Assigning Papers to Referees
Objectives, Algorithms, Open Problems

Kurt Mehlhorn
Max-Planck-Institut für Informatik
Saarbrücken
Germany

joint work with

Naveen Garg and Amit Kumar, IIT Delhi
Telikepalli Kavitha, IISC Bangalore
Julian Mestre, MPI Informatik
Overview

• Motivation
• Informal Problem Definition
• Formal Definition
• Algorithms and Hardness
• Truthfulness

Slides and paper are available at my home page
Motivation

I was program chair of ESA 2008.

After submission closes and before reviewing starts, the PC chair assigns the papers to the PC members (called reviewers in the sequel).

What constitutes a good assignment?
Informal Problem Definition I

- $n$ reviewers, $r$ indexes reviewers
- $m$ papers, $p$ indexes papers
- $v_{rp}$, the value of paper $p$ for reviewer $r$
  - the interest of reviewer $r$ in paper $p$
  - the qualification of reviewer $r$ for paper $p$
  - the rank of paper $p$ for reviewer $r$

- valuations can be determined in many different ways:
  - the PC chair invents them
  - papers and reviewers provide key words, $v_{rp}$ is a function of the number of common key words
  - reviewers provide values in \{NO, LOW, MEDIUM, HIGH\}
  - a combination of the above (our recommendation)
  - EasyChair (Andrei Voronkov), the system used for ESA 2008, asks the reviewers for bids
Informal Problem Definition II

- $n$ reviewers, $r$ indexes reviewers
- $m$ papers, $p$ indexes papers

- edge-labelled bipartite graph $G = (\text{papers} \cup \text{reviewers}, E)$
- $(r, p) \notin E$ means that $r$ cannot review $p$
- conflict of interest

- for $e = (r, p) \in E$, $v_{rp} \in \{1, \ldots, \Delta\}$ is the rank of $r$ for $p$

- an assignment $M$ is a subset of the edges
Informal Problem Definition II

- $n$ reviewers, $r$ indexes reviewers
- $m$ papers, $p$ indexes papers

- edge-labelled bipartite graph $G = (\text{papers} \cup \text{reviewers}, E)$

- $(r, p) \notin E$ means that $r$ cannot review $p$

- for $e = (r, p) \in E$, $v_{rp} \in \{1, \ldots, \Delta\}$ is the rank of $r$ for $p$

- an assignment $M$ is a subset of the edges

Objectives

- **Coverage**: each paper is reviewed (at least) $k$ times

- **Load-Balance**: load is shared evenly among reviewers;
  every rev. reviews $h = \lceil mk/n \rceil$ or $h - 1$ papers; today: $mk/n \in \mathbb{N}$
Informal Problem Definition II

- $n$ reviewers, $r$ indexes reviewers
- $m$ papers, $p$ indexes papers
- edge-labelled bipartite graph $G = (\text{papers} \cup \text{reviewers}, E)$
- $(r, p) \notin E$ means that $r$ cannot review $p$
- for $e = (r, p) \in E$, $v_{rp} \in \{1, \ldots, \Delta\}$ is the rank of $r$ for $p$
- an assignment $M$ is a subset of the edges

Objectives

- **Coverage**: each paper is reviewed (at least) $k$ times
- **Load-Balance**: load is shared evenly among reviewers; every rev. reviews $h = \lceil mk/n \rceil$ or $h - 1$ papers; today: $mk/n \in \mathbb{N}$
- **Quality**: papers are reviewed by qualified reviewers and reviewers get the papers that they are interested in
Informal Problem Definition II

- $n$ reviewers, $r$ indexes reviewers
- $m$ papers, $p$ indexes papers
- edge-labelled bipartite graph $G = (\text{papers} \cup \text{reviewers}, E)$
- $(r, p) \notin E$ means that $r$ cannot review $p$
- for $e = (r, p) \in E$, $v_{rp} \in \{1, \ldots, \Delta\}$ is the rank of $r$ for $p$
- an assignment $M$ is a subset of the edges

Objectives
- **Coverage**: each paper is reviewed (at least) $k$ times
- **Load-Balance**: load is shared evenly among reviewers; every rev. reviews $h = \lceil mk/n \rceil$ or $h - 1$ papers; today: $mk/n \in \mathbb{N}$
- **Quality**: papers are reviewed by qualified reviewers and reviewers get the papers that they are interested in
- **Fairness**: papers are treated fairly, reviewers are treated fairly
Quality w.r.t. a Reviewer (Paper)

