Knowledge Representation for the Semantic Web

Lecture 7: Answer Set Programming II

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partially based on slides by Thomas Eiter

D5: Databases and Information Systems
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WS 2017/18
Unit Outline

More about Logic Programs

ASP Paradigm

Programming Techniques

Answer Set Solvers
Strong Negation

- Default negation “not a” means “a cannot be proved (derived) using rules,” and that a is false by default (believed to be false).
Strong Negation

- Default negation “\textit{not a}” means “\textit{a} cannot be proved (derived) using rules,” and that \textit{a} is false by default (believed to be false).

- Strong negation \( \neg a \) (also \( -a \)) means that \textit{a} is (provably) false

- Both default and strong negation can be used in ASP
Strong Negation

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- Both default and strong negation can be used in ASP.

“At a railroad crossing, cross the rails if no train approaches.”

We may encode this scenario using one of the following two rules:

$$walk \leftarrow at(X), crossing(X), not \ train\_approaches(X). \quad (r_1)$$

$$walk \leftarrow at(X), crossing(X), \neg train\_approaches(X). \quad (r_2)$$

- $r_1$ fires if there is no information that a train approaches.
- $r_2$ fires if it is explicitly known that no train approaches.
Constraints

- **Constraints** are rules with empty head which exclude invalid models.

\[ \leftarrow q_1, \ldots, q_m, \text{not } r_1, \ldots, \text{not } r_n. \]

Kills answer sets that

- contain \( q_1, \ldots, q_m \), and
- do not contain \( r_1, \ldots, r_n \).
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This kills answer sets that

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- do not contain \( r_1, \ldots, r_n \).

An equivalent version of the above rule is with a fresh predicate \( p \):

\[
p \leftarrow q_1, \ldots, q_m, \text{not } r_1, \ldots, \text{not } r_n, \text{not } p.
\]
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\[ p \leftarrow q_1, \ldots, q_m, \text{not } r_1, \ldots, \text{not } r_n, \text{not } p. \]

**Example:** adjacent nodes cannot be colored with the same color

\[ \bot \leftarrow \text{edge}(X, Y), \text{colored}(X, Z), \text{colored}(Y, Z). \]
Disjunction

The use of disjunction is natural

- to express indefinite knowledge:

\[
\text{female}(X) \lor \text{male}(X) \leftarrow \text{person}(X).
\]

\[
\text{broken}(\text{left\_arm}, \text{robot1}) \lor \text{broken}(\text{right\_arm}, \text{robot1}).
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\]

- to express a “guess” and to create non-determinism.

\[
\text{ok}(C) \lor \neg\text{ok}(C) \leftarrow \text{component}(C).
\]
Minimality

- Semantics: disjunction is minimal (different from classical logic):

\[ a \lor b \lor c. \]
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Minimal models: \{a\}, \{b\}, and \{c\}.
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• But minimality is not necessarily exclusive:

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Minimal models: \( \{a, b\}, \{a, c\}, \text{ and } \{b, c\} \).
Extended Logic Programs

An extended disjunctive logic program (EDLP) is a finite set of rules

\[ a_1 \lor \cdots \lor a_k \leftarrow b_1, \ldots, b_m, \text{not } c_1, \ldots, \text{not } c_n \quad (k, m, n \geq 0) \quad (1) \]

where all \( a_i, b_j, c_l \) are atoms or strongly negated atoms.
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**Semantics:**

- Stable models (answer sets) of EDLPs are defined similarly as for LPs, viewing \( \neg p \) as a new predicate.
Extended Logic Programs with Disjunctions

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Semantics:

- **Stable models (answer sets)** of EDLPs are defined similarly as for LPs, viewing \( \neg p \) as a new predicate.
- **Differences:**
  - \( I \) must not contain atoms \( p(c_1, \ldots, c_n), \neg p(c_1, \ldots, c_n) \) (consistency)
  - \( I \) is a model of ground rule (1), if either \( \{b_1, \ldots, b_m\} \not\subseteq I \) or \( \{a_1, \ldots, a_k, c_1, \ldots, c_n\} \cap I \neq \emptyset \).
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  - Condition “\( M \) is the least model of \( P^M \)” is replaced by “\( M \) is a minimal model of \( P^M \)” (\( P^M \) may have multiple minimal models).
Example

Let $P$ be the following program:

$$\text{man}(\text{dilbert}).$$

$$\text{single}(X) \lor \text{husband}(X) \leftarrow \text{man}(X).$$
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\text{man}(\text{dilbert}).
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Let $P$ be the following program:

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- As $P$ is "not"-free, $\text{grnd}(P)^M = \text{grnd}(P)$ for every $M$.

- Answer sets:
  - $M_1 = \{\text{man}(\text{dilbert}), \text{single}(\text{dilbert})\}$, and
  - $M_2 = \{\text{man}(\text{dilbert}), \text{husband}(\text{dilbert})\}$. 
ASP Paradigm
**ASP Paradigm**

**General idea: stable models are solutions!**
Reduce solving a problem instance $I$ to computing stable models of a LP

1. **Encode** $I$ as a (non-monotonic) logic program $P$, such that solutions of $I$ are represented by models of $P$

2. **Compute** some model $M$ of $P$, using an ASP solver

3. **Extract** a solution for $I$ from $M$.

Variant: Compute multiple models (for multiple / all solutions)
ASP Paradigm (cont’d)

Compared to SAT solving, ASP offers more features:

- transitive closure
- negation as failure
- predicates and variables
ASP Paradigm (cont’d)

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- negation as failure
- predicates and variables

Generic problem solving by separating the

- specification of solutions (“logic” \( PS \))
- concrete instance of the problem (data \( D \))

\[ \text{Problem} \rightarrow \text{Spec. } PS \]
\[ \text{Data } D \rightarrow \text{Encoding: Program } P_{PS} \]
\[ \text{Encoding: Program } P_{D} \rightarrow \text{ASP Solver} \rightarrow \text{Model(s)} \]
\[ \text{Solution(s)} \]
The “Guess and Check” Methodology

Important element of ASP: **Guess and Check** methodology (or **Generate-and-Test** [Lifschitz, 2002]).

1. **Guess**: use unstratified negation or disjunctive heads to create candidate solutions to a problem (program part $G$), and

2. **Check**: use further rules and/or constraints to test candidate solution if it is a proper solution for our problem (program part $C$).
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From another perspective:

- $G$: defines the search space
- $C$: prunes illegal branches.

Example: 3-Coloring

Problem specification $P_{PS}$ (compact encoding)

$$g(X) \lor r(X) \lor b(X) \leftarrow node(X) \ } \text{Guess}$$

$$\leftarrow b(X), b(Y), edge(X, Y)$$
$$\leftarrow r(X), r(Y), edge(X, Y)$$
$$\leftarrow g(X), g(Y), edge(X, Y) \ } \text{Check}$$

Data $P_D$: Graph $G = (V, E)$

$P_D = \{\text{node}(v) | v \in V\} \cup \{\text{edge}(v, w) | (v, w) \in E\}$. Correspondence: 3-colorings $\leftrightarrow$ models: $v \in V$ is colored with $c \in \{r, g, b\}$ iff $c(v)$ is in the model of $P_{PS} \cup P_D$. 
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g(X) &\lor r(X) &\lor b(X) &\leftarrow \text{node}(X) \quad \text{Guess} \\
\leftarrow b(X), b(Y), \text{edge}(X, Y) \\
\leftarrow r(X), r(Y), \text{edge}(X, Y) \\
\leftarrow g(X), g(Y), \text{edge}(X, Y) \quad \text{Check}
\end{align*}
\]

