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Online Checkpointing

with

Improved Worst-Case Guarantees

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Problem

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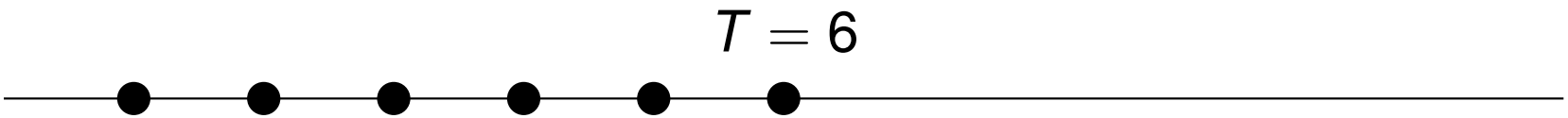
$$\text{Discr}(A) := (k+1) \sup_{T \geq t_k} \frac{\bar{\ell}_T}{T}$$

$\bar{\ell}_T$ is the longest interval at time T.
$T/(k+1)$ is the size of an interval in the equidistant setting.

Example $k = 6$



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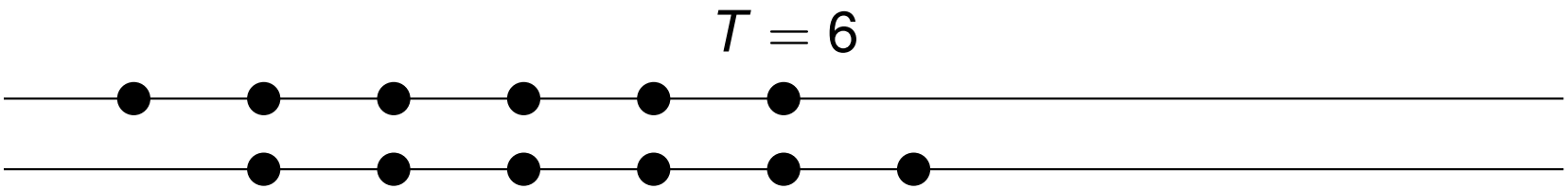

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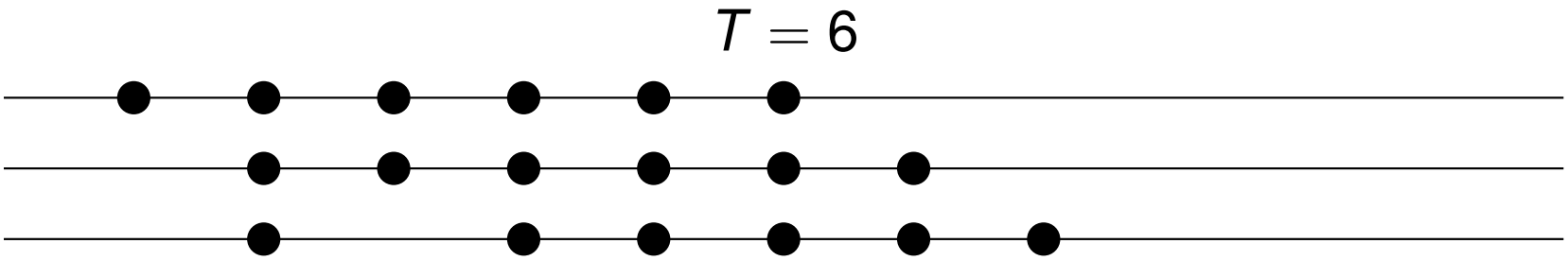

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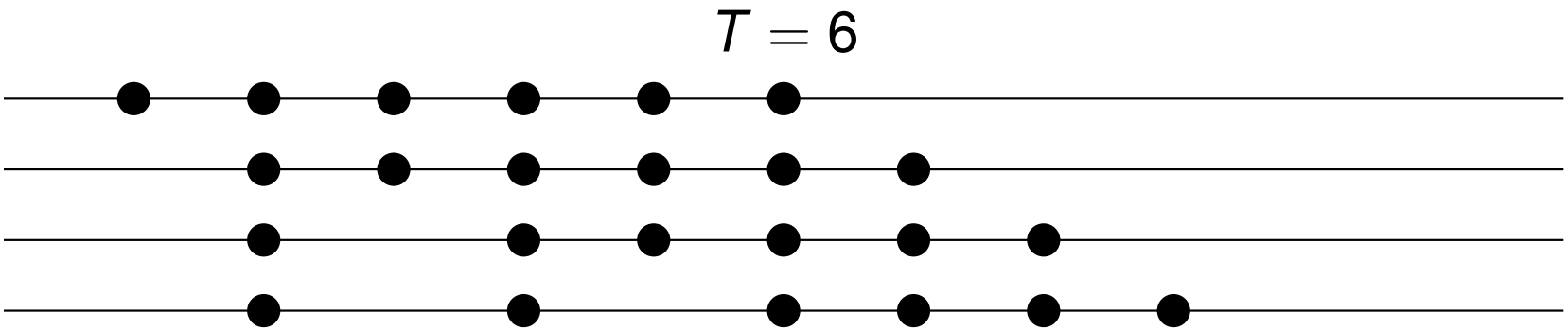

