NRGA: Gravitational Approach for Non-Rigid Point Set Registration

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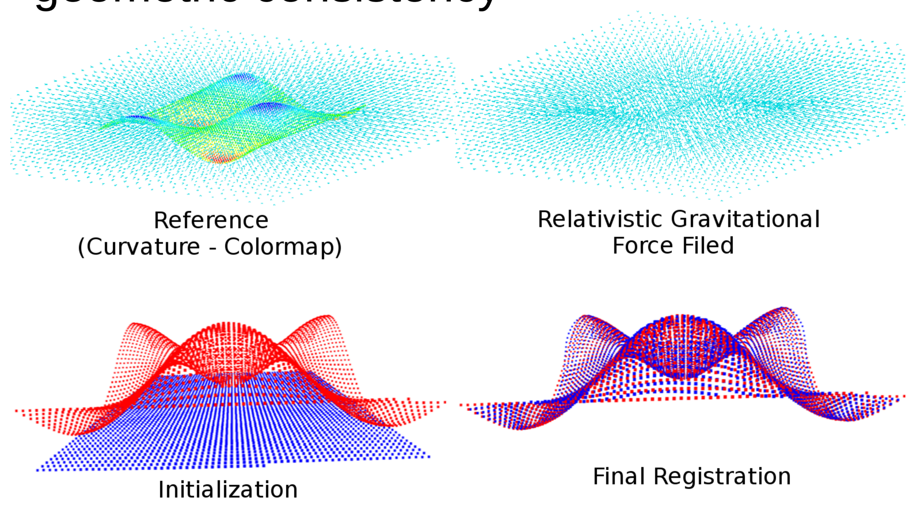
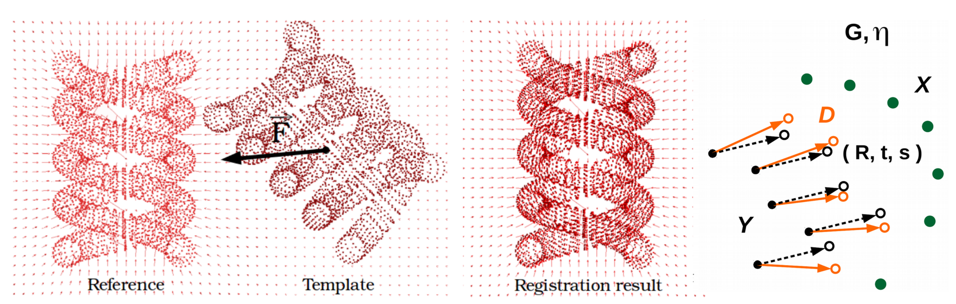
Overview

- **NRGA** is physics-based non-rigid point set registration method which estimates **correspondences** and **transformations** between *'template'* and *'reference'* point sets
- **Distributed** and **collision-less N-body** simulation is performed on several regions of point sets; the attractive gravitational force has relativistic effect; the **Coherent Collective Motion** operator [3] regularizes distributed position updates
- Motivation: a paralleizable non-rigid point set registration method which is robust against missing input data and noise; better balances correspondence accuracy and geometric consistency

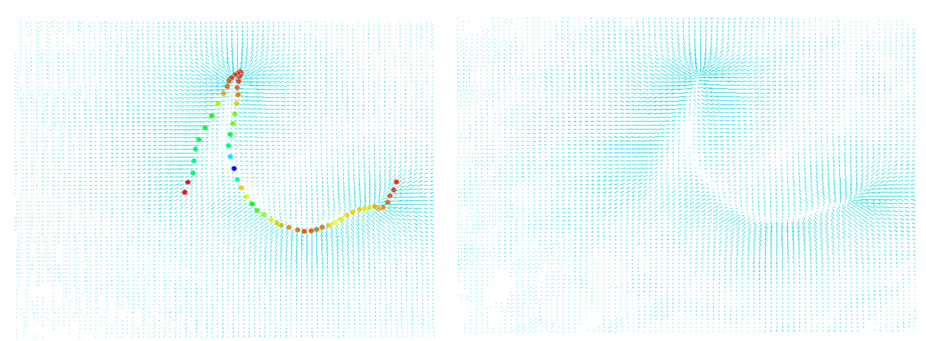


Related Work

NRGA is related to its **rigid** counterpart **GA** [1] which estimates a single global rigid transformation parameter using **Absolute Orientation** method or **Kabsch Algorithm** [4]