- balanced assignment: \( k \) reviews per paper, \( h \) reviews per reviewer

- signature of reviewer \( r \):
  \[ s^r = (s^r_\Delta, \ldots, s^r_1) \]
  \( s^r_\ell \) = number of papers with valuation \( \ell \) assigned to \( r \)

- order signatures either
  - lexicographically or
  - by weight

\[
w(s^r) = \sum_{1 \leq \ell \leq \Delta} w_\ell s^r_\ell
\]

where \( w_\ell = \ell \) or \( w_\ell = 2^\ell \) or \ldots

- reviewers prefer assignments that give them a high signature (selfish view)
EasyChair’s Solution

- convert the $v_{rp}$’s to numbers (LOW = 1, MEDIUM = 2, HIGH = 3)
- compute an maximum weight balanced assignment
  EasyChair computes an approximation
- value of assignment = sum of the values of the reviewers
  $$\sum_r w(s^r)$$
- LEDA running time: 0.1 sec for ESA instance
- maximum weight assignments are not necessarily “fair”
Fairness

- four papers, two reviewers, each paper needs to be reviewed once
- reviewers agree in their valuation: two papers are H, two papers are L
- consider
  
  Assignment A: reviewer 1: L L reviewer 2: H H
  
  Assignment B: reviewer 1: L H reviewer 2: L H

- both assignments have weight $2w(H) + 2w(L)$, but Assignment B is more fair than Assignment A
- whenever valuation $v_{rp}$ depends only on $p$, all assignments have the same weight
Max Weight Assignment
Formalization of Fairness

- PC work is a group effort; therefore special attention should be given to the reviewer that is least satisfied by an assignment.

- recall $s^r(M) =$ signature of reviewer $r$ in assignment $M$

- signatures are ordered (lexicographically or by weight)

- for an assignment $M$

  $$\min_r s^r(M)$$

  is the worst signature of any reviewer $r$

- we want the balanced assignment that maximizes the minimum signature

  $$\max_M \min_r s^r(M)$$

- and among these assignments?

- the one that maximizes the second smallest signature, and among these, the one ...

  leximax solution
Results

$\Delta \geq 3$: problem is NP-complete

all $\Delta$: approx. such that every reviewer looses at most $\Delta$ wrt optimum

$\Delta = 2$: efficient algorithm

experiments: good solutions for ESA data
Signatures are Ordered by Weight

• inspired by allocation of indivisible goods (Santa Claus problem)
• sources
  • Bezakova, Dani: ACM SIGecom 2005
  • Lenstra, Schmoys, Tardos: Math Program. 1990
• the values $v_{rp}$ are numbers and it makes sense to add them
• binary variables $x_{rp}$ with $x_{rp} = 1$ iff paper $p$ is assigned to reviewer $r$
• load and coverage constraints:
  • $\sum_p x_{rp} = h$ for every reviewer $r$
  • $\sum_r x_{rp} = k$ for every (real) paper $p$

$$S_r := \sum_p v_{rp} x_{rp}$$ is value (utility) for reviewer $r$
A Hardness Result

- goal: maximize the smallest signature

- It is NP-hard to compute a balanced assignment approximating the minimum signature within less than $\Delta/2$ for all $\Delta \geq 3$
An Approximation Result

- fractional balanced assignments
  - every fractional balanced assignment gives rise to a vector $(t_1, t_2, \ldots, t_n)$, where $t_i =$ utility for reviewer $i$
  - let $(t_1^*, t_2^*, \ldots, t_n^*)$ be an optimal fractional assignment, i.e., it maximizes $\text{sort}(t_1^*, \ldots, t_n^*)$ (sort in increasing order)
  - $(t_1^*, t_2^*, \ldots, t_n^*)$ is unique and efficiently computable
- fractional assignment: we may assign papers fractionally, e.g., 0.3 to reviewer 1, 0.5 to reviewer 2, 0.2 to reviewer 3.
An Approximation Result