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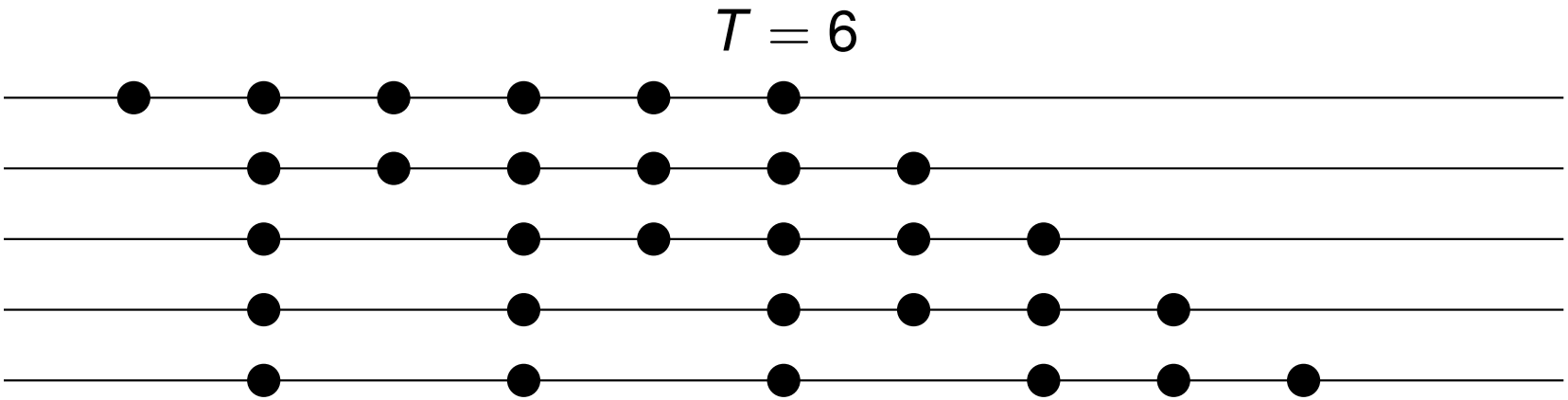

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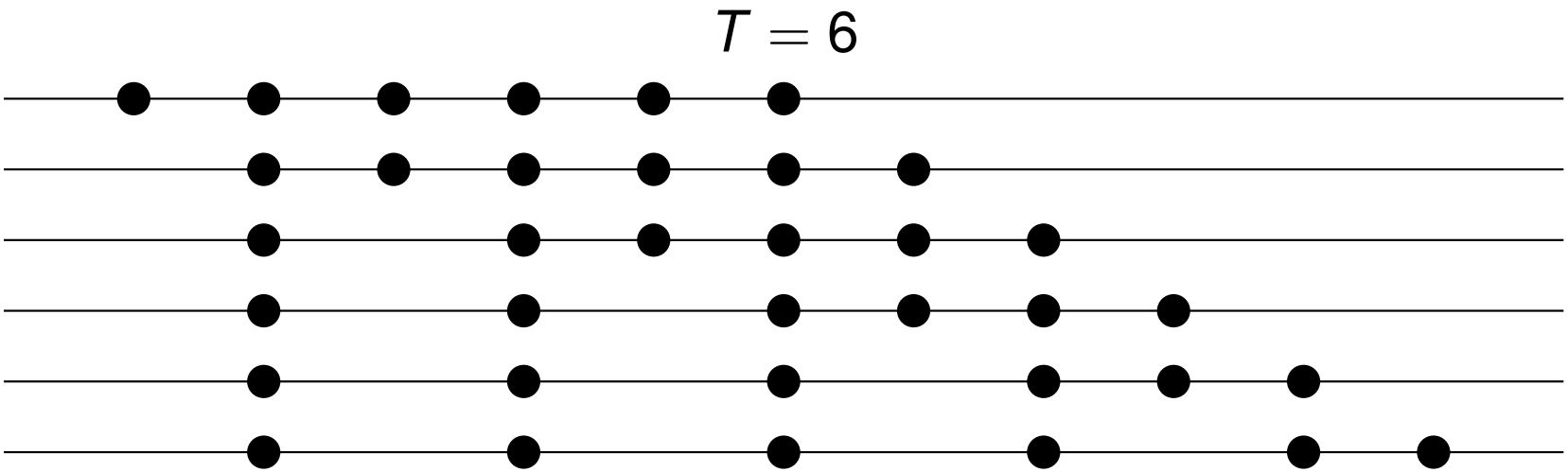