The gravitational force function used in [1] is altered in NRGA by relativistic effect as in [2]



References

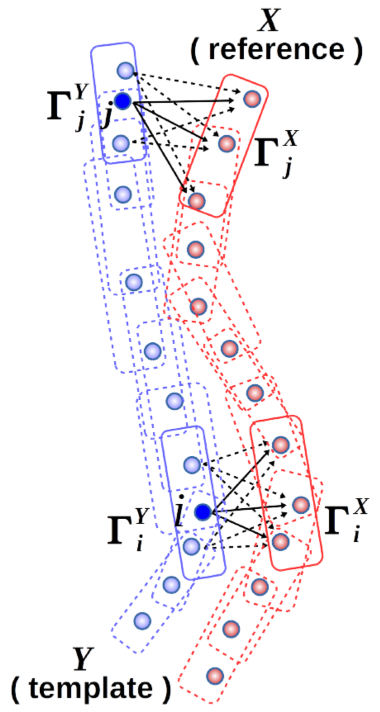
[1] V. Golyanik, S. A. Ali, and D. Stricker. Gravitational approach for point set registration. In *Computer Vision and Pattern Recognition (CVPR)*, 2016.
[2] F. Diacu. The classical n-body problem in the context of curved space. *Canadian Journal of Mathematics*, 69:790 – 806, 2017.
[3] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet. Novel type of phase transition in a system of self-driven particles. *Physical Review Letters (PRL)*, 75:1226 – 1229, 1995.
[4] W. Kabsch. A solution for the best rotation to relate two sets of vectors. *Acta Crystallographica Section A,* 32(5):922 – 923, 1976.

Proposed NRGA

Distributed N-Body Simulation

Gravitational Potential Energy is the Distance Transform function ⇒

$$\arg\min_{\mathbf{T}} \sum_{i=1}^{M} \sum_{j=1}^{|\mathcal{N}(i)|} \omega_{ij} \left(d\left(\mathcal{T}\left(\mathbf{Y}_i, \mathbf{T}_i\right) - \mathbf{X}_j\right)\right)^{-1}$$



i) Locally Multiply-Linked N-Body Interaction

$$\mathbf{F}^a_{p\in\Gamma^{\mathbf{Y}}_k} = -Gm_p \sum_{q\in\Gamma^{\mathbf{X}}_k} \frac{m_q\left(\mathbf{r}_p\left(1-\frac{\kappa_q\mathbf{r}^2_{pq}}{2}\right)-\mathbf{r}_q\right)}{\left(\|\mathbf{r}_p-\mathbf{r}_q\|^2+\epsilon^2\right)^{\frac{3}{2}}\left(1-\frac{\kappa_q\mathbf{r}^2_{pq}}{4}\right)^{\frac{3}{2}}}$$