Data $P_D$: Graph $G = (V, E)$

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P_D = \{\text{node}(v) \mid v \in V\} \cup \{\text{edge}(v, w) \mid (v, w) \in E\}.
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Correspondence: 3-colorings $\Leftrightarrow$ models:
$v \in V$ is colored with $c \in \{r, g, b\}$ iff $c(v)$ is in the model of $P_{PS} \cup P_D$. 
Example: 3-Coloring (cont’d)

\[ P_D = \{ \text{node}(a), \text{node}(b), \] \\
\[ \text{node}(c), \text{edge}(a, b), \] \\
\[ \text{edge}(b, c), \text{edge}(a, c) \} \]
Example: 3-Coloring (cont’d)

\[ P_D = \{ \text{node}(a), \text{node}(b), \]

\[ \text{node}(c), \text{edge}(a, b), \]

\[ \text{edge}(b, c), \text{edge}(a, c) \} \]
Example: Hamiltonian Path/Cycle

Input: a directed graph represented by $\text{node}(\_)$ and $\text{edge}(\_,\_)$ and a starting node $\text{start}(\_)$.

Problem: find a path beginning at the starting node which visits all nodes of the graph exactly once.
Example: Hamiltonian Path/Cycle

Input: a directed graph represented by node(\_) and edge(_, _) and a starting node start(_).

Problem: find a path beginning at the starting node which visits all nodes of the graph exactly once.

\[
inPath(X, Y) \lor outPath(X, Y) \leftarrow edge(X, Y). \] \textit{Guess}

\[
\leftarrow inPath(X, Y), inPath(X, Y_1), Y \neq Y_1.
\leftarrow inPath(X, Y), inPath(X_1, Y), X \neq X_1.
\leftarrow node(X), \text{not reached}(X). \] \textit{Check}

\[
reached(X) \leftarrow start(X).
reached(X) \leftarrow reached(Y), inPath(Y, X). \] \textit{Auxiliary Predicate}
Example: Hamiltonian Path/Cycle

Input: a directed graph represented by \texttt{node(\_)} and \texttt{edge(\_, \_)} and a starting node \texttt{start(\_)}.

Problem: find a path/cycle\(^1\) beginning at the starting node which visits all nodes of the graph exactly once.

\begin{align*}
inPath(X, Y) \lor\ & outputPath(X, Y) \leftarrow edge(X, Y). \} \text{Guess} \\
\leftarrow inPath(X, Y), \ inPath(X, Y_1), \ Y \neq Y_1. \\
\leftarrow inPath(X, Y), \ inPath(X_1, Y), \ X \neq X_1. \\
\leftarrow node(X), \ not\ reached(X). \\
\leftarrow not\ start\_reached. \\
reached(X) \leftarrow start(X). \\
\text{Auxiliary Predicate}
\end{align*}

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\(^1\)The encoding for the Hamiltonian cycle contains red lines along with green ones.
Example: Hamiltonian Path/Cycle (cont’d)

\[ P_D = \{\text{node}(a), \text{node}(b), \text{node}(c), \text{node}(d), \text{edge}(a, b), \text{edge}(a, c), \text{edge}(c, b), \text{edge}(b, c), \text{edge}(b, d), \text{edge}(d, c), \text{edge}(d, a), \text{edge}(a, d), \text{start}(a)\} \]
Example: Hamiltonian Path/Cycle (cont’d)

\[ P_D = \{ \text{node}(a), \text{node}(b), \]
\[ \text{node}(c), \text{node}(d), \]
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\[ \text{edge}(b, d), \text{edge}(d, c), \]
\[ \text{edge}(d, a), \text{edge}(a, d), \]
\[ \text{start}(a) \} \]
Example: Course Assignment

Input: information about members and courses of a computer science (CS) dept \( cs \)

Problem:
- assign courses to members of the CS dept
- teachers must like the assigned course
- each member must teach 1-2 courses
Example: Course Assignment

