Walking the Dog Fast in Practice: Algorithm Engineering of the Fréchet Distance

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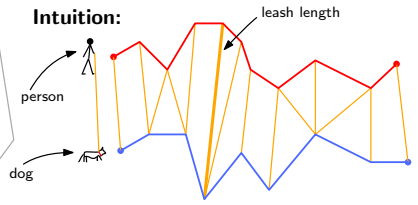
Fréchet Distance

- Curve similarity measure
- Applications:
  - Signature verification
  - Analysis of animal trajectories
  - Trajectory clustering
- Hard in theory
  - Algorithm: $\mathcal{O}(n^2)$ [AG'95]
  - Conditional lower bound: no $n^{1.99}$ algorithm [B'14]
- **Easy to compute in practice?**

Intuition:



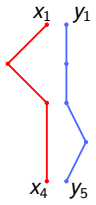
Formal Definition:

$$d_F(\pi,\sigma) := \min_{f\in\mathcal{T}_\pi, g\in\mathcal{T}_\sigma} \max_{t\in[0,1]} \left\|\pi_{f(t)} - \sigma_{g(t)}\right\|$$

$\pi, \sigma =$ polygonal curves
$\mathcal{T}_\pi =$ monotone, continuous traversals of π
$\mathcal{T}_\sigma =$ monotone, continuous traversals of σ

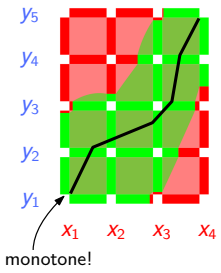
Free-Space Diagram

Curves:



Decision distance δ:

Corresponding Free-Space Diagram:



Algorithm

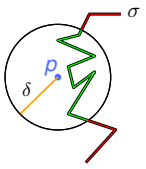
- Divide & conquer
- Use pruning rules to reduce search space
- **Main challenge:** develop such pruning rules
- Practical runtime depends on input structure

Pseudocode

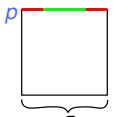
```
1: procedure DecideFréchetDistance(π, σ)
2:     ComputeOutputs(π, σ, [1, n] × [1, m])
3:     return [(n, m) ∈ R]

4: procedure ComputeOutputs(π, σ, B)
5:     if B is a cell then
6:         compute outputs by cell propagation
7:     else
8:         use rules I to IV to compute outputs of B
9:         if not all outputs have been computed then
10:            Split B horizontally/vertically into B1, B2
11:            ComputeOutputs(π, σ, B1)
12:            ComputeOutputs(π, σ, B2)
```

Tool: Simple Intervals

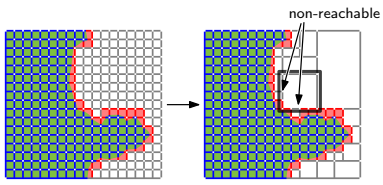
Point and subcurve:



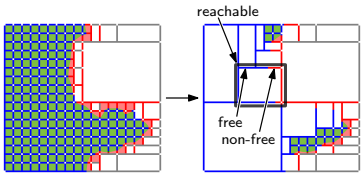
Free-space box:



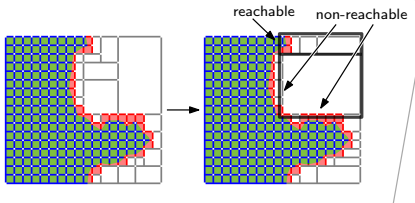
Pruning Rule I:



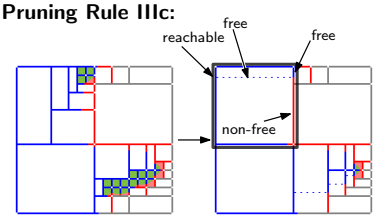
Pruning Rule IIIb:



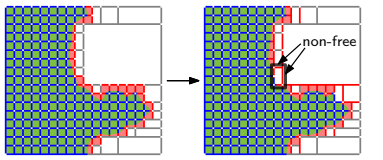
Pruning Rule II:



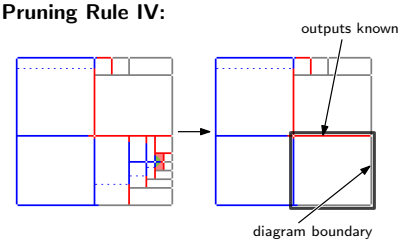
Pruning Rule IIIc:



Pruning Rule IIIa:



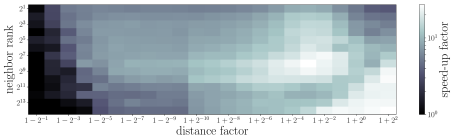
Pruning Rule IV:



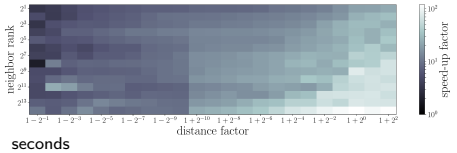
Experiments

- Comparison to winners of SIGSPATIAL GIS Cup 2017
- Experiments on 3 data sets
- Up to **more than two orders of magnitude faster**

SIGSPATIAL:



GEOLIFE:



output size

k	0	1	10	100	1000
GIS1	0.094	0.123	0.322	1.812	8.408
GIS2	0.421	0.618	1.711	7.86	35.704
GIS3	0.197	0.188	0.643	5.564	76.144
ours	0.017	0.007	0.026	0.130	0.490

seconds

k	0	1	10	100	1000
GIS1	0.298	0.741	4.327	33.034	109.44
GIS2	3.627	6.067	26.343	120.509	415.548
GIS3	2.614	4.112	16.428	166.206	1352.19
ours	0.027	0.089	0.341	1.108	3.642

Reference

Bringmann, Künnemann, Nusser. Walking the Dog Fast in Practice: Algorithm Engineering of the Fréchet Distance. In SoCG 2019.

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