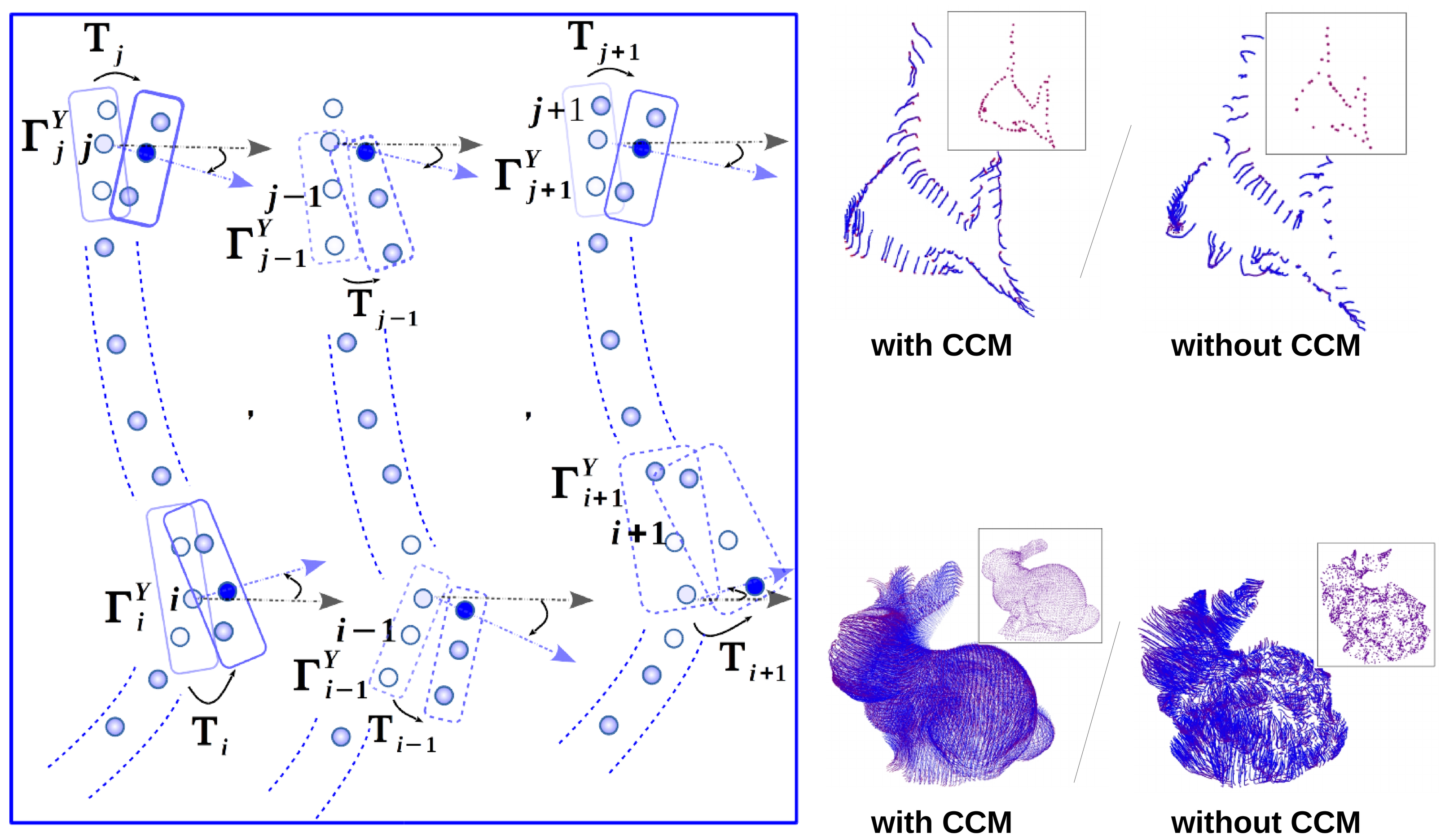
Relativistic Gravitation Force (RGF) field parameterized by per-point **Gaussian Curvature** of *'reference'*

$$\mathscr{F}^t_k = \left[\ldots, \mathbf{F}^a_p - \eta v^t_p, \ldots\right]^T$$
$$\mathscr{V}^t_k = \left[\ldots, v^t_p, \ldots\right]$$

$$\mathscr{V}^{t+1}_k = \mathscr{V}^t_k + \Delta t\, \mathscr{F}^t_k \circ \left[\ldots, m_p^{-1}, \ldots\right]^T$$
$$\mathscr{D}^{t+1}_k = \Delta t\, \mathscr{V}^{t+1}_k$$

⇐ Motion Update using Euler time integration

Coherent Collective Motion (CCM)



ii) Ξ (Coherent Collective Motion)