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**Problem:**
- assign courses to members of the CS dept
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$$P_D:$$

\[ \text{member(sam, } cs) \quad \text{member(bob, } cs) \quad \text{member(tom, } cs) } \]
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$P_D:$

```prolog
member(sam, cs).
member(bob, cs).
member(tom, cs).
course(java, cs).
course(ai, cs).
course(c, cs).
course(logic, cs).
```
Example: Course Assignment

Input: information about members and courses of a computer science (CS) dept \( cs \)

Problem:
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\[ P_D: \]

\[
\text{member}(\text{sam}, \text{cs}). \quad \text{member}(\text{bob}, \text{cs}). \quad \text{member}(\text{tom}, \text{cs}). \\
\text{course}(\text{java}, \text{cs}). \quad \text{course}(\text{ai}, \text{cs}). \quad \text{course}(\text{c}, \text{cs}). \\
\text{course}(\text{logic}, \text{cs}). \\
\text{likes}(\text{sam}, \text{java}). \quad \text{likes}(\text{sam}, \text{c}). \quad \text{likes}(\text{tom}, \text{ai}). \\
\text{likes}(\text{bob}, \text{java}). \quad \text{likes}(\text{bob}, \text{ai}). \quad \text{likes}(\text{tom}, \text{logic}).
\]
Example: Course Assignment (cont’d)

Problem specification $P_{PS}$:

% assign a course to a teacher who likes it, by default
$teach(X, Y) ← member(X, cs), course(Y, cs),$
$likes(X, Y), not ¬ teach(X, Y).$

% determine when a course should not be assigned to a teacher
$¬ teach(X, Y) ← member(X, cs), course(Y, cs),$
$teach(X_1, Y), X_1 ≠ X.$
Example: Course Assignment (cont’d)

Problem specification $P_{PS}$:

% assign a course to a teacher who likes it, by default
\[
\text{teach}(X,Y) \leftarrow \text{member}(X,cs), \text{course}(Y,cs),
\text{likes}(X,Y), \text{not} - \text{teach}(X,Y).
\]

% determine when a course should not be assigned to a teacher
\[
-\text{teach}(X,Y) \leftarrow \text{member}(X,cs), \text{course}(Y,cs),
\text{teach}(X_1,Y), X_1 \neq X.
\]

% check each cs member has some course
\[
\text{has.course}(X) \leftarrow \text{member}(X,cs), \text{teach}(X,Y).
\]
\[
\leftarrow \text{member}(X,cs), \text{not has.course}(X).
\]
Example: Course Assignment (cont’d)

Problem specification $P_{PS}$:

% assign a course to a teacher who likes it, by default
\[ \text{teach}(X, Y) \leftarrow \text{member}(X, cs), \text{course}(Y, cs), \]
\[ \text{likes}(X, Y), \neg \text{teach}(X, Y). \]

% determine when a course should not be assigned to a teacher
\[ \neg \text{teach}(X, Y) \leftarrow \text{member}(X, cs), \text{course}(Y, cs), \]
\[ \text{teach}(X_1, Y), X_1 \neq X. \]

% check each cs member has some course
\[ \text{has.course}(X) \leftarrow \text{member}(X, cs), \text{teach}(X, Y). \]
\[ \leftarrow \text{member}(X, cs), \neg \text{has.course}(X). \]

% check each cs member has at most 2 courses
\[ \leftarrow \text{teach}(X, Y_1), \text{teach}(X, Y_2), \text{teach}(X, Y_3), \]
\[ Y_1 \neq Y_2, Y_1 \neq Y_3, Y_2 \neq Y_3. \]
Programming Techniques
Programming Techniques

- With the “guess and check paradigm”, one may use different techniques to solve particular tasks

E.g.,

- choice of exactly one element from a set
- computing maximum / minimum values (use double negation)
- testing a property for all elements in a set (iteration over a set)
- testing a co-NP hard property (saturation)
- modularization

- We do not discuss here saturation (see [Eiter et al., 2009])

Note: extensions of ASP allow to test properties of a set / choose elements elegantly
Selecting One Element from a Set

- **Task:** given a set, defined by predicate $p(X)$, select exactly one element from it (if nonempty).

