D5: Databases and Information Systems
Knowledge Representation
for the Semantic Web, WS 2017
Assignment 1

max planck institut informatik

*Please checkmark exercises that you solved before **11.01.2018**. The details of the checkmarking process will be available on the course website from **11.12.2017** onwards. Be sure to tick only those exercises which you can solve and explain on the blackboard. Do not leave the exercise work for the very last moment. Start preparing solutions as early as possible!*

Description Logic Syntax

Problem 1. Suppose that C and D are concept names and s, r are role names. State for every expression from below whether it is

- a concept
- a concept equivalence
- a concept inclusion
- a role inclusion

(1) $C \sqcap D$

(2) $C \sqcap D \sqcup C$

(3) $\neg C$

(4) $C \sqsubseteq D$

(5) $\forall r.(C \sqcup D)$

(6) $C \equiv \exists r^{-}.(C \sqcup D)$

(7) $\exists s.(\exists r.C) \sqsubseteq D$

(8) $C \sqcap D \sqsubseteq C$

(9) $C \equiv C \sqcap D$

(10) $r \sqsubseteq s$

Problem 2. Let *Author*, *Book* and *Article* be concept names, and let *writes* be a role name.

- Express the following statements in natural language:
  (1) $\exists writes.Book \sqsubseteq Author$
  (2) $\forall writes.Book$
  (3) $\exists writes.Book$
  (4) $\exists writes^{-}.\top \sqsubseteq Book \sqcup Article$
  (5) $Author \sqsubseteq \exists writes.\top$

(6) $Author \sqsubseteq \exists writes.\bot$

(7) $\geq 11\ writes.\top \sqsubseteq Author$

(8) $\geq 9\ writes.Book \sqsubseteq Author$

(9) $\forall writes.\top \sqsubseteq \exists writes.Book$

(10) $\exists writes.\top \sqsubseteq (\geq 5 writes.\top)$

(11) $\geq 5\ writes.\top \sqsubseteq \exists writes.\top$

(12) $\leq 1\ writes.\top \sqsubseteq \neg Author$

- For every expression from above state whether it is (a) a concept, (b) a concept inclusion, (c) a role inclusion, (d) none of the above.

Problem 3. Write the following statements in description logic $\mathcal{SHOIQ}$. Explicitly mention, which of the used symbols are concept names, role names and nominals.

(1) Every student at Saarland university is a person;

(2) MPI for Informatics has at least 500 students;

(3) Every citizen of Germany is a European;

(4) There are at least 150.000 people in Saarland;

(5) The domain of the relation "lives in"comprises of people;

(6) The range of the relation "has nationality" comprises countries;

(7) Bob lived in at least 3 countries;

(8) Everybody who has a happy friend is also happy;

(9) Brad and Angelina played in at least 2 movies together;

(10) Brad and Charlie did not work together.

Description Logic Semantics

Problem 4. Consider the description logic statements from **Problem 2**.

- For every concept inclusion $\mathcal{E}$, check whether $\mathcal{E}$ follows from the empty TBox (i.e., check whether $\emptyset \models \mathcal{E}$), and if it is not the case, construct an interpretation $\mathcal{I}$, such that $\mathcal{I} \not\models \mathcal{E}$.
- For every concept $\mathcal{E}$, test whether $\mathcal{E}$ is satisfiable. If this is the case, define an interpretation $\mathcal{I}$ such that $\mathcal{E}^{\mathcal{I}} \neq \emptyset$.

Problem 5. Consider the following interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \circ^{\mathcal{I}})$ given as

- $\Delta^{\mathcal{I}} = \{a, b, c\}$
- $C^{\mathcal{I}} = \{a\}$
- $D^{\mathcal{I}} = \{b, c\}$
- $s^{\mathcal{I}} = \{(c, a), (a, a), (a, b)\}$

Compute the extension of the following concepts under $\mathcal{I}$:

(1) $(C \sqcup D)^{\mathcal{I}}$;

(2) $(C \sqcap D)^{\mathcal{I}}$;

(3) $(\top \sqcap \neg(C \sqcup \neg D))^{\mathcal{I}}$;

(4) $(\forall s.(C \sqcup D))^{\mathcal{I}}$;

(5) $(\forall s.(C \sqcap D))^{\mathcal{I}}$;

(6) $(\forall s.C \sqcap \exists s.D)^{\mathcal{I}}$;

(7) $(\neg(\neg C \sqcup \neg D))^{\mathcal{I}}$;

(8) $(\exists s.(\exists s.(\exists s.(C \sqcup D))))^{\mathcal{I}}$;

(9) $(\exists s^{-}.D)^{\mathcal{I}}$;

(10) $(\forall s^{-}.C)^{\mathcal{I}}$.

Problem 6. Consider the following TBox $\mathcal{T} = \{Author \sqsubseteq \exists writes.Book, Novelist \sqsubseteq Author\}$. Formally prove that $\mathcal{T} \not\sqsubseteq Author \sqsubseteq Novelist$ by constructing an interpretation $\mathcal{I}$, such that $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \not\models Author \sqsubseteq Novelist$.

Problem 7. Suppose you are given the following ABox consisting of the following axioms:

- $takesCourse(olly, databases)$;
- $takesCourse(olly, data\_structures)$;
- $takesCourse(olly, data\_modeling)$;
- $takesCourse(olly, kr)$;
- $\leq 2\ takesCourse(olly)$.

Is this ABox satisfiable? If the answer is yes, then construct an interpretation that satisfies it. Under which assumption it is not satisfiable?

Problem 8. Consider the following ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, where the TBox $\mathcal{T}$ and the ABox $\mathcal{A}$ are given as:

- $\mathcal{T} = \{E \sqsubseteq F, B \sqsubseteq \neg(\neg C \sqcap \neg E), A \sqsubseteq \neg\forall s.\neg B\}$
- $\mathcal{A} = \{\neg\exists s.F(j_1), A(j_1)\}$

Is the given knowledge base satisfiable? Formally prove your answer

- using semantics and
- using the $\mathcal{ALC}$ tableu algorithm.

Reasoning

Problem 9. Suppose that r is a role name and i, j are individual names. Prove formally that the following expressions are not valid by constructing appropriate countermodels:

(1) $\exists r.\{i\} \sqcap \exists r.\{j\} \equiv \geq 2r.\{i, j\}$;

(2) $\exists r.\{i\} \sqcap \exists r.\{j\} \equiv \exists r.\{i, j\}$.

Problem 10. Consider the following two concept definitions:

- $\mathit{BinaryTree} \equiv \leq 2\ \mathit{hasBranch} \sqcap \forall \mathit{hasBranch}.\mathit{BinaryTree}$;
- $\mathit{List} \equiv \leq 1\ \mathit{hasBranch} \sqcap \forall \mathit{hasBranch}.\mathit{List}$

Show formally that for any interpretation $\mathcal{I}$, we have $\mathcal{I} \models \mathit{List} \sqsubseteq \mathit{BinaryTree}$.