- The CCM is a locally-aware global topology preserving operator
- The nature of collective particle motion is similar to **Smoothed Particle Hydrodynamics**

$$\Xi(v^t_i) = |v^t_i|(\vartheta)\left(\sum_{k\in\mathbf{\Psi}_i} v^t_k\right)$$

⇐ Gives directional Coherency

$$\Xi(\mathbf{T}_i) = \left\{(\vartheta)\left(\sum_{k\in\mathbf{\Psi}_i}\mathbf{R}_k\right), (\vartheta)\left(\sum_{k\in\mathbf{\Psi}_i}\mathbf{t}_k\right), (\vartheta)\left(\sum_{k\in\mathbf{\Psi}_i}\mathbf{s}_k\right)\right\}$$

⇐ Gives Orientation Coherency

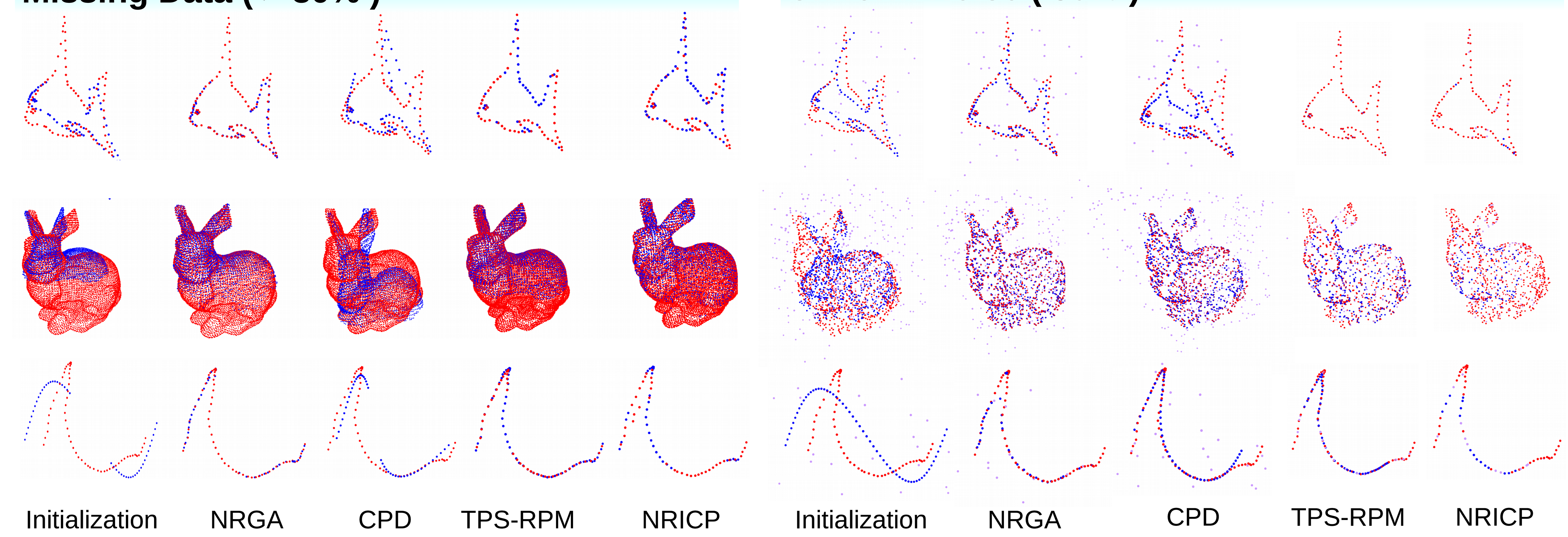
Complexity Analysis

$$\mathcal{O}(\underbrace{\xi}_{\text{iterations}}(\underbrace{M\log M}_{\text{k-d tree}} + \underbrace{M\rho M\,\rho N}_{\text{NRGA: N-body}} + \underbrace{M}_{\text{NRGA: CCM}})) = \mathcal{O}\left(M^2N\right)$$

- Distributed N-body simulation on the regions can be solved using Fast Multi-Pole Method (FMM) or Barnes-Hut algorithm

Results

Missing Data (> 30%)   Uniform Noise (30%)