- fractional balanced assignments
  - every fractional balanced assignment gives rise to a vector $(t_1, t_2, \ldots, t_n)$, where $t_i =$ utility for reviewer $i$
  - let $(t_1^*, t_2^*, \ldots, t_n^*)$ be an optimal fractional assignment, i.e., it maximizes $\text{sort}(t_1^*, \ldots, t_n^*)$ (sort in increasing order)
  - $(t_1^*, t_2^*, \ldots, t_n^*)$ is unique and efficiently computable

- in polynomial time on can compute an integral assignment $M$ such that
  $$S_r > t_r^* - \Delta$$
  for all $r$
  i.e., each reviewer is within $\Delta$ of its utility in optimal fractional assignment
An Approximation Result

- fractional balanced assignments
  - every fractional balanced assignment gives rise to a vector $(t_1, t_2, \ldots, t_n)$, where $t_i =$ utility for reviewer $i$
  
- let $(t_1^*, t_2^*, \ldots, t_n^*)$ be an optimal fractional assignment, i.e., it maximizes $\text{sort}(t_1^*, \ldots, t_n^*)$ (sort in increasing order)
  
- $(t_1^*, t_2^*, \ldots, t_n^*)$ is unique and efficiently computable

- in polynomial time one can compute an integral assignment $M$ such that

\[ S_r > t_r^* - \Delta \quad \text{for all } r \]

- approach: compute optimal fractional assignment and round
An Approximation Result

- fractional balanced assignments
  - every fractional balanced assignment gives rise to a vector \((t_1, t_2, \ldots, t_n)\), where \(t_i = \text{utility for reviewer } i\)
  - let \((t_1^*, t_2^*, \ldots, t_n^*)\) be an optimal fractional assignment, i.e., it maximizes \(\text{sort}(t_1^*, \ldots, t_n^*)\) (sort in increasing order)
  - \((t_1^*, t_2^*, \ldots, t_n^*)\) is unique and efficiently computable

- in polynomial time on can compute an integral assignment \(M\) such that

\[
S_r > t_r^* - \Delta \quad \text{for all } r
\]

- remark: ESA instance \((\Delta = 3)\): \(h = 58\) and \(58 \leq S_r \leq 174\), but \(k = 4\) and \(4 \leq S_p \leq 12\)
Finding the Optimum Fractional Solution

- proceed in rounds: in \( j \)-th round we compute \( j \)-th entry of \((s_1^*, \ldots, s_n^*) := \text{sort}(t_1^*, \ldots, t_n^*)\)
- assume that we know the first \( j - 1 \) entries of \( s^* \) and the reviewers \( r_1 \) to \( r_{j-1} \) defining them
- consider the following LP: maximize \( q \) subj. to
  - \( x \) guarantees coverage and load balance
  - \( \sum_p v_{ri}x_{ri} = s_i^* \) for \( 1 \leq i < j \)
  - \( \sum_p v_{rp}x_{rp} \geq q \) for the remaining \( r \)
- let \( q^* \) be the optimal value.
- find the reviewer(s) that cannot do better than \( q^* \)
  - change one of the \( \geq q \) into a \( > q \) and check feasibility
- set \( s_j^* \) to \( q^* \) and \( r_j \) to this reviewer
Rounding Fractional Solutions

- let $x(e)$, $e \in E$ be any fractional solution satisfying the load and coverage constraints
- let $s_r(x) := \sum_p v_{rp}x_{rp}$, value for reviewer $r$
- let $s_p(x) := \sum_r v_{rp}x_{rp}$, value for paper $p$

- in polynomial time, we can find an integer assignment $y(e)$, $e \in E$, such that
  - $y$ satisfies the load and coverage constraints
  - $s_r(y) > s_r(x) - \Delta$ for all reviewers $r$
  - $s_p(y) > s_p(x) - \Delta$ for all papers $p$.
- observe that we have a guarantee for reviewers and papers
The Rounding Scheme

• given a fractional assignment \( x(e), e \in E \) round to \( y(e) \in \{0, 1\} \)
The Rounding Scheme

• given a fractional assignment \( x(e), e \in E \) round to \( y(e) \in \{ 0, 1 \} \)

• consider a fixed reviewer \( r \), order the incident edges in order of decreasing weight, say \( w(e^r_1) \geq w(e^r_2) \geq \ldots \)