- **More general variant:** $p(\vec{X}, \vec{Y})$, where $\vec{X} = X_1, \ldots, X_n$, $\vec{Y} = Y_1, \ldots, Y_m$, select for each $\vec{X}$ exactly one $\vec{Y}$ (if possible)
  - Implicitly, already done in the above course assignment problem
Selecting One Element from a Set

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### Select one element from a set: Normal rule encoding

\[
\begin{align*}
\text{sel}(\vec{X}, \vec{Y}) & \leftarrow p(\vec{X}, \vec{Y}), \text{not} \ - \text{sel}(\vec{X}, \vec{Y}). \\
-\text{sel}(\vec{X}, \vec{Y}) & \leftarrow p(\vec{X}, \vec{Y}), \text{sel}(\vec{X}, \vec{Z}), Y_i \neq Z_i. \quad i = 1, \ldots, m
\end{align*}
\]

where $\vec{Z} = Z_1, \ldots, Z_m$. 
Selecting One Element from a Set (cont’d)

Example: Course assignment

- \( p(X, Y) \) is \( \text{member}(Y, cs) \), \( \text{course}(X, cs) \), \( \text{likes}(Y, X) \) and \( \text{sel}(X, Y) \) is \( \text{teach}(Y, X) \).
- could define an auxiliary rule

\[
p(X, Y) \leftarrow \text{member}(Y, cs), \text{course}(X, cs), \text{likes}(Y, X)
\]

Select one element from a set: Disjunctive rule encoding

\[
\text{sel}(\vec{X}, \vec{Y}) \leftarrow p(\vec{X}, \vec{Y}), \text{not } \neg \text{sel}(\vec{X}, \vec{Y}).
\]

\[
\neg \text{sel}(\vec{X}, \vec{Y}) \lor \neg \text{sel}(\vec{X}, \vec{Z}) \leftarrow p(\vec{X}, \vec{Y}), p(\vec{X}, \vec{Z}), Y_i \neq Z_i. \quad i = 1, \ldots, m
\]
Selecting One Element from a Set (cont’d)

Example: Course assignment

- $p(X, Y)$ is $\text{member}(Y, cs)$, $\text{course}(X, cs)$, $\text{likes}(Y, X)$ and $\text{sel}(X, Y)$ is $\text{teach}(Y, X)$.
- could define an auxiliary rule

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Select one element from a set: Disjunctive rule encoding

\[ \text{sel}(\vec{X}, \vec{Y}) \leftarrow p(\vec{X}, \vec{Y}), \text{not } \neg \text{sel}(\vec{X}, \vec{Y}). \]
\[ -\text{sel}(\vec{X}, \vec{Y}) \lor -\text{sel}(\vec{X}, \vec{Z}) \leftarrow p(\vec{X}, \vec{Y}), p(\vec{X}, \vec{Z}), Y_i \neq Z_i, \quad i = 1, \ldots, m \]

In some answer set solvers, special syntax is available (see ASP-Core2):

\[ 1\{ \text{sel}(\vec{X}, \vec{Y}) : p(\vec{X}, \vec{Y}) \} 1 \leftarrow p(\vec{X}, \vec{Z}) \]
Use of Double Negation

Defining a predicate \( p \) in terms of its negation \( \neg p \)

Greatest Common Divisor — Euclid-style

% base case
\[
gcd(X, X, X) \leftarrow \text{int}(X), X > 1.
\]

% subtract smaller from larger number
\[
gcd(D, X, Y) \leftarrow X < Y, gcd(D, X, Y_1), Y = Y_1 + X.
\]
\[
gcd(D, X, Y) \leftarrow X > Y, gcd(D, X_1, Y), X = X_1 + Y.
\]

This is not easy to come up with and needs more care in Prolog.
Use of Double Negation (cont’d)