Expression Matching

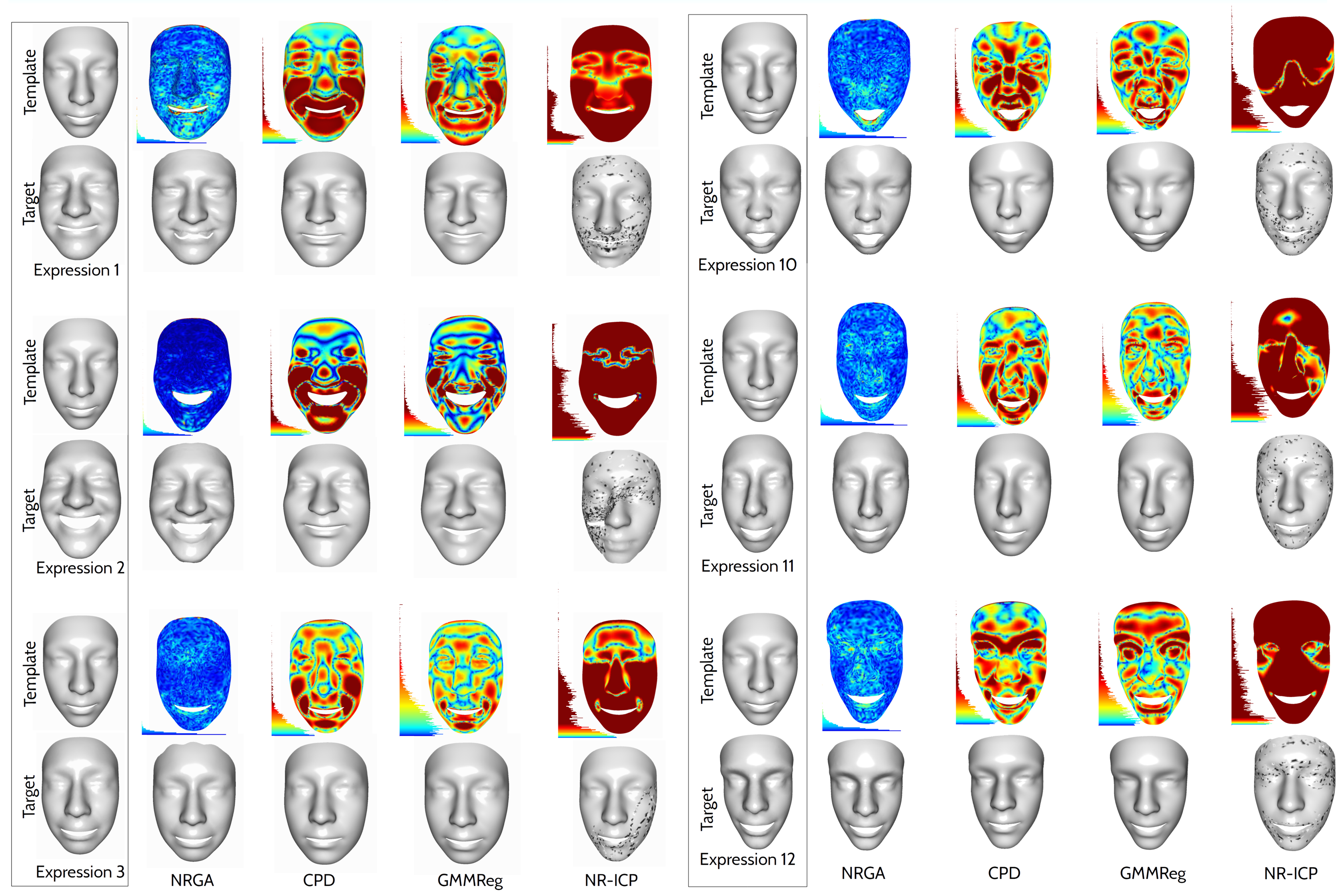