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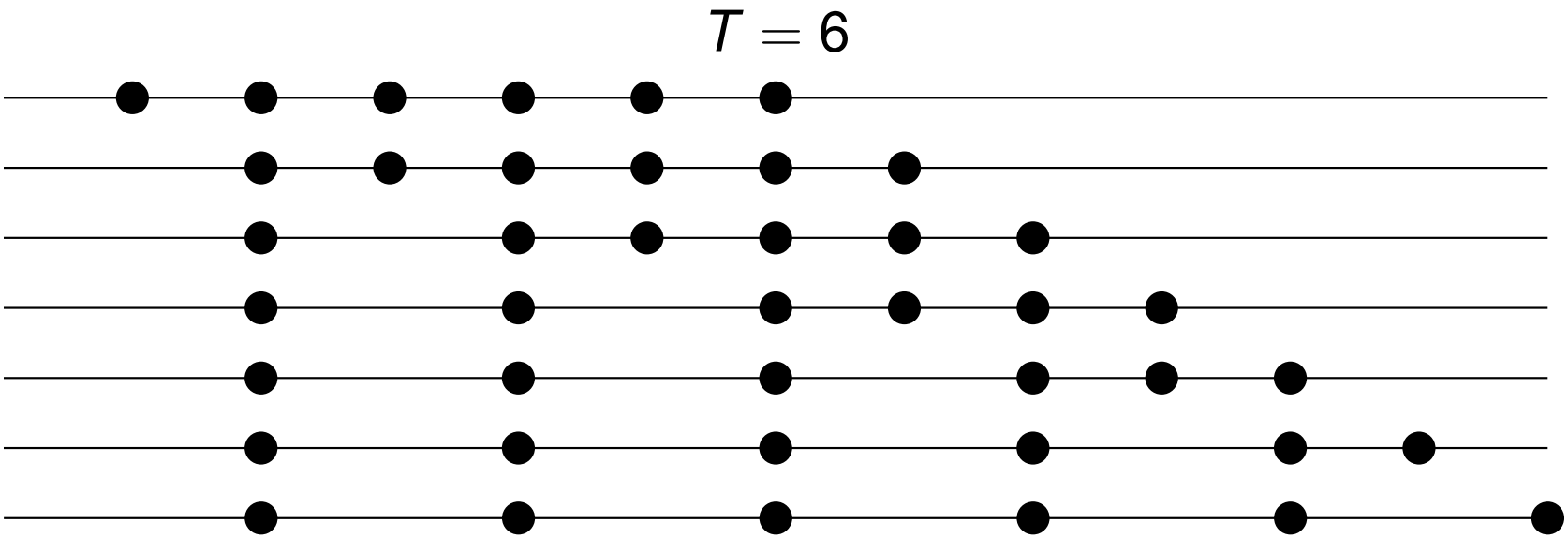

Results

Ahlroth, Pottonen, Schumacher (COCOON '11) show $1 + 1/k \leq p_k \leq 2$. We improve to

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- Algorithms
  - LINEAR: For all k,

$$\text{Discr}(\text{LINEAR}) = 1.59 + O(k^{-1})$$

  - BINARY: For powers of two,

$$\text{Discr}(\text{BINARY}) = \ln(4) + o(1) \approx 1.39$$

  - Results for fixed k: $p_k < 1.5$ for $k \in [5, 60]$
- Lower Bound: $p_k \geq 2 - \ln(2) - o(1) \approx 1.3$