• visualize the values \( x(e^r_1), x(e^r_2), \ldots \)
The Rounding Scheme

- Given a fractional assignment $x(e), e \in E$ round to $y(e) \in \{0, 1\}$
- Consider a fixed reviewer $r$, order the incident edges in order of decreasing weight, say $w(e'_1) \geq w(e'_2) \geq \ldots$
- Visualize the values $x(e'_r), x(e'_2), \ldots$

$$
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
\mid & \mid & \mid & \mid \\
x(e'_1) & x(e'_2) & x(e'_3) & x(e'_4) & x(e'_5) \\
\end{array}
$$

- Goal: at least one of $y(e'_1), y(e'_2)$ is one,
  at least two of $y(e'_1), \ldots, y(e'_4)$ are one, \ldots
- More generally: $x(e'_1) + \ldots + x(e'_\ell) \geq j \Rightarrow y(e'_1) + \ldots + y(e'_\ell) \geq j$
- Such an integral solution exists and it yields the desired approximation
The Approximation Quality

- given a fractional solution $x(e), e \in E$, round to $y(e) \in \{0, 1\}$
- reviewer $r$, order incident edges by weight $w(e'_1) \geq w(e'_2) \geq \ldots$
- assume: $x(e'_1) + \ldots + x(e'_\ell) \geq j \Rightarrow y(e'_1) + \ldots + y(e'_\ell) \geq j$

how much can we lose by rounding?
The Approximation Quality

- given a fractional solution \( x(e), e \in E \), round to \( y(e) \in \{0, 1\} \)
- reviewer \( r \), order incident edges by weight \( w(e_1^r) \geq w(e_2^r) \geq \ldots \).
- assume: \( x(e_1^r) + \ldots + x(e_\ell^r) \geq j \) \( \Rightarrow \) \( y(e_1^r) + \ldots + y(e_\ell^r) \geq j \)

- how much can we loose by rounding? No more than

\[
(w(e_1^r) - w(e_2^r)) + (w(e_2^r) - w(e_4^r)) + \ldots
\]

since fractional value of \([1, 2]\) at most \( w(e_2^r) \) and integral value at least \( w(e_4^r) \)
The Approximation Quality

- given a fractional solution \( x(e), e \in E \), round to \( y(e) \in \{0, 1\} \)
- reviewer \( r \), order incident edges by weight \( w(e'_r) \geq w(e'_2) \geq \ldots \)
- assume: \( x(e'_1) + \ldots + x(e'_l) \geq j \Rightarrow y(e'_1) + \ldots + y(e'_l) \geq j \)
  
  \[
  \begin{array}{ccccccc}
  0 & 1 & 2 & 3 & \ldots & h \\
  \hline
  x(e'_1) & x(e'_2) & x(e'_3) & x(e'_4) & x(e'_5) & \ldots \\
  \end{array}
  \]

- how much can we loose by rounding? No more than

\[
(w(e'_1) - w(e'_2)) + (w(e'_2) - w(e'_4)) + \ldots
\]

since fractional value of \([1, 2]\) at most \( w(e'_2) \) and integral value at least \( w(e'_4) \)

- this telescopes to no more than \( \Delta \)
Existence: A Flowproblem

- have $h$ nodes for each reviewer (supply one) and $k$ nodes for each paper (demand one)
- $e'_4 = rp$ belongs to second and third group with respect to $r$ and first and second group with respect to $p$.
- fractional flow is feasible
- all capacities are integral $\Rightarrow$ integral flow exists
- flow out of $\{r_1, \ldots, r_i\}$ is at least $i$, flow into $\{p_1, \ldots, p_j\}$ is at least $j$.
Our Assignment (leximax)
Two Ranks

- ordering signatures by weight or lexicographically is the same
- consider (reviewers are ordered by signature)

```
H H H H H H H H L L L L
H H H H H H L L L L L L
H H H H H H L L L L L L
H H H H H L L L L L L L
H H H L L L L L L L L L
```

- we want the assignment for which the \textit{H — L – staircase} is as far to the right as possible
- this is the same as saying that the \textit{H — L – staircase} is as far down as possible.
- we will next see a polynomial time alg for the case of two ranks
A Polynomial Time Algorithm for Two Ranks

- the following alg computes an assignment for any value of $\Delta$
  - for $\Delta = 2$, it computes leximax solution
  - for $\Delta \geq 3$, it also seems to work well
- ranks are in $\{1, \ldots, \Delta\}$, large ranks are better than small ranks
- we view the assignment as proceeding in rounds:

<table>
<thead>
<tr>
<th>revs</th>
<th>papers</th>
<th>revs</th>
<th>ranks (sorted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3  7  4 9 1</td>
<td>1</td>
<td>5  5  3  1 1</td>
</tr>
<tr>
<td>2</td>
<td>5  4  2 3 7</td>
<td>2</td>
<td>5  4  2  2 2</td>
</tr>
<tr>
<td>3</td>
<td>3  1  4 7 9</td>
<td>3</td>
<td>3  1  1  1 1</td>
</tr>
</tbody>
</table>

- signature of a round:
  (# of rank $\Delta$ papers, # of rank $\Delta - 1$ papers, \ldots, # of rank 1 papers)
Rank-Maximality

• we view the assignment as proceeding in rounds:

• signature of a round:
  (# of rank $\Delta$ papers, # of rank $\Delta - 1$ papers, . . . , # of rank 1 papers)

• objective:
  • maximize signature of first round and subject to this signature of second round and . . .
  • for two ranks: objective yields lex-max solution
  • for more than two ranks: ????

• polynomial time algorithm via
  • weighted bipartite matching problem with exponentially large weights
  • running time, 1 sec for ESA instance
The Weighted Bipartite Matching Problem

- vertex $r_\ell$ represents reviewer $r$ in round $\ell$, $1 \leq \ell \leq h$
- vertex $p_c$ represents copy $c$ of paper $p$, $1 \leq c \leq k$
- for an edge $e = (r, p)$ of rank $d$, we have vertices $(e, R)$ and $(e, P)$ and the edges shown

- If $p$ is assigned to $r$ in round $\ell$, $(r_\ell, (e, R))$ and $((e, P), p_c)$ are in $M$.
- If $p$ is not assigned to $r$ in any round, $((e, R), (e, P)) \in M$.
- the edges from nodes $r_\ell$ to the nodes $(e, R)$ are weighted
The Weights

• if $e = (r, p)$ has rank $d$, we give the edge connecting $r_\ell$ and $(e, R)$ weight

$$(n + 1)^d W^{k-\ell+1} \quad \text{where } W = (n + 1)^{r+1}$$

• weights for a single round:
  
  • a paper of rank $d$ contributes weight $(n + 1)^d$ to the weight of a round; because then
  
  • $n$ rank $d - 1$ assignments cannot make up for one rank $d$ assignment
  
  • maximum weight of a round: $n(n + 1)^r$, set $W = (n + 1)^{r+1}$

• total weight of assignment $= w_1 W^k + w_2 W^{k-1} + \ldots + w_k W^0$

  $$w_\ell = \text{weight of round } \ell \text{ and } k \text{ is the number of rounds}$$
Our Assignment (rank-maximal)

first 22 rounds are perfect
Truthfulness

- the goal of a reviewer is to maximize his signature
does any of the strategies induce reviewers to reveal their true valuations?
Truthfulness

- the goal of a reviewer is to maximize his signature

does any of the strategies induce reviewers to reveal their true valuations?

- NO
  - assume we have three reviewers, three papers and each paper needs to be reviewed twice.
  - the reviewers have equal valuations: they rate papers 1 and 2 high and paper 3 medium.
  - assume reviewers 2 and 3 tell the truth; then
  - reviewer 1 should lie about paper 3 and state a low rating.
  - he will get papers 1 and 2.
Truthfulness

• the goal of a reviewer is to maximize his signature

Does any of the strategies induce reviewers to reveal their true valuations?

• NO
  • assume we have three reviewers, three papers and each paper needs to be reviewed twice.
  • the reviewers have equal valuations: they rate papers 1 and 2 high and paper 3 medium.
  • assume reviewers 2 and 3 tell the truth; then
  • reviewer 1 should lie about paper 3 and state a low rating.
  • he will get papers 1 and 2.

• more extreme: a reviewer declares a conflict for all but $h$ papers
What Next?

- are these the right objectives; alternative objectives?
- more algorithms (exact and approximate)
- a deeper investigation of truthfulness
- a better way to determine valuations? bids + keywords + wisdom of PC chair
- more experiments in collaboration with Andrei Voronkov (EasyChair)
- incorporation into EasyChair