Algorithms and Data Structures **Summer 2008**
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Exercise 2

Motivation

The first part of these exercises are to remind you about asymptotic notation and some of its intricacies.
The second part is to reinforce the results about divide-and-conquer recursive relations that you saw in class.

1. Order the following functions by their asymptotic growth rate. How would you prove that your ordering is indeed correct?

$$3^{n/logn} \quad n\log n \quad n^{3/2} \quad n\cdot 2^{\sqrt{\log n}} \quad n^{1-\varepsilon} \quad n/\log\log n \quad n^{3/2}\log n \quad 2^{n/2^{\log^{1+\varepsilon} n}}$$

   Here all logarithms are with respect to base 2, and ε stands for arbitrary small positive real number, say $0 \leq \varepsilon \leq 0.1$.

2. Prove that the following equivalences hold for all functions f and g in AEP (almost everywhere positive):

$$g \in \Theta(f) \iff f \in \Theta(g) \iff O(f) = O(g) \iff o(f) = o(g)$$

3. Give good asymptotic upper bounds for the following functions defined by divide-and-conquer type inequalities. (Base cases are omitted.)

$$f(n) \leq 3\cdot f(n/5) + c\cdot n$$

$$g(n) \leq 5\cdot g(n/3) + c\cdot n$$
$$h(n) \leq 4\cdot h(n/2) + n^2$$
$$k(n) \leq 4\cdot k(n/2) + n^2/\log n$$

Have fun with the solution!