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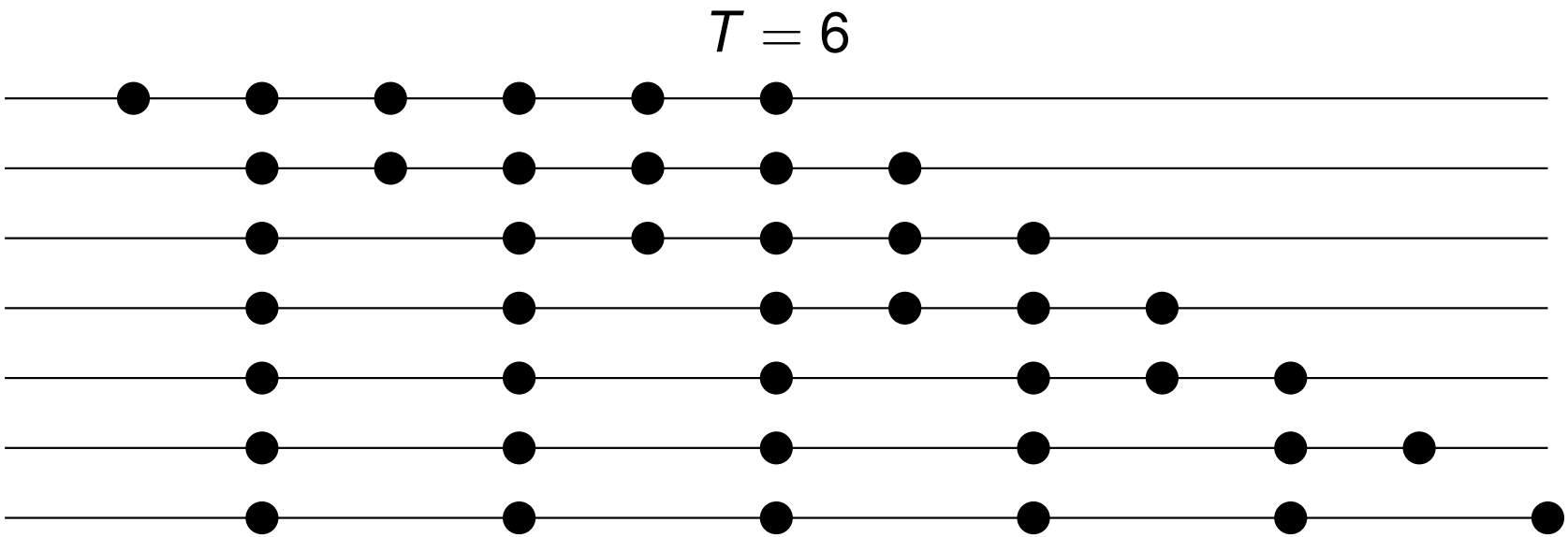

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Performance

For cyclic algorithms it suffices to look at one period to find the performance.

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- Intervals to consider

$$\left\{ \frac{t_1 - 0}{t_3}, \frac{t_2 - t_1}{t_3}, \frac{t_3 - t_2}{t_3} \right\}$$
$$\cup \left\{ \frac{t_2 - 0}{t_4}, \frac{t_3 - t_2}{t_4}, \frac{t_4 - t_3}{t_4} \right\}$$
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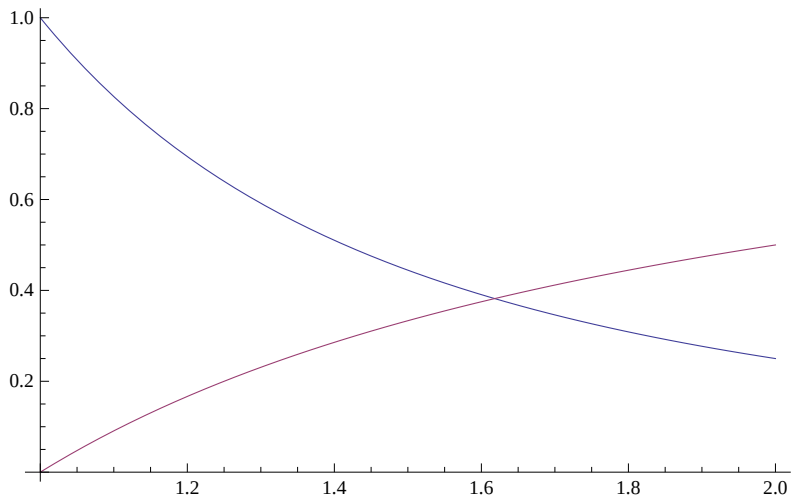

