Smallest tree grammar by recompression

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Compression and grammars

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Grammars based compression

- CFG defining unique word
- Straight Line Programs (SLP)
- easy to work on
- natural in many applications
- Block-based compression translates to SLPs.

Smallest grammar

Problem

Given w return smallest CFG G_w such that $L(G_w) = w$.

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- decision problem: NP-hard
- lower bound for approximation ratio

Best approximation ratio

$\mathcal{O}(\log(n/g))$, where g is the size of the optimal grammar.

Tree grammars

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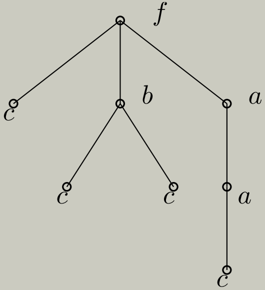
Definition (labelled trees = terms)

- (ranked) alphabet $\Sigma = \bigcup_{i \geq 0} \Sigma_i$
- $\text{rank} : \Sigma \mapsto \mathbb{N}$, $\text{rank}(\Sigma_i) = \{i\}$
- rooted trees, nodes labelled with elements of Σ
- node a has $\text{rank}(a)$ children (ordered)



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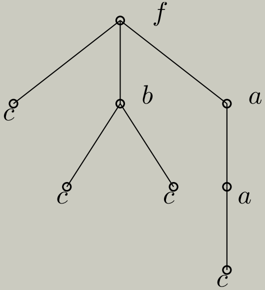

Grammar: different generalisations.

SLPs for strings

Definition (SLP: Straight Line Programme)

CFG with

- ordered nonterminals $X_1, X_2, \ldots$
- Chomsky normal form
- one rule for nonterminal
- for $X_i \to X_j X_k$ we have $j, k < i$

From string to trees

Simplest

- ordered nonterminals $X_1, X_2, \ldots$
  each generates a tree
- rules $X_i \rightarrow f(X_j, \ldots, X_k)$ we have $j, \ldots, k < i$

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DAGs

- those are exactly DAGs
- smallest one can be found

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Not a good candidate

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- rules $X_i(y_1, y_2, \ldots, y_m) \to t$, where
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  - t contains a single leaf $y_1, \ldots, y_m$
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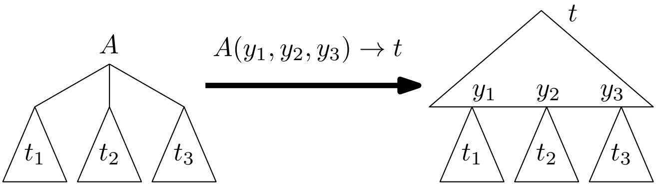

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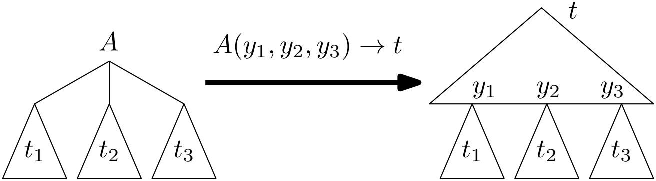
- if y_i may be multiplied: turns very difficult.



Properties and intuition

Grammar for text: 1 parameter (text to the right)
$A \rightarrow w \Leftrightarrow A(y) \rightarrow w(y)$

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Lemma (Lohrey, Maneth, Schauss-Schmidt)

Without loss of generality each nonterminal has 0 *or* 1 *parameter.*
Size increases $\mathcal{O}(r)$ *times.*

Proof.

Rightmost derivation. $\square$

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Theorem (NEW!)

The smallest tree grammar can be $\mathcal{O}(r \log N)$ approximated, where r is the maximal rank.

- based on local compression rules
- analysis modifies the optimal grammar

Leaf compression

- 'Absorb' each leaf by its father (and change the labels).
- Replace $f(c_1, t_2, t_3, \ldots, c_2, \ldots, t_k)$ with $f'(t_2, t_3, \ldots, t_k)$

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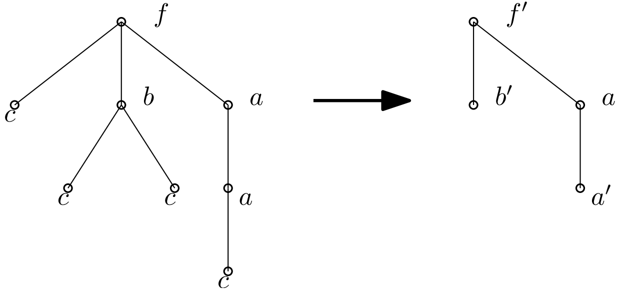

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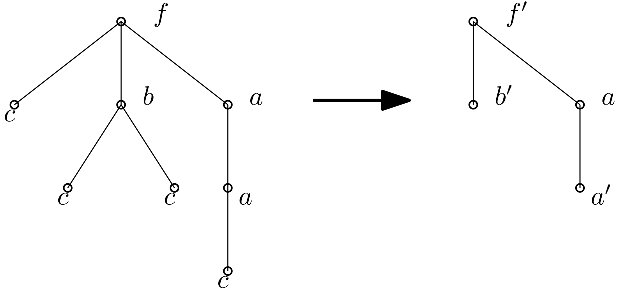

- half of nodes are leaves
- $\mathcal{O}(\log n)$ rounds

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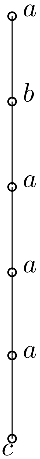

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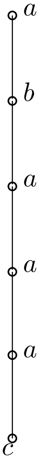

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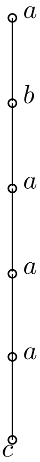

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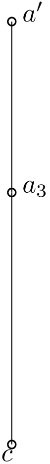
Lemma

1/4 of the nodes are compressed.



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  find partition of Σ into Σ_ℓ and Σ_r
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    compress 2-chain ab ▷ 2-chains compression
  $L_0 \leftarrow$ list of constants, $L_{\geq 1} \leftarrow$ list of other letters in T
  for $f \in L_{\geq 1}$ and $1 \leq \ell \leq \text{rank}(f)$ and $a \in L_0$ **do**
    perform all leaf compressions for $f\ a$

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Time

- enough if one phase takes linear time.
- compressions: grouping done by sorting (RadixSort)

Size analysis

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Mental experiment

- We take the smallest SLCF
- We perform the compression step on it
  - it always generates the current tree
- Some changes of the SLCF are needed
- The number of nonterminals depends on the SLCF, not tree.

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- each letter has credit
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Ensure that this is OK

- $X(y) \to aa(y)$, $Y \to X(b)$: ab not there

Rules modification (recompression)

Modification of the rules

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- Total cost: the issued credit
- $\mathcal{O}(rg)$ credit per phase. Essentially: $\mathcal{O}(r)$ per nonterminal.
- $\mathcal{O}(rg \log N)$ in total

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More general grammar — hardness?