency $1/(\pi\tau)$ and with $1/k^2$ from then onward. It is important to note that for small G_n this approximation does not hold and the exact Fourier coefficients have to be used.

B. Noise

The input referred white noise is being sampled by the ISF, down-converting broadband noise from higher frequencies down to the signal band (c.f. [7]). Assuming the ISF is solely due to the non-linearity of the clamping circuit (i.e. the amplifier is perfectly linear), the input noise becomes amplified by the stage gain g_i and then (additional) noise power is down-converted from higher frequencies. As the exact calculation of the ISF is not possible without knowing the exact circuit we approximate the ISF by a pulse train of rectangular pulses with width τ_w. This results in the ISF's Fourier coefficients $\widehat{\Gamma}[k]$ being constant up to a frequency of $1/(\pi\tau_w)$ after which they decay with $1/k$. To ensure convergence, we further assume there is an upper frequency $\nu_{\max}$ of after which the ISF decays

[1]While we use here a similar notation as Collins did, please be aware that g_i and G_n are slightly but subtly different. While Collins uses the slope gain of his circuits, we use here the gain of the amplifier.

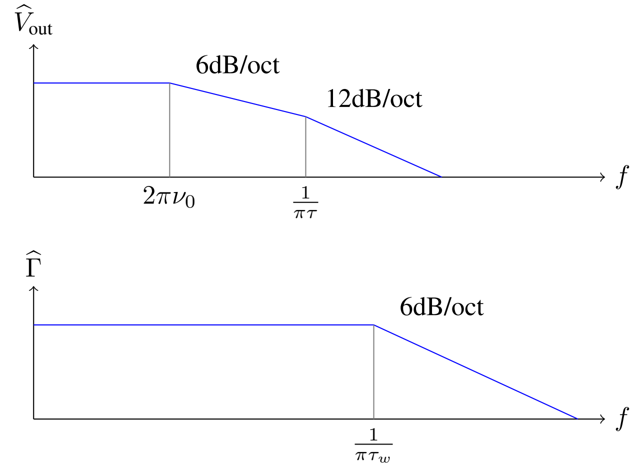

Fig. 2. Comparison of the Fourier transform of the output signal $\widehat{V}_{\text{out}}$ and the Fourier transform of the ISF $\widehat{\Gamma}$. While $\widehat{V}_{\text{out}}$ first decays with $1/f$ up to the frequency of $1/(\pi\tau)$ and then decays with $1/f^2$, the ISF is constant up to the frequency of $1/(\pi\tau_w)$ and decays with $1/f$ afterwards. Generally, $\tau > \tau_w$ can be assumed.

faster than $1/k$. This is a safe assumption for real circuits, as invariably all components will have a finite switching time, thus limiting the steepness of the ISF and guaranteeing finite and non-zero rise and fall times. Thus the output noise power becomes:

$$N_{\text{out}}^2 = v_{\text{n}}^2 g_i \sum_{k=1}^{\infty} \widehat{\Gamma}[k] \tag{3}$$

For most practical circuits τ_w will be smaller than τ, i.e. the corner frequency for the ISF will be larger than the corner frequency for the output signal.

C. Jitter and the Effect of the Filter

Comparing the Fourier transforms of both the output signal V_{out} and the ISF reveals that the frequency range between $2\pi\nu_0$ and $1/(\pi\tau)$ contributes the most to increasing the slew rate ρ. At frequencies higher than $1/(\pi\tau)$, the contribution of each harmonic is still not negligible[2], but the decay of the harmonics with $1/k^2$ and thus the contribution to the slew rate with $1/k$ quickly becomes much lower than the contribution of the ISF harmonics, which remain constant up to the frequency of $1/(\pi\tau_w)$. From this can be easily concluded, that an optimal filter should cut off the output signal harmonics and ISF at $1/(\pi\tau)$ for optimal jitter J^2. This is the frequency domain equivalent to Collins' result that the optimum half-level crossing time k is equal to 1. Thus, the same result for optimal distribution of gain between stages, namely $g_{n-1} = \sqrt{2g_n}$ holds true for this frequency domain analysis as well. It also becomes evident that a higher order filter will immediately give benefits in terms of jitter performance, as it will reduce down-sampling of broadband noise much more than it will reduce the slew rate of the output signal.

[2] $\sum_{k=1}^{\infty} k \cdot 1/k^2 = \sum_{k=1}^{\infty} 1/k = \infty$

There are two things to note, though. First, this last, graphical analysis used the trapezoid signal approximation for V_{out}. While this holds true for circuits where the cumulative, total gain G_n is large, it does not for longer chains. E.g. Collins calculated for a 6 stage chain with a total gain of 10^6 the gains for the first three stages to be 2.3, 2.7, and 3.7 respectively. This results in a total gain at each stage of 2.3, 6.2, and 23.0 respectively. These gains are low enough that the trapezoid approximation produces significant errors.

Second, it becomes evident, that for optimal performance the harmonic components of the ISF have to be limited. Using the circuit model from above, where the ISF is generated due to the non-linearity of the clamping circuit, it can be easily achieved by placing the filter before the clamping circuit instead of after, thus limiting the harmonic contents of the signal reaching the clamping circuit, which in turn reduces the harmonics of the ISF generated. Obviously, if there is no explicit clamping circuit, but the clamping is part of the amplifier itself (e.g. by using a differential pair driven into saturation), then the filter has to become a part of the amplifier in order to be able to modify the ISF.

As mentioned earlier, in a practical design the offset voltage of each amplifier stage should be closely controlled. While once a high G_n has been reached, all remaining stages will have the same duty cycle and thus can be controlled using an earlier stage, the first few stages have to be individually controlled in order to limit even order harmonics generation and thus flicker noise amplification in these stages.

VI. Conclusion

While Collins [3] already gives a good handle on how to design the gains of a multi-stage limiting amplifier, it does not easily allow to design the same circuit with higher order filters, as the time domain analysis cannot be easily adapted to this case.

We showed how to transform the analysis into the frequency domain using the ISF formalism introduced by Hajimir and Lee. This analysis nicely reproduces the earlier results by Collins.

Equivalently to Collins, we find that the optimal filter cut-off frequency should be chosen as $1/(\pi\tau)$ with $\tau = \arccos(1/G_n)$ for the n-th stage. Filters with higher order and thus steeper roll-offs will benefit jitter performance.

For low cumulative gains G_n the trapezoid signal approximation does lead to errors and exact formulas for the harmonics of the output signal should be used instead.

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