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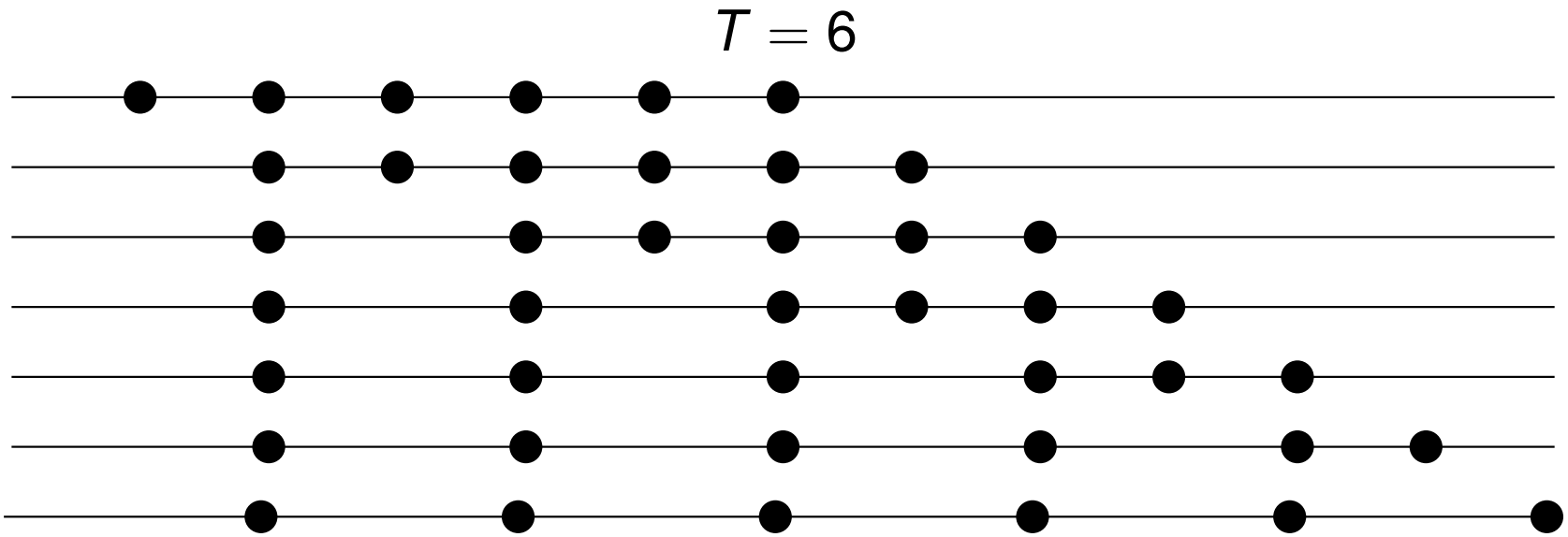
Performance: $4 * 1/\phi^2 \approx 1.53$

First Example Again



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Pattern $P = (1, 2, \ldots, k)$. Checkpoints placed at times

