Motivation

We continue our study of cycle bases.

NP-Completeness

Three-dimensional Matching Let $X, Y, Z$ be disjoint sets of equal cardinality and let $T \subseteq X \times Y \times Z$, i.e., each element of $T$ is a triple whose first component is in $X$, second component is in $Y$, and third component belongs to $Z$. Is there a subset $M \subseteq T$ such that no two elements of $M$ agree in any coordinate and such that $|M| = |X|$.

Establish NP-completeness.

Hint: Reduce 3-SAT to three dimensional matching.

Exact Set Cover Let $t$ be an integer and let $S = \{S_1, \ldots, S_k\}$ be a family of 3-element subsets of $T = [1, \ldots, t]$. Is there a subfamily $S' \subseteq S$ of pairwise disjoint sets such that any element of $T$ belongs to exactly one member of the subfamily.

Establish NP-completeness.

Addition and Multiplication in 2-Complement

Let $k$ be an integer. For digits $d_\ell \in \{0, 1\}$, $0 \leq \ell \leq k$, let

$$D = \sum_{0 \leq \ell < k} d_\ell 2^\ell - d_k 2^k.$$

- Which numbers $D$ can be represented in this way.

- Show that the following algorithm can be used to add two such numbers.
  
  - Add the numbers as usual binary numbers, i.e., ignore the fact that $d_k$ contributes $-d_k 2^k$ to $D$.
  
  - If there is a carry into position $k + 1$, then declare overflow, i.e., the result cannot be represented. Otherwise, return the result.

- Show that the following algorithm can be used to multiply two such numbers.
– Multiply the numbers as usual binary numbers, i.e., ignore the special interpretation of position $k$.

– Return the result, if ??? (this is for you to fill in).

**Remark:** You probably learned the solution to this exercise in your introductory course on machine organization and computer architecture.

Have fun with the solution!