Selected Topics in Algorithms
K. Mehlhorn
Exercise 7

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We will discuss this exercise sheet on July 10th and July 13th.

String Matching

In class, we extended the pattern $P[0]\ldots P[m-1]$ by a new character $P[m] = \$$ that does not match any other character and that does not appear elsewhere in either text or pattern. Extend the pattern by a new character $P[-1] = \star$ that matches every character. Does this simplify the code?

Planar Graphs

Consider a connected graph $G = (V, E)$ embedded into the plane with no edges crossings. Removal of the embedding from the plane splits the plane into regions called faces. One face is unbounded, all other faces are unbounded.

1. Let f be the number of faces, m the number of edges and n the number of nodes. Prove $f - m + n = 2$. This equality is called Euler's formula. Hint: Since G is connected, we have $m \geq n-1$. If $m = n-1$, G is a tree and $f = 1$. This is the base case of an induction.

2. Conclude from item 1) that in a planar graph $m \leq 3n-6$. Hint: Consider the pairs (e, f), where e is an edge, f is a face, and e belongs to the boundary of f. Let N be their number. Show $N \leq 2m$ and $N \geq 3f$. Then use Euler's formula.

3. Use item 2) to prove that $K_{3,3}$ is not planar. $K_{3,3}$ is the complete bipartite graph with 3 nodes on each side.

4. Use item 2) to prove that every planar graph contains a node of degree 5 or less.

5-Coloring Planar Graphs

A node-coloring of a graph $G = (V, E)$ with k colors is a mapping $c : V \rightarrow [1\ldots c]$ such that $c(u) \neq c(v)$ for any $e = uv \in E$.

- Design a recursive algorithm for 5-coloring any planar graph. Hint: Let v be a vertex of degree 5 or less. If v has degree four or less, color $G - v$ and then use a color for v that is not used by any neighbor of v. If v has degree 5, ...

- What does your algorithm do, if the input graph is non-planar?

- In what sense is your algorithm certifying?

Your favorite algorithm

Consider an algorithm of your choice. Is the concept of certifying algorithm applicable to it?