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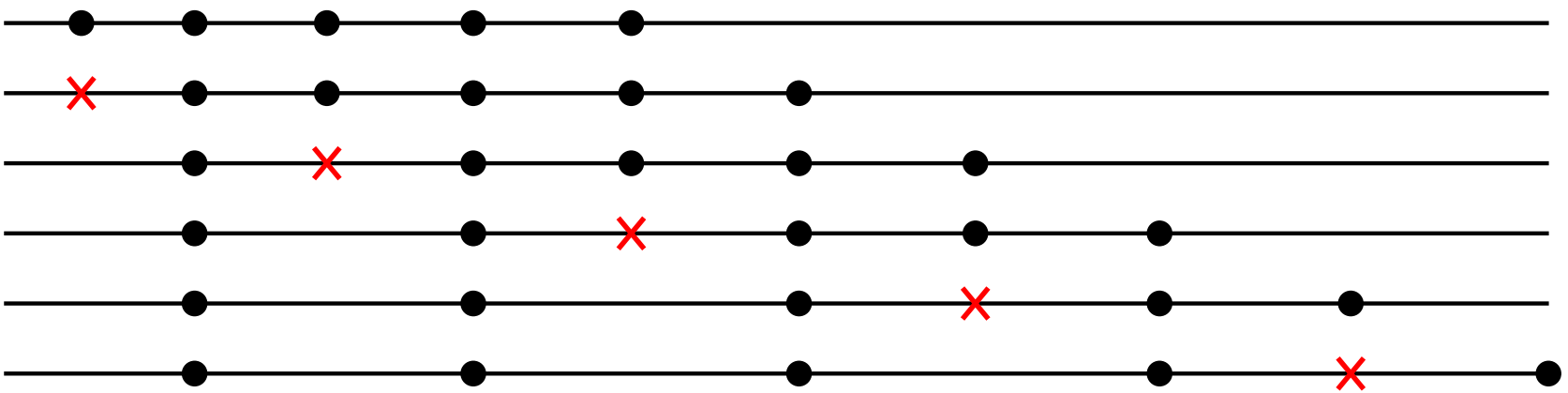
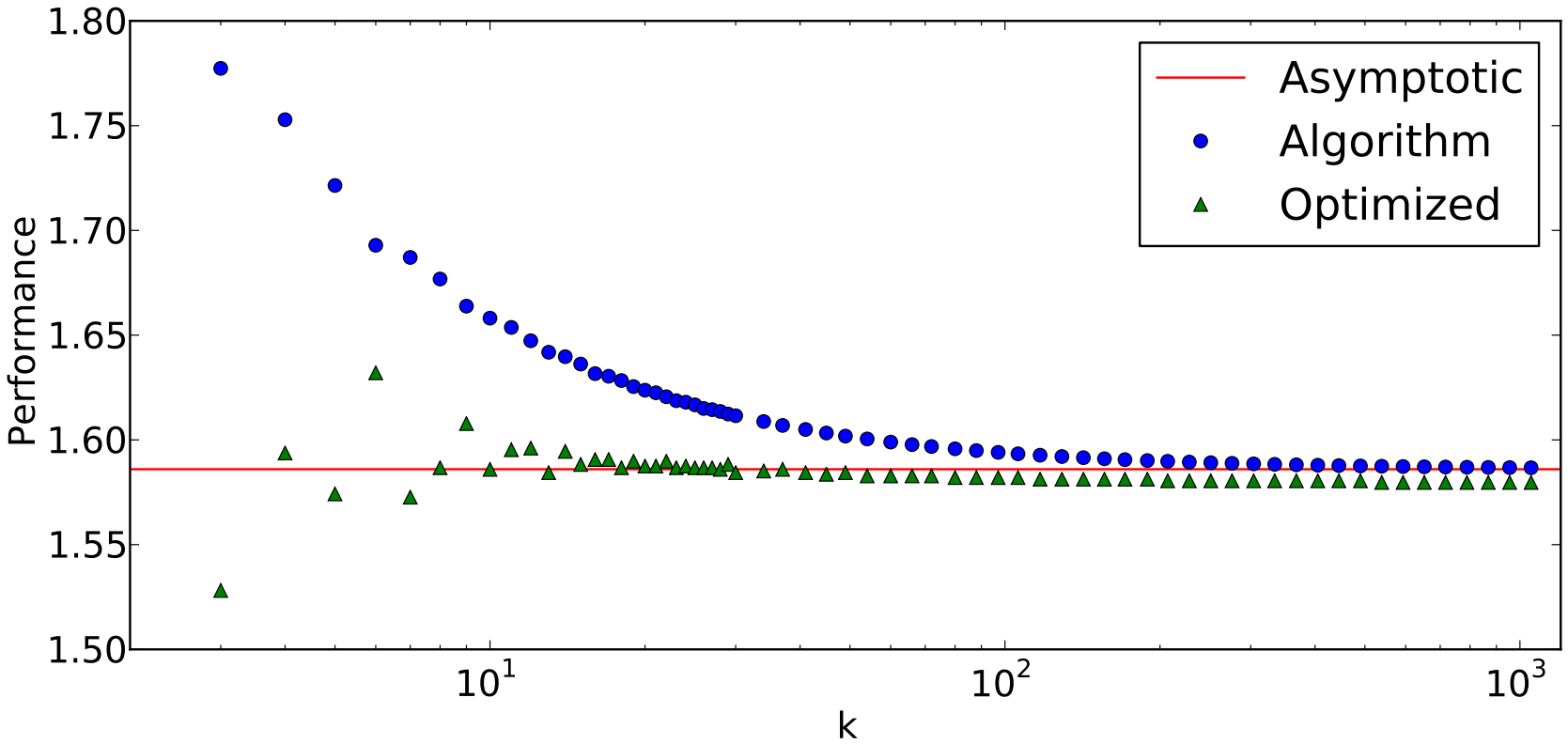


