Bejeweled, Candy Crush and other Match-Three Games are (NP-)Hard

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Given a game $A$ that is engaging to play, it is often the case that each problem $B$ in the complexity class $NP$ (or in $PSPACE$) can be transformed (i.e. reduced) in polynomial time to an instance $m$ of $A$ such that you can solve $B$ by playing $m$. (Kendall ’08, Hearn ’09, Forišek ’10, Viglietta ’12).
Casual Games

New public of casual players looking for "soft" gaming:

- easy to play
  ↓
  simple rules;
- engaging
  ↓
  complex structure.
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A big class of Casual Games:
Match Three Games.
The Match-Three Game: Bejeweled

- Played on a $8 \times 8$ grid filled with gems of 6 types.
- Three or more vertically or horizontally aligned and contiguous gems are said to form a match.
The Match-Three Game: Bejeweled

The game mechanic:

- **Moving phase:** The player swap two (vertically or horizontally) adjacent gems provided that doing so will create a match, then the popping phase take place. If it is not possible to make such a move, the game is over.

- **Popping phase:** As there is any match, the matched gems pop simultaneously and the remaining gems fall filling the empty space; when there are no more matches, the moving phase take place.
The Reduction - Preliminaries

General Bejeweled
Bejeweled played on a $n \times n$ grid (still 6 gems only!).

Main decision problem
Can we pop a specific gem?
Implies:
- Can we get a score of at least $s$?
- Can we get a score of at least $s$ in less than $k$ moves?
- Can we cause at least $p$ gems to pop?
- Can we play for at least $t$ turns?
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Main difficulty
Pops affects everything above.
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Strategy
- Preserve structure by modularity
- Make swaps irreversible
The Reduction from 1-in-3 Positive SAT

Instance
- $n$ variables $x_1, \ldots, x_n$;
- $m$ clauses with at most 3 variables each.

Goal
An assignment that satisfies all clauses by setting exactly one variable to true for each of them.

Example
Instance:
$$(x_1 \lor x_2 \lor x_3) \land (x_2 \lor x_4 \lor x_5)$$

Bad assignment:
$$\begin{align*}
x_1, x_2 & \leftarrow true \\
x_3, x_4, x_5 & \leftarrow false
\end{align*}$$

Good assignment:
$$\begin{align*}
x_2 & \leftarrow true \\
x_1, x_3, x_4, x_5 & \leftarrow false
\end{align*}$$
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1-in-3 positive SAT embedding in Bejeweled:
The Gadgets - (Partial) Overview

Choice wire

Sequencer

Goal wire
Choice wire
The choice wire is either activated or skipped by swapping a gem in the choice activator. The activation shift some clause column by two rows, constructing a truth assignment.

Sequencer

Goal wire
The Gadgets - (Partial) Overview

Choice wire

Sequencer

The sequencer make possible only to swap gems placed in the choice activator (from the topmost to the bottommost).

Goal wire
The Gadgets - (Partial) Overview

Choice wire

Sequencer

Goal wire

Toward the end of the game, an activating gem ends up in the check point. A sequence of swaps/pops reaching the goal gem along the goal wire will take place if and only if all clause are satisfied.
In the filling pattern no match can be formed, even if the column fall by different amounts.
The Gadgets - Filler and Sequencer

In the filling pattern no match can be formed, even if the column fall by different amounts.

The sequencer controls the order in which the other gadgets are activated.
Each choice wire corresponds to a variable $x_i$; if activated, it sets $x_i$ to true by making all clauses containing $x_i$ fall by some number $l \equiv 2 \pmod{6}$ (while others fall by multiples of 6).
Once we have traversed all the variable gadgets, the sequencer gives the "control" to the goal wire.

The goal wire ensure that the goal gem can be reached from the check point only if the choice wire activations result in a satisfying assignment.
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Open Questions

What about the complexity of Bejeweled with only 5 kind of gems (or even less)?
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Is Bejeweled PSPACE-complete?