Greatest Common Divisor — ASP-style

% Declare when $D$ divides a number $N$.
\[ \text{divisor}(D, N) \leftarrow \text{int}(D), \text{int}(N), \text{int}(M), N = D \ast M. \]

% Declare common divisors
\[ \text{cd}(T, N_1, N_2) \leftarrow \text{divisor}(T, N_1), \text{divisor}(T, N_2). \]

% Single out non-maximal common divisors $T$
\[ -\text{gcd}(T, N_1, N_2) \leftarrow \text{cd}(T, N_1, N_2), \text{cd}(T_1, N_1, N_2), T < T_1. \]

% Apply double negation: take non non-maximal divisor
\[ \text{gcd}(T, N_1, N_2) \leftarrow \text{cd}(T, N_1, N_2), \text{not} - \text{gcd}(T, N_1, N_2). \]
Iteration over a Set

• Testing a property, say Prop, for all elements of a set S without negation

• This may be needed in some contexts:
  • in combination with other techniques, e.g., saturation (see [Eiter et al., 2009]), or
  • if negation could lead to undesired behavior (e.g., cyclic negation).
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- walk through all elements of set $S$, from the first to the last element;
- check whether property $Prop$ holds up to the current element $y$
  $\Leftrightarrow$ holds for $y$ and holds up to for $x$, where $y$ is the successor of $x$;
- when $Prop$ holds up to the last element, it holds for all elements of $S$
**Iteration over a Set**

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- Note: this is a form of finite induction

- Use an enumeration of $S$ with predicates $\text{first}(\_)$, $\text{succ}(\_, \_)$, $\text{last}(\_)$
  - Easy for static $S$, more involved for dynamically computed $S$
Example: Hamiltonian Path 2/Reachability

• Variant: no use of negation in checking that all nodes are reached (do not immediately kill stable model candidate):

  \[
  \leftarrow \text{node}(X), \text{not reached}(X).
  \]

• Check that all nodes of the graph are reached via the selected edges \(\text{inPath}(X, Y)\) by iteration \((S = \text{nodes of the graph})\)

• Use
  
  - \text{all_reached_uppto(\_)}
  
  - \text{all_reached}

• Supply in the input an enumeration of the nodes via \(\text{first(\_), succ(\_, \_), last(\_)}\)

  - Alternative: build the enumeration \textit{dynamically} in the program, using e.g. string comparison.
Example: Hamiltonian Path 2 (cont’d)

\[
\begin{align*}
\text{inPath}(X, Y) \lor \text{outPath}(X, Y) & \leftarrow \text{edge}(X, Y). \quad \{\text{Guess}\} \\
& \leftarrow \text{inPath}(X, Y), \text{inPath}(X, Y_1), Y \neq Y_1. \\
& \leftarrow \text{inPath}(X, Y), \text{inPath}(X_1, Y), X \neq X_1. \quad \{\text{Check}\} \\
\text{reached}(X) & \leftarrow \text{start}(X). \\
\text{reached}(X) & \leftarrow \text{reached}(Y), \text{inPath}(Y, X). \quad \{\text{Auxiliary Predicates}\} \\
\text{all_reached_upto}(X) & \leftarrow \text{first}(X), \text{reached}(X). \\
\text{all_reached_upto}(X) & \leftarrow \text{all_reached_upto}(Y), \text{succ}(Y, X), \text{reached}(X). \\
\text{all_reached} & \leftarrow \text{last}(X), \text{all_reached_upto}(X). \quad \{\text{reached} = \text{nodes}\}
\end{align*}
\]
Example: Hamiltonian Path 2 (cont’d)

\[
P_D = \{ \text{edge}(a, b), \text{edge}(a, c), \\
\text{edge}(c, b), \text{edge}(b, c), \\
\text{edge}(b, d), \text{edge}(d, c), \\
\text{edge}(d, a), \text{edge}(a, d), \\
\text{first}(a), \text{succ}(a, b), \\
\text{succ}(b, c), \text{succ}(c, d), \\
\text{last}(d), \text{start}(a) \}\
\]
Example: Hamiltonian Path 2 (cont’d)