Experiment


1.80
1.75
1.70
1.65
1.60
1.55
1.50
Performance
10^1
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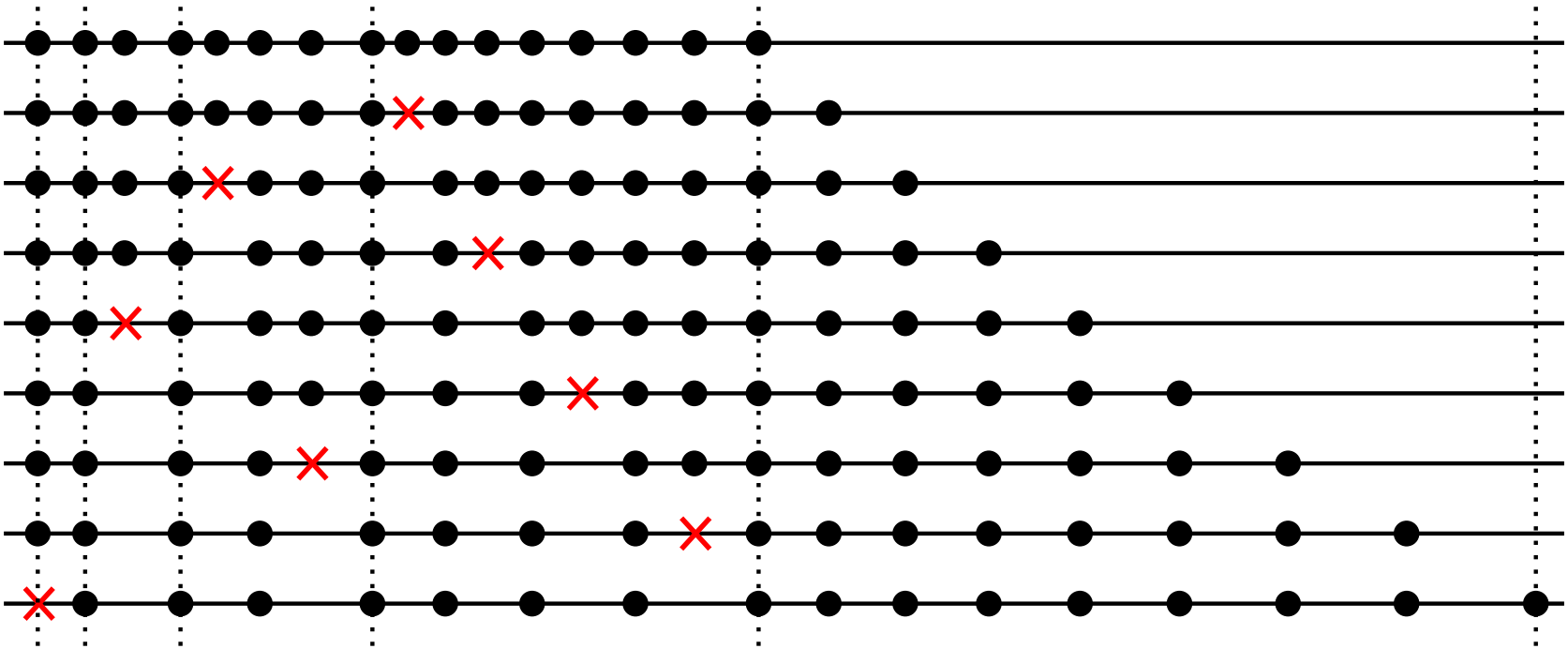

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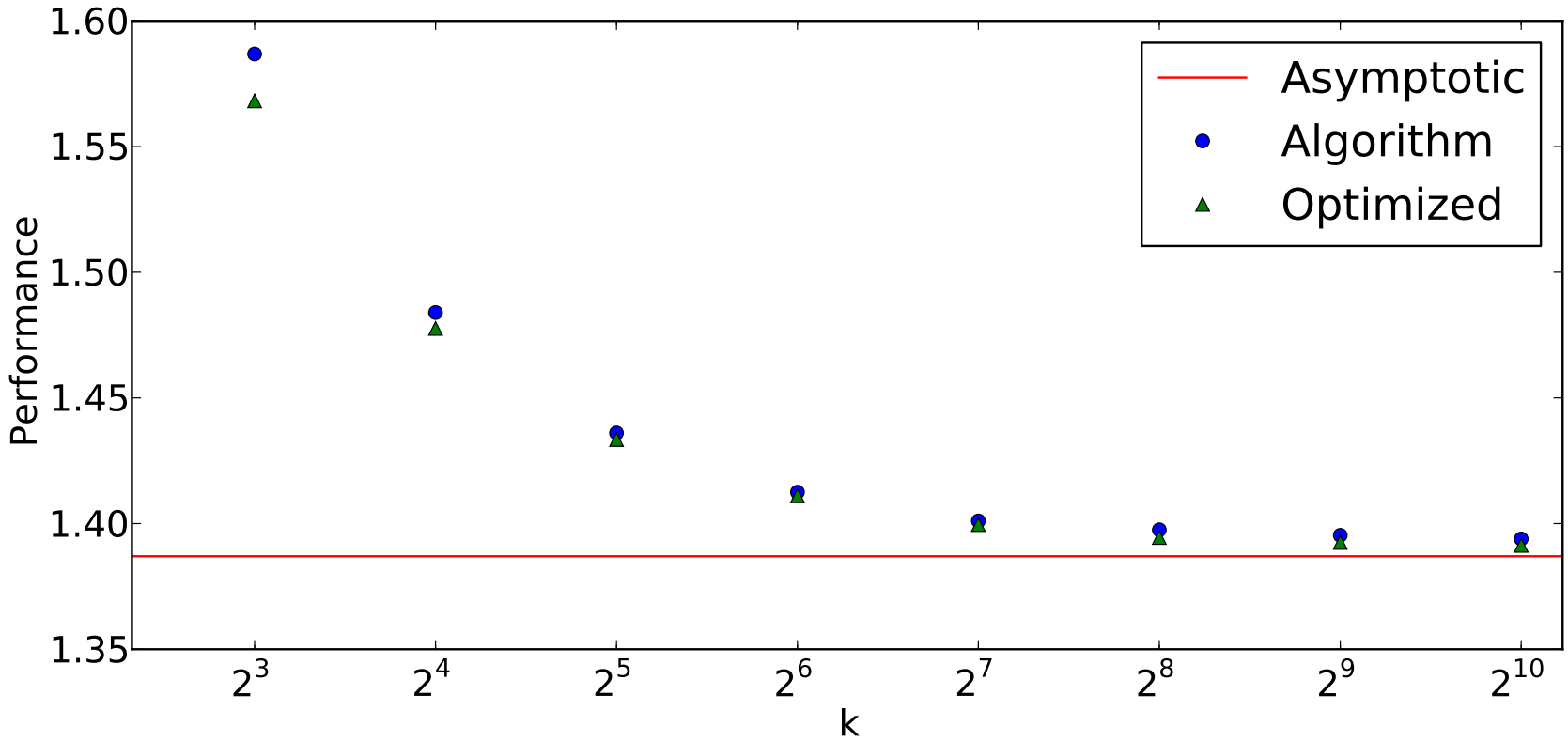
1.60
1.55
1.50
1.45
1.40
1.35
Performance
2^3
2^4
2^5
2^6
2^7
2^8
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- Enforce a performance of λ

