Exercise Sheet 4
complete until Thursday, November 16th

Throughout this exercise sheet, A, B, and F refer to the suffix, backward, and forward array, respectively, as defined in the lecture (slides are on the Wiki).

Exercise 1
Consider the functions b and f with \( b(i) = C[\sigma] + \text{Occ}[\sigma, i] \) where \( \sigma = B[i] \) and \( C[\sigma] = |\{ j : B[j] < \sigma \}| \) and \( \text{Occ}[\sigma, i] = |\{ j : j \leq i \text{ and } B[j] = \sigma \}| \), and \( f(i) = F[i] \).

By the definition of \( F \) from the lecture, \( A[f(i)] = A[i] + 1 \) for all \( i \).

Show that \( A[b(i)] = A[i] - 1 \) for all \( i \).

From that infer that \( b \) and \( f \) are inverses of each other, that is, \( b(f(i)) = f(b(i)) = i \) for all \( i \).

Exercise 2
Let \( n_B \) be the number of maximal intervals \([i..j]\) with the property that \( B[i] = \ldots = B[j] \) (maximal same-letter subsequences, for example, \( \text{aaa} \) and \( \text{bb} \) and \( \text{11} \) and \( \text{aa} \) in \( \text{aaab11aa} \)), and let \( n_F \) be the number of maximal intervals \([i..j]\) with the property that \( F[i], \ldots, F[j] = F[i], F[i] + 1, F[i] + 2, \ldots \) (maximal runs of consecutive integers, for example, \( \text{7} \) and \( \text{3, 4, 5} \) and \( \text{2 and 8, 9 in 7, 3, 4, 5, 2, 8, 9} \)).

Show that \( n_F \leq n_B \leq n_F + |\Sigma| \), where \( \Sigma \) is the alphabet.

Exercise 3
Prepare yourself to give a short (5-10 minutes) blackboard presentation of your proof of either Exercise 1 or Exercise 2. Mark your choice on the Wiki following the instructions given there.