\[ P_D = \{ edge(a, b), edge(a, c), \]
\[ edge(c, b), edge(b, c), \]
\[ edge(b, d), edge(d, c), \]
\[ edge(d, a), edge(a, d), \]
\[ first(a), succ(a, b), \]
\[ succ(b, c), succ(c, d), \]
\[ last(d), start(a) \} \]
Example: Hamiltonian Path 2 (cont’d)

Some path guesses not reaching all nodes from $a$:

- $all\_reached\_upto(c)$
  - $d \rightarrow c$
  - $a \rightarrow b$

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  - $d \rightarrow c$
  - $a \rightarrow b$

- $all\_reached\_upto(a)$
  - $d \rightarrow c$
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- $all\_reached\_upto(c)$
  - $d \rightarrow c$
  - $a \rightarrow b$

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Modularization

- Do not reinvent the wheel: reuse solutions to basic problems.
- Program Splitting: syntactic means to
  - develop larger programs by combining parts, and to
  - compute answer sets layer by layer (by composition).
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**Program splitting**

Suppose (ground) $P$ can be split into $P = P_1 \cup P_2$, such that every atom $A$ that occurs in $P_1$ ("bottom part") occurs in $P_2$ ("top part") only in rule bodies (i.e., $A$ is "defined" entirely in $P_1$). Then

$$AS(P) = \bigcup_{M \in AS(P_1)} AS(P_2 \cup M).$$

$AS(P) = \text{set of answer sets of } P$

- **Examples**: "stratified" programs, like GCD; guess&check
- Versions of ASP with modules, macros etc. are available
Answer Set Solvers
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(see also http://en.wikipedia.org/wiki/Answer_set_programming)

ASPERIX  www.info.univ-angers.fr/pub/claire/asperix/
ASSAT    assat.cs.ust.hk/
CLASP 1  potassco.sourceforge.net/#clasp/
CMODELS www.cs.utexas.edu/users/tag/cmodels/
DLV 2   www.dbai.tuwien.ac.at/proj/dlv/
ASPTOOLS research.ics.aalto.fi/software/asp/
ME-ASP  www.mat.unical.it/ricca/me-asp/
OMIGA   www.kr.tuwien.ac.at/research/systems/omiga
SMODELS www.tcs.hut.fi/Software/smodels/
WASP    www.mat.unical.it/ricca/wasp/
XASP    xsb.sourceforge.net/,
        distributed with XSB

1 + CLASP\textsuperscript{D}, CLINGO, CLINGCON etc. (http://potassco.sourceforge.net/)
2 + DLV\textsuperscript{HEX}, DLV\textsuperscript{DB}, DLT, DLV-COMPLEX, ONTO-DLV etc.
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**ASPERIX**  [www.info.univ-angers.fr/pub/claire/asperix/](http://www.info.univ-angers.fr/pub/claire/asperix/)
**ASSAT**    [assat.cs.ust.hk/](http://assat.cs.ust.hk/)
**CLASP**  ^1 [potassco.sourceforge.net/#clasp/](http://potassco.sourceforge.net/#clasp/)
**CMODELS** [www.cs.utexas.edu/users/tag/cmodels/](http://www.cs.utexas.edu/users/tag/cmodels/)
**DLV** ^2 [www.dbai.tuwien.ac.at/proj/dlv/](http://www.dbai.tuwien.ac.at/proj/dlv/)
**ASPTOOLS** [research.ics.aalto.fi/software/asp/](http://research.ics.aalto.fi/software/asp/)
**ME-ASP** [www.mat.unical.it/ricca/me-asp/](http://www.mat.unical.it/ricca/me-asp/)
**OMIGA** [www.kr.tuwien.ac.at/research/systems/omiga](http://www.kr.tuwien.ac.at/research/systems/omiga)
**SMODELS** [www.tcs.hut.fi/Software/smodels/](http://www.tcs.hut.fi/Software/smodels/)
**WASP** [www.mat.unical.it/ricca/wasp/](http://www.mat.unical.it/ricca/wasp/)
**XASP** [xsb.sourceforge.net/](http://xsb.sourceforge.net/), distributed with XSB