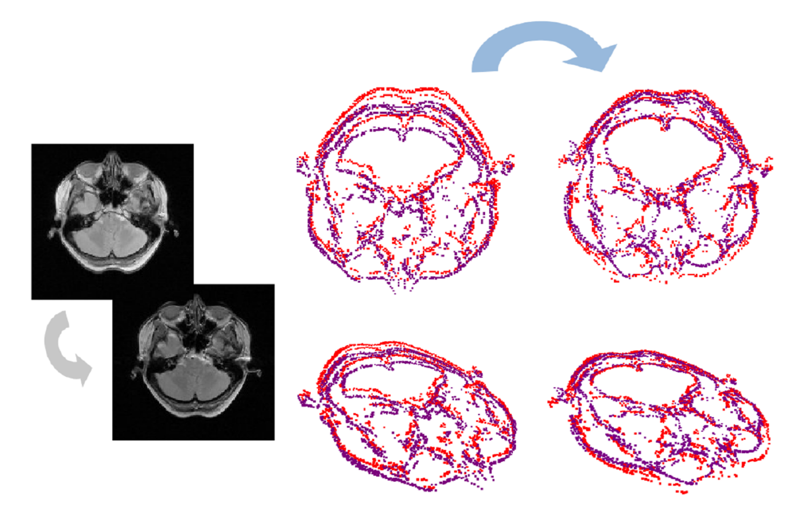
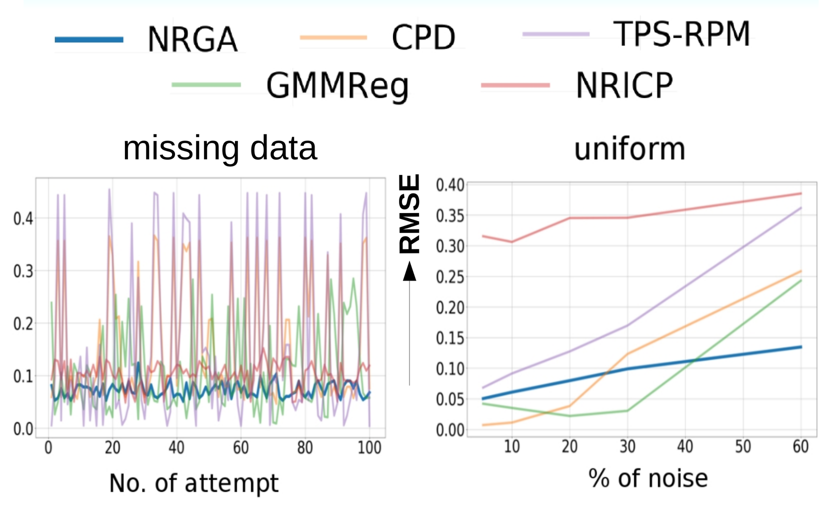


