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To all algorithmists
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Preface

Algorithms are at the heart of every nontrivial computer application. Therefore every computer scientist and every professional programmer should know about the basic algorithmic toolbox: structures that allow efficient organization and retrieval of data, frequently used algorithms, and generic techniques for modeling, understanding, and solving algorithmic problems.

This book is a concise introduction to this basic toolbox, intended for students and professionals familiar with programming and basic mathematical language. We have used the book in undergraduate courses on algorithmics. In our graduate-level courses, we make most of the book a prerequisite, and concentrate on the starred sections and the more advanced material. We believe that, even for undergraduates, a concise yet clear and simple presentation makes material more accessible, as long as it includes examples, pictures, informal explanations, exercises, and some linkage to the real world.

Most chapters have the same basic structure. We begin by discussing a problem as it occurs in a real-life situation. We illustrate the most important applications and then introduce simple solutions as informally as possible and as formally as necessary to really understand the issues at hand. When we move to more advanced and optional issues, this approach gradually leads to a more mathematical treatment, including theorems and proofs. This way, the book should work for readers with a wide range of mathematical expertise. There are also advanced sections (marked with a *) where we recommend that readers should skip them on first reading. Exercises provide additional examples, alternative approaches and opportunities to think about the problems. It is highly recommended to take a look at the exercises even if there is no time to solve them during the first reading. In order to be able to concentrate on ideas rather than programming details, we use pictures, words, and high-level pseudocode to explain our algorithms. A section "implementation notes" links these abstract ideas to clean, efficient implementations in real programming languages such as C++ and Java. Each chapter ends with a section on further findings that provides a glimpse at the state of the art, generalizations, and advanced solutions.

Algorithmics is a modern and active area of computer science, even at the level of the basic toolbox. We have made sure that we present algorithms in a modern
way, including explicitly formulated invariants. We also discuss recent trends, such as algorithm engineering, memory hierarchies, algorithm libraries, and certifying algorithms.

We have chosen to organize most of the material by problem domain and not by solution technique. The final chapter on optimization techniques is an exception. We find that presentation by problem domain allows a more concise presentation. However, it is also important that readers and students obtain a good grasp of the available techniques. Therefore, we have structured the final chapter by techniques, and an extensive index provides cross-references between different applications of the same technique. Bold page numbers in the Index indicate the pages where concepts are defined.

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# Contents

1 Appetizer: Integer Arithmetics ........................................... 1  
  1.1 Addition ................................................................. 2  
  1.2 Multiplication: The School Method ................................. 3  
  1.3 Result Checking ...................................................... 6  
  1.4 A Recursive Version of the School Method ....................... 7  
  1.5 Karatsuba Multiplication ........................................... 9  
  1.6 Algorithm Engineering ............................................. 11  
  1.7 The Programs .......................................................... 13  
  1.8 Proofs of Lemma 1.5 and Theorem 1.7 ........................... 16  
  1.9 Implementation Notes ............................................. 17  
  1.10 Historical Notes and Further Findings .......................... 18  

2 Introduction ................................................................. 19  
  2.1 Asymptotic Notation ................................................. 20  
  2.2 The Machine Model .................................................. 23  
  2.3 Pseudocode ............................................................ 26  
  2.4 Designing Correct Algorithms and Programs .................... 31  
  2.5 An Example – Binary Search ....................................... 34  
  2.6 Basic Algorithm Analysis ........................................... 36  
  2.7 Average-Case Analysis ............................................. 41  
  2.8 Randomized Algorithms ............................................ 45  
  2.9 Graphs ................................................................. 49  
  2.10 P and NP .............................................................. 53  
  2.11 Implementation Notes ............................................. 56  
  2.12 Historical Notes and Further Findings ......................... 57  

3 Representing Sequences by Arrays and Linked Lists ............ 59  
  3.1 Linked Lists ......................................................... 60  
  3.2 Unbounded Arrays ................................................... 66  
  3.3 Amortized Analysis .................................................. 71  
  3.4 Stacks and Queues ................................................... 74
9 Graph Traversal ...................................................... 175
9.1 Breadth-First Search ........................................... 176
9.2 Depth-First Search ............................................. 178
9.3 Implementation Notes ......................................... 188
9.4 Historical Notes and Further Findings ....................... 189

10 Shortest Paths ..................................................... 191
10.1 From Basic Concepts to a Generic Algorithm .............. 192
10.2 Directed Acyclic Graphs ....................................... 195
10.3 Nonnegative Edge Costs (Dijkstra’s Algorithm) .......... 196
10.4 *Average-Case Analysis of Dijkstra’s Algorithm ........... 199
10.5 Monotone Integer Priority Queues .......................... 201
10.6 Arbitrary Edge Costs (Bellman–Ford Algorithm) ........ 206
10.7 All-Pairs Shortest Paths and Node Potentials ............. 207
10.8 Shortest-Path Queries ......................................... 209
10.9 Implementation Notes ......................................... 213
10.10 Historical Notes and Further Findings ...................... 214

11 Minimum Spanning Trees ....................................... 217
11.1 Cut and Cycle Properties ..................................... 218
11.2 The Jarník–Prim Algorithm ................................... 219
11.3 Kruskal’s Algorithm ............................................ 221
11.4 The Union–Find Data Structure .............................. 222
11.5 *External Memory ............................................. 225
11.6 Applications ...................................................... 228
11.7 Implementation Notes ......................................... 231
11.8 Historical Notes and Further Findings ...................... 231

12 Generic Approaches to Optimization .......................... 233
12.1 Linear Programming – a Black-Box Solver .................. 234
12.2 Greedy Algorithms – Never Look Back ..................... 239
12.3 Dynamic Programming – Building It Piece by Piece ....... 243
12.4 Systematic Search – When in Doubt, Use Brute Force .... 246
12.5 Local Search – Think Globally, Act Locally ................. 249
12.6 Evolutionary Algorithms ...................................... 259
12.7 Implementation Notes ......................................... 261
12.8 Historical Notes and Further Findings ...................... 262

A Appendix ............................................................. 263
A.1 Mathematical Symbols .......................................... 263
A.2 Mathematical Concepts ......................................... 264
A.3 Basic Probability Theory ........................................ 266
A.4 Useful Formulae ................................................... 269
Contents

References ................................................................. 273

Index ................................................................. 285