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1 + **CLASP**^D, **CLINGO**, **CLINGCON** etc. ([http://potassco.sourceforge.net/](http://potassco.sourceforge.net/))
2 + **DLV**^DB, **DLT**, **DLV**-**COMPLEX**, **ONTO-DLV** etc.

---

- Many ASP solvers are available (mostly function-free programs)
- **clasp** was first ASP solver competitive to top SAT solvers
- another state-of-the-art solver is **dlv**
Evaluation Approaches

- Different methods and evaluation approaches:
  - resolution-based
  - forward chaining
  - lazy grounding AsperiX, Omiga
  - translation-based (see below)
  - meta-interpretation
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Predominant solver approach

intelligent grounding + model search (solving)
2-Level Architecture

1. **Intelligent grounding**

   Given a program $P$, generate a (subset) of $grnd(P)$ that has the same models
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   More complicated than in SAT/CSP Solving:
   - candidate generation (classical model)
   - model checking (stability, foundedness!)
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   More complicated than in SAT/CSP Solving:
   - candidate generation (classical model)
   - model checking (stability, foundedness!)
     - for SAT, model checking is feasible in logarithmic space
     - for normal propositional programs, model checking is PTime-complete
     - for disjunctive propositional programs, model checking is co-NP-complete
Intelligent Grounding

- Grounding is a hard problem

\[
\text{bit}(0). \quad \text{bit}(1). \\
p(X_1, \ldots, X_n) \leftarrow \text{bit}(X_1), \ldots, \text{bit}(X_n).
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  • dlv’s grounder (built-in);
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  - domain-restriction (smodels)
  - deductive db methods: semi-naive evaluation, magic sets, ...
Solving: Model Search

- Applied to ground programs.
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- Early solvers (e.g. smodels, dlv): native methods
  - inspired by Davis-Putnam-Logemann Loveland (DPLL) for SAT
    - 3 basic operations: decision, propagate, backtrack
  - special propagation for ASP, e.g.,
    - dlv: must-be-true propagation (supportedness), ...

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\begin{align*}
a & : \neg b. \\
b & : \neg a. \\
c & : \neg c, a. \\
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a:− not b.
b:− not a.
c:− not c, a.
not aa
c not c
not bb not b
c not c
b

• important: heuristics (which atom/rule is next?)
• chronological backtrack-search improved by backjumping and look-back heuristics
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- Stability check: unfounded sets, reductions to UNSAT (disj. ASP)
ASP Solving Approaches

- Predominant to date: modern SAT techniques (*clause driven conflict learning, CDCL*)
- Export of techniques from ASP to SAT (optimization issues)
- **Genuine conflict-driven ASP solvers**
  - clasp, wasp

- **Translation based solving**: to
  - SAT: assat, cmodels, lp2sat (multiple SAT solver calls)
  - SAT modulo theories (SMT) aspmt
  - Mixed Integer Programming (CPLEX backend)

- **Cross translation**: intermediate format to ease cross translation
  - SAT modulo acyclicity
    - interconnect graph based constraints with clausal constraints
    - can postpone choice of the target format to last step solver.

- **Portfolio solvers**
  - claspfolio: combines variants of clasp
  - ME-ASP: multi-engine portfolio ASP solver
Summary

1. More about logic programs
   - Strong negation, disjunction

2. The answer set programming paradigm
   - The guess and check methodology

3. Programming techniques
   - Element selection
   - Use of double negation
   - Iteration over a set
   - Modularization

4. Answer set solvers
   - Intelligent grounding and solving
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