$$\tau_{i+1}^j - \tau_i^j \leq \lambda \tau_k^j / (k+1)$$

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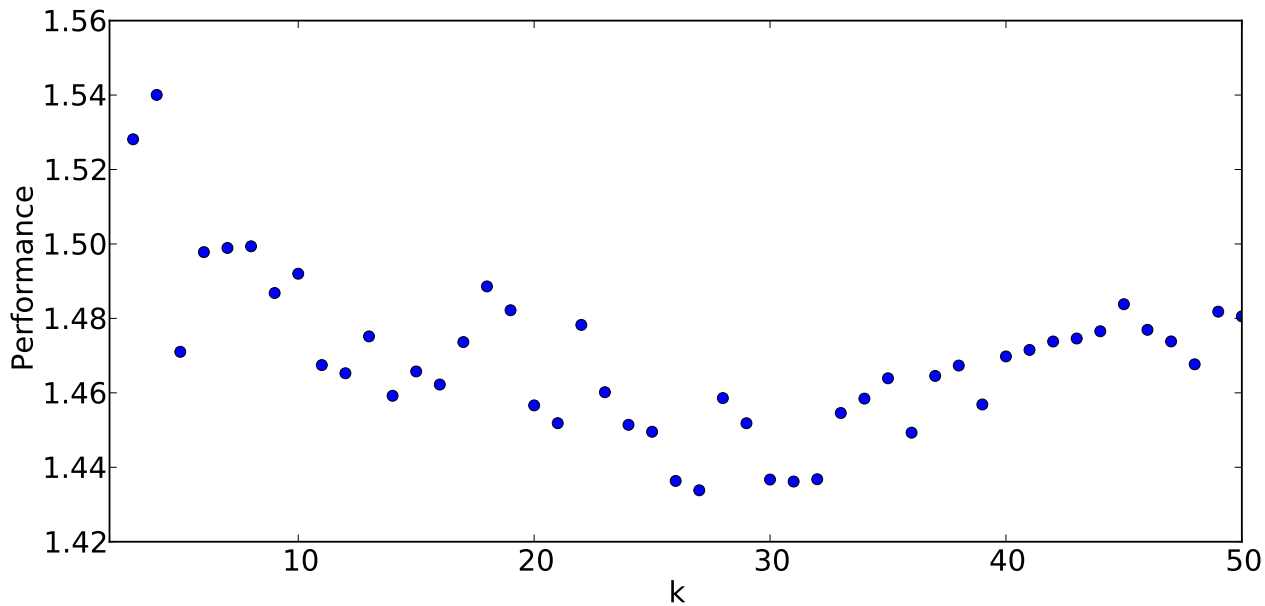

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Proof Sketch

Let A be an algorithm with performance p.

- Analyze A until $k/(2p)$ initial checkpoints are removed
- Assume intervals from storing new points are always optimal
- Bound the sizes of
  - Unchanged intervals
  - Intervals from deletion
  - Intervals from insertion

Optimal Algorithms Exist

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The infimum can be replaced by a minimum.

Proof sketch

- Starting positions with quality p_k exist
- Two algorithms can be combined losslessly
- There always are algorithms that make a few steps without worsening quality
- Combine a sequence of such algorithms to get optimal A^*.

Summary

We show for all k

$$1.3 \leq p_k \leq 1.6$$

- First lower bound asymptotically larger than one.
- First upper bound asymptotically smaller than two.

For $k = 2^i$ we show

$$1.3 \leq p_k \leq 1.39$$

only about 6% room for improvement left!

Open Problems

- Different performance measures, e.g. expected recomputation time
- Analysis of greedy algorithms