Motivation

We fill in some details of de Pina’s algorithm for minimum cycle basis.

de Pina’s Algorithm

de Pina suggested the following algorithm.

\[ B := \emptyset \]

\[ \text{while } |B| < m - (n - 1) \text{ do} \]

\[ \text{compute a non-zero } S \in k^E \text{ such that } \langle C, S \rangle = 0 \text{ for all } C \in B. \]

\[ \text{compute a minimum weight (isometric) circuit } C \text{ with } \langle C, S \rangle \neq 0. \]

\[ \text{add } C \text{ to } B. \]

end while

Correctness: Show that both versions of the algorithm (with and without the adjective isometric) computes a minimum weight \( k \)-basis.

Finding a Minimum Weight Circuit

For the field of two elements (undirected cycle basis), the following method computes a minimum weight circuit.

Set up an auxiliary graph \( G_A \). For each vertex \( v \) of \( G \), we have vertices \( (v,0) \) and \( (v,1) \) in \( G_A \). For each edge \( e = uv \in G \), we have the edges \( ((u,i),(v,i+S_e)) \) for \( i = 0,1 \) in \( G_A \). Here, addition is modulo two.

- Illustrate this definition by a small example.

- Consider a path in \( G_A \) from \( (v,0) \) to \( (v,1) \). Argue that it corresponds to a circuit \( C \) in \( G \) with \( \langle C,S \rangle \neq 0 \).

- Derive an alg for computing a minimum weight circuit with \( \langle C,S \rangle \neq 0 \).

Have fun with the solution!