Optimising for Scale in Globally Multiply-Linked Gravitational Point Set Registration Leads to Singularities

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**GRavitational METHODS FOR POINT SET ALIGNMENT**

- **Rigid Gravitational Approach (Second-Order ODEs)** [3]
- **Modified Rigid Gravitational Approach with Shape Descriptors** [4]
- **Non-Rigid Gravitational Approach (Second-Order ODEs)** [5]

**ANALYSIS OF THE ENERGY LANDSCAPE WITH CALCULUS**

- **Unit Circle** $S^1$
  \[ E_a = \int_0^1 (1 - \cos \theta) \, d\theta \]
- **Unit Sphere** $S^2$
  \[ E_a = \frac{1}{2} \int_0^1 \sin \theta \, d\theta \]

**WHAT IS A SCALE SINGULARITY?**

- A singularity is a state when a template collapses to a single point, and scale $\sim 0$.
- A singularity can arise in 7DoF point set alignment approaches due to numerical reasons (e.g., it can be observed in CPD [2]).

**ELLIPITIC INTEGRALS**

... of the second kind in the Legendre form are integrals of type

\[ E(k, \phi) = \int_0^\phi \sqrt{1 - \sin^2 \theta} \, d\theta \]

- Elliptic integrals arise in the study of the arc length problem for ellipses.
- As a rule, they cannot be simplified and analytically evaluated.

**APPENDIX**

\[ I_1 = \int_0^\pi \sqrt{1 + 2 \cos \phi \sin^2 \theta} \, d\phi \]

**REFERENCES**

2. A. Myronenko and X. Song. Point Set Registration: Coherent Point Drift. TPAMI, 2010.