Assignment 11
due on Wednesday, August 4, 2018

Name:

Exercise 1 (20 points).
Simultaneously diagonalize the following three matrices:
\[
\begin{pmatrix}
5 & -30 & -8 \\
-18 & 182 & 48 \\
69 & -690 & -182 \\
\end{pmatrix},
\begin{pmatrix}
-21 & 88 & 24 \\
156 & -569 & -156 \\
-594 & 2178 & 597 \\
\end{pmatrix},
\begin{pmatrix}
-42 & 70 & 20 \\
300 & -527 & -150 \\
-1140 & 1995 & 568 \\
\end{pmatrix}.
\]

Exercise 2 (10 points).
Let \( A = \mathbb{C}[x, y]^2 \) and let \( V = \mathbb{C}[A]^2 \) be the polynomial \( GL_2 \)-representation from the lecture. For \( \alpha_1, \alpha_2 \in \mathbb{C}^\times \), determine the element \( \rho(\text{diag}(\alpha_1, \alpha_2)) \in GL(V) \).

Exercise 3 (10 points).
Let \( A = \mathbb{C}[x, y]^2 \). Determine a weight decomposition of the 6-dimensional \( T_2 \)-representation \( \mathbb{C}[A]^2 \).

Exercise 4 (10 points).
Consider the group homomorphism \( \varphi : T_2 \to T_2 \) given by \( \varphi(t_1, t_2) = (t_1^2, t_2) \). For \( A = \mathbb{C}[x, y]^2 \) let \( V = \mathbb{C}[A]^2 \) with the action \( \rho : GL_2 \to GL(V) \) from the lecture. Consider the action \( \rho' \) of \( T_2 \) on \( \mathbb{C}[A]^2 \) given by \( \rho'(t) = \rho(\varphi(t)) \) for \( t \in T_2 \). Determine a weight decomposition of this 6-dimensional \( T_2 \)-representation.

Exercise 5 (10 points).
Prove that there are exactly \( k \) pairwise non-isomorphic irreducible representations of the cyclic group \( \mathbb{Z}/k\mathbb{Z} \).

Exercise 6 (20 points).
Prove that every irreducible representation of the product group \( (\mathbb{Z}/k\mathbb{Z}) \times (\mathbb{Z}/\ell\mathbb{Z}) \) is 1-dimensional.