

# Low-Cost Preference Judgment via Ties

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## Background

Two rankings from system  $S_1$  and  $S_2$ :

$$S_1: d_3, d_1, d_2$$

$$S_2: d_5, d_4, d_2, d_1$$

Graded judgments on five documents:

$$d_1: 0, d_2: 1, d_3: 1, d_4: 1, d_5: 2$$

Preference judgments:

$$d_1 < d_2, d_1 < d_3, \dots, d_3 \sim d_4, d_3 < d_5, d_4 < d_5$$

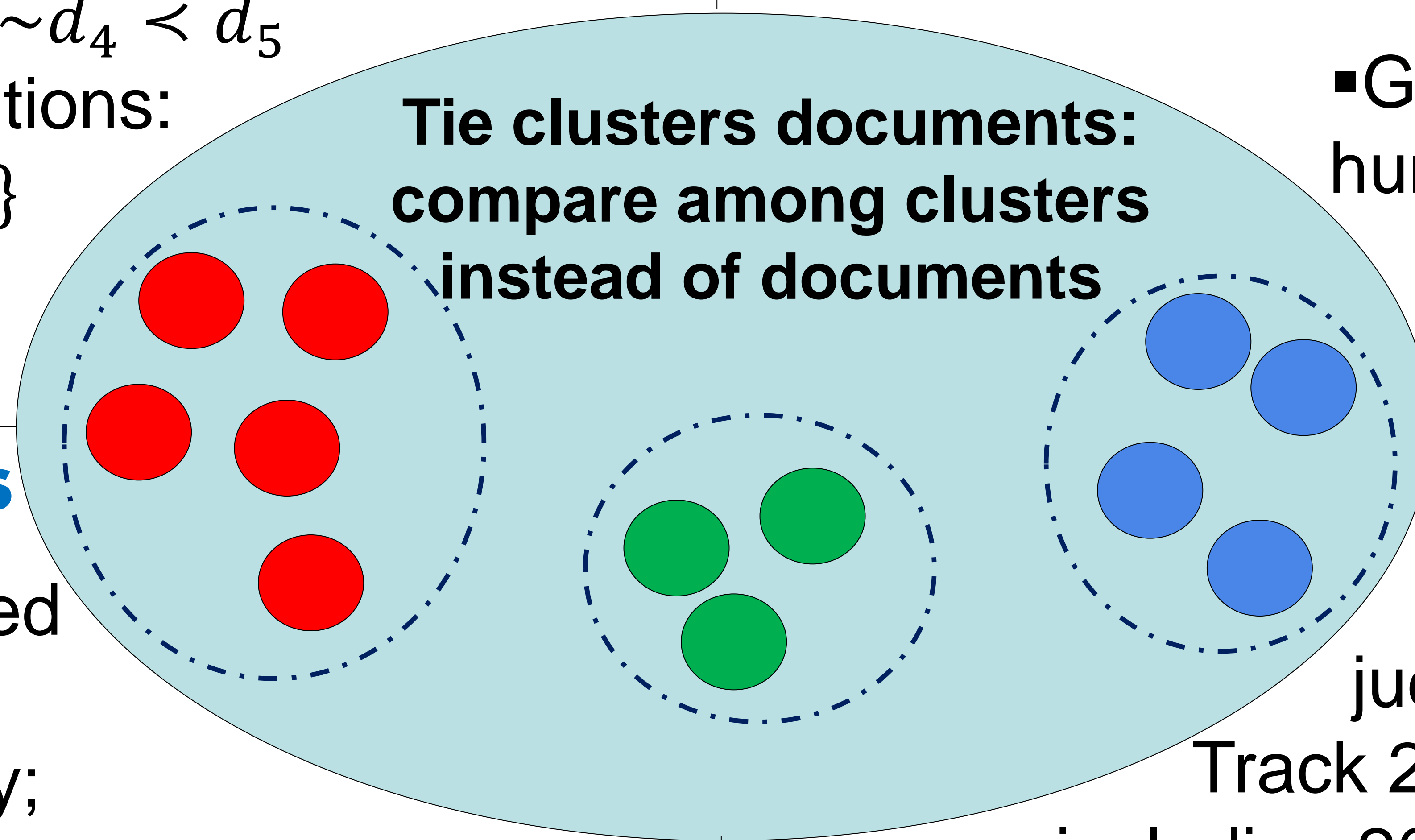
A ground-truth document ranking:

$$d_1 < d_2 \sim d_3 \sim d_4 < d_5$$

Represented as tie partitions:

$$\{d_1\} < \{d_2, d_3, d_4\} < \{d_5\}$$

**Tie clusters documents:  
compare among clusters  
instead of documents**



## Motivation

- Preference judgments achieve better quality;

- $O(N_d^2)$  judgments due to quadratic nature;

- Assume transitivity reduces the number of judgments to  $O(N_d \log N_d)$ , which is still too expensive;

- Ground-truth ranking of hundred documents should include ties.

## Theoretical Analysis

- Analysis with randomized Quick-Sort algorithm;
- Assume strict transitivity;
- $N_d$ : the number of documents;
- $N_t$ : the number of tie partitions;
- Total number of judgments equals the sum of tie and non-tie judgments

We demonstrate that:

$$E(N_{jud}) = E(N_{ntie}) + E(N_{tie})$$

$$< 2N_d \log N_t + N_d$$

Beyond Quick-Sort, by clustering tied documents simultaneously:

$$E(N_{jud}) = E(N_{ntie}) + E(N_{tie})$$

$$< 2N_t \log N_t + N_d$$

## Empirical Analysis

**Dataset:** graded judgments from TREC Web Track 2011–2014 for ad-hoc task including 200 queries.

### Number of judgments required to obtain ground truth.

