Exercise 1 (10 points).
Use the discriminant polynomial to show that the Waring rank of $X^2 + XY + Y^2$ is at least 2.

Exercise 2 (10 points).
We have seen polynomials whose Waring rank exceeds their border Waring rank. In contrast to this observation prove that the set of Waring rank 1 polynomials is $\mathbb{C}$-closed.

Exercise 3 (10 points).
Consider the action of $\mathbb{C}^{N\times N}$ on $\mathbb{C}[X_1, \ldots, X_N]_d$ defined in the lecture. Compute the following polynomial in the standard monomial basis:

$$
\begin{pmatrix}
2 & 3 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{pmatrix}
(X_1X_2^2 + X_3)
$$

Exercise 4 (10 points).
Let $GL_n$ denote the group of invertible complex $n \times n$ matrices. Let $G = GL_n \times GL_n$ and let $V = \mathbb{C}^{n\times n}$. Define an action of $G$ on $V$ by

$$(g_1, g_2)v := g_1 \cdot v \cdot g_2^t,$$

where \( \cdot \) is the product of matrices. Let $v \in V$ have rank exactly $k$. Prove that

$$Gv = \{ w \in V \mid \text{rk}(w) = k \}.$$