Tutorials for “Automated Deduction for Equational Logic”
Exercise sheet 4

Exercise 4.1:
The proof for the following proposition remains to be done from the lecture of May 26 (p. 4):

The relation $\preceq$ ("more general") is defined by $\sigma \preceq \tau$ iff $\sigma \circ \rho = \tau$ for some $\rho$.

Proposition:
(i) $\preceq$ is a quasi-ordering on substitutions (i.e. reflexive and transitive).
(ii) If $\sigma \preceq \tau$ and $\tau \preceq \sigma$, then there is a bijective variable renaming $\rho$ such that $\sigma \rho = \tau$.

Exercise 4.2:
Given the substitutions $\sigma = \{x \mapsto f(y)\}$ and $\tau = \{x \mapsto f(z)\}$, show that they are $\preceq$-incomparable: Neither $\sigma \preceq \tau$ nor $\tau \preceq \sigma$ hold.

Exercise 4.3:
Show that $s \hookrightarrow_R t$ implies $s\sigma \hookrightarrow_R t\sigma$ for any term rewrite system $R$, substitution $\sigma$ and terms $s$ and $t$.

Exercise 4.4:
Prove inductively (some of) the following elementary properties of replacements (cf. lecture of April 28, p. 10):

(i) Embedding: $s[t]_p/pq = t/q$
(ii) Associativity: $(s[t]_p)[t']_q = s[t[t']_q]_p$
(iii) Distributivity: $s[t]_p/p = (s/p)[t]_q$
(iv) Dominance: $s[t]_p[q] = s[t']_q$
(v) Persistency: $s[t]_p/q = s/q$ for $p \parallel q$
(vi) Commutativity: $s[t]_p[q] = s[t']_q[t]_p$ for $p \parallel q$

Exercise 4.5:
Consider the signature $\Sigma = \{\{f/2,g/1,h/2,k/3,a/0,b/0\}, \emptyset\}$. Check for the following term
pairs \((s, t)\) whether \(s\) and \(t\) are unifiable, and whether there is a match from the \(s\) to \(t\) or a match from \(t\) to \(s\).

\[
\begin{array}{ccc}
  s & t \\
a) & f(x, g(b)) & f(a, y) \\
b) & f(h(x, b), y) & f(h(a, y), x) \\
c) & f(x, y) & f(h(x, b), y) \\
d) & k(x, h(x, y), h(y, h(x, y))) & k(x, w, h(z, w)) \\
e) & k(x, h(x, y), h(y, h(x, y))) & k(x, w, h(w, z)) \\
f) & k(a, x, g(h(y, a))) & k(z, g(z), g(w)) \\
\end{array}
\]

**Exercise 4.6:**

Can you think of three terms such that each two of them are unifiable, but no substitution makes all three of them equal?

Put your solution into the mail box at the door of room 617 in the MPI building (46.1) before the upcoming lecture, or submit it to the lecturer at the beginning of that lecture.

Note: In case of group work, write the names of all group members (not more than three!) on a single solution sheet. Do not submit several identical solution sheets.