Exercise 1 (10 points).
Let $X \in K^{m \times m}$, $Y \in K^{m \times n}$, $Z \in K^{n \times m}$, and $W \in K^{n \times n}$ with $\det W \neq 0$. Prove that
\[
\det \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} = \det(W) \det(X - YW^{-1}Z),
\]

Exercise 2 (20 points).
In the proof of Strassen’s lower bound for 3-slice tensors, we used the tensor $T^\Lambda_A$. What happens if we use $T^S_A$ (defined by projecting onto $S^2 A$) instead?

Exercise 3 (10 points).
Generalize Strassen’s lower bound: If $T = (I, X, Y)$ is a 3-slice tensor with slices in $K^{b \times b}$, then
\[
\mathcal{R}(T) \geq b + \frac{1}{2} \mathrm{rk}(XY - YX).
\]