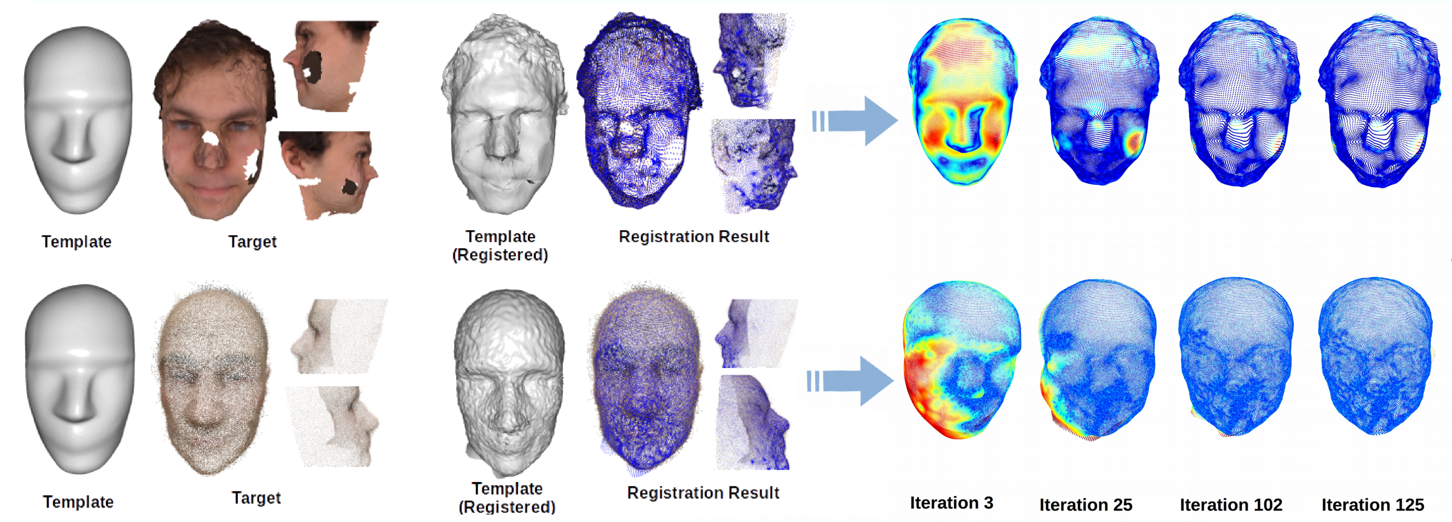
Image Registration

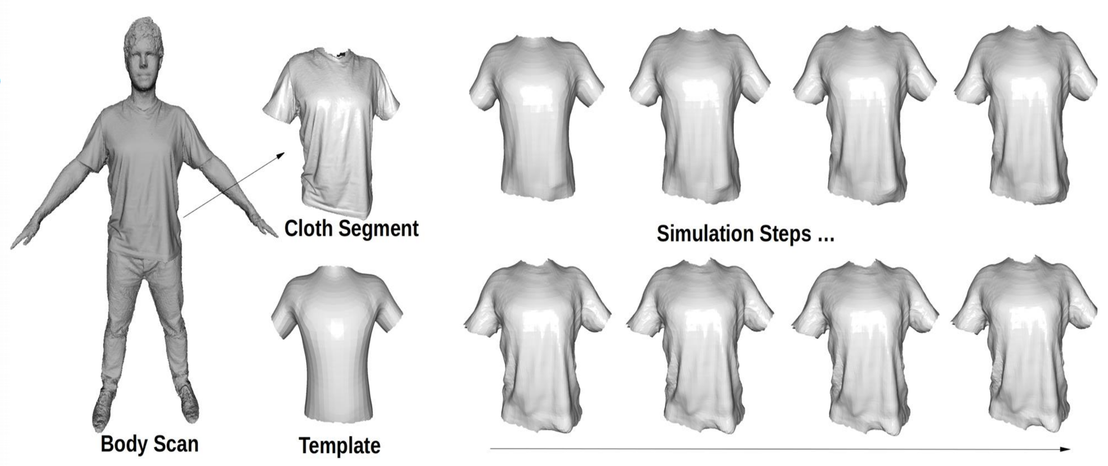


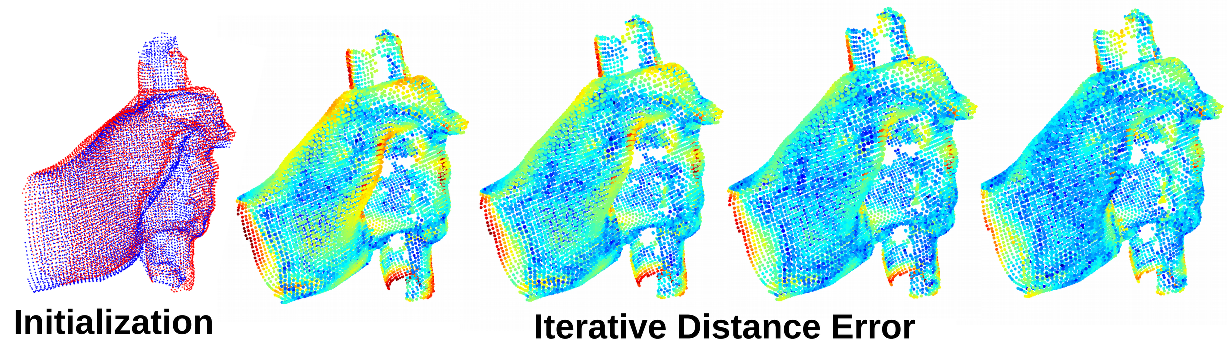
Performance Evaluation



Real Dataset (Template → Scan Fitting)







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