A Quantum Computational Approach to Correspondence Problems on Point Sets

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http://gvv.mpi-inf.mpg.de/projects/QA/

Motivation and Contributions

- Quantum computers are already used to solve difficult combinatorial optimisation problems, and they can be useful in computer vision.
- We show that the classical problem of finding optimal transformation and correspondence between two point sets can be efficiently solved on a quantum computer. The quantum annealing time is constant and does not depend on the size of the inputs in a given dimension.
- We show how to formulate point set alignment as a quadratic binary constrained optimisation problem (QUBOP) of the form

\[
\arg \min_{P} \langle Q | P \rangle
\]

and overcome the difficulty of rotation parameterisation.

What is a Quantum Computer?

- Quantum computers take advantage of quantum mechanical effects, i.e., quantum superposition, entanglement and tunneling [1-2].
- They can perform all operations which classical computers can perform, plus multiple algorithms which have lower complexity class compared to their classical counterparts (e.g., prime number factorisation [3]).
- Quantum computers can be classified into two models – gate model and quantum annealers.

Adiabatic Quantum Annealers

- The accuracy of QA under random initial misalignments.
- The final QUBOP and \( P \):

\[
\arg \min_{P} \langle Q | P \rangle \quad P = \Phi^T \Phi
\]

State Preparation:

- Transformation Estimation

Reference point set: \( \{ x \}_N \in \mathbb{R}^{2 \times N} \)
- Template point set:

\[
S = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \end{bmatrix}, \quad \text{(MD)}
\]

Cayley-Hamilton Theorem:

\[
S^2 + \Phi^T \Phi = 0 \quad (\text{MD})
\]

Exponential map for \( R \) with power series:

\[
R = \exp(S) = \left( I + \frac{\sin(\theta)}{\theta} S + \frac{1 - \cos(\theta)}{\theta^2} S^2 \right)
\]

For each template point:

\[
\Phi_k = \begin{bmatrix} -Q_1 y_1 & -Q_2 y_1 & \ldots & -Q_N y_1 \\
-Q_1 x_1 & -Q_2 x_1 & \ldots & -Q_N x_1 \\
\vdots & \vdots & \ddots & \vdots \\
-Q_1 x_N & -Q_2 x_N & \ldots & -Q_N x_N \\
\end{bmatrix}
\]

Unembedding

Unembedding is the decoding of the solution to QUBOP to the basis states of particles (reference + template) with local point linking:

\[
R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Experiments on D-WAVE 2000Q (2D)

K = 10 (top row), 30 (bottom row)

Minor-Embedding and Quantum Annealing

The region with the ground state of the embedded problem

Tests on CPU Sampler and Spectral Gap Analysis (2D)

The metrics as the functions of A/\( \theta \), the size of the point interaction region parameterised by K/\( \theta \), the angle of initial misalignment \( \theta \) on the template noise ratio.

Examples of \( P = \Phi^T \Phi \) in QA.

Complexity of state preparation:

\[
O(2^{CN}) \quad (\text{transformation estimation})
\]

QUBOP of the form:

\[
\min_{R} \langle R | R \rangle
\]

Error metrics:

\[
\epsilon_{\text{misalignment}} = \frac{\langle R | R \rangle_{\text{approx}} - \langle R | R \rangle_{\text{exact}}}{\langle R | R \rangle_{\text{exact}}}
\]

\[\epsilon_{\text{transformation}} = \frac{\langle R | R \rangle_{\text{approx}} - \langle R | R \rangle_{\text{exact}}}{\langle R | R \rangle_{\text{exact}}}
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References


Quantum notions and their classical counterparts.

The accuracy of QA under random initial misalignments.

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Graphical Notation

Condition

\[ P = \Phi^T \Phi \]

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