Distributed Set Reachability

**Definition.** Given a directed graph \( G(V, E) \), a \( k \)-vertex-disjoint partitioning of \( G \) as \( G = \{G_1, G_2, \ldots, G_k\} \), a source set \( S \subseteq V \), and a target set \( T \subseteq V \), a DSR query \( S \leadsto T \) returns all reachable pairs, i.e.,

\[
S \leadsto T = \{(s, t) | s \in S \text{ and } t \in T\}
\]

Example: \( S = \{a, d, g\} \) and \( T = \{l, q\} \).

\[
S \leadsto T = \{(a, l), (a, q), (d, q), (g, l), (g, q)\}
\]

**Related work.** Distributed reachability [Fan et al. [1]], centralized multi-source multi-target reachability [Gao et al. [2], Then et al. [3]].

**Our Approach: Indexing**

**Query: \( S \leadsto T \)**

**Step 0.** Partition \( S, T \) into \( \{S_1 \leadsto T_1, \ldots, S_k \leadsto T_k\} \)

For e.g.,

\[
\begin{align*}
G_1 & : \{a, d\} \leadsto \emptyset \\
G_2 & : \{g\} \leadsto \{l\} \\
G_3 & : \emptyset \leadsto \{p\}
\end{align*}
\]

**Step 1.** Compute local reachability \( S_i \leadsto T_i \) and \( S_i \leadsto F_j \), \( \forall j \neq i \)

Example:

\[
\begin{align*}
S_i & : T_i : \{a, d\} \leadsto \emptyset \\
S_i & : F_i : \{a, d\} \leadsto \{c, g, h, m, n\} \\
& \quad \{g\} \leadsto \{f, m, n\} \leadsto \emptyset \leadsto \{f, c, g, h\}
\end{align*}
\]

**Step 2.** Communicate reachability \( S_i \leadsto F_i \) to other partitions

Example:

\[
\begin{align*}
1 & \rightarrow 2 : \{(a, [a, d]), (g, [a, d]), (b, [a, d])\} \rightarrow 1 : \{(f, [g])\} \rightarrow 3 : \emptyset \rightarrow 2 : \{(f, m, n), (g, [g])\} \rightarrow 3 : \emptyset \\
1 & \rightarrow 3 : \{(m, [a, d]), (n, [a, d])\} \rightarrow 2 : \{(m, [g]), (n, [g])\} \rightarrow 3 : \emptyset
\end{align*}
\]

**Step 3.** Compute local reachability from boundaries to targets

Example:

\[
\begin{align*}
G_1 & : \{f\} \leadsto \emptyset \\
G_2 & : \{c, g, h\} \leadsto \{l\} \\
G_3 & : \{m, n\} \leadsto \{p\}
\end{align*}
\]

**References**