On Generalizing Decidable Standard Prefix Classes of First-Order Logic

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Generalized BSR fragment:
relational sentences
\[ \forall \vec{x}_1 \exists \vec{y}_1 \ldots \exists \vec{x}_n \exists \vec{y}_n \psi \]
where
the set of atoms in \( \psi \)
can be partitioned into
At_1(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_{n-1}, \vec{x}_n),
At_2(\vec{y}_1, \vec{x}_2, \ldots, \vec{x}_{n-1}, \vec{x}_n),
\ldots
At_{n-1}(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_{n-2}, \vec{y}_{n-1}),
At_n(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_{n-1}, \vec{y}_n),
such that
\( \text{vars}(\text{At}_i) \cap \text{vars}(\text{At}_j) \cap \vec{x}_\ell = \emptyset \)
for all \( i \neq j \) and every \( \ell \).

Example from GBSR \( \cap \) GAF:
\[ \exists u \forall x \exists y \forall z. \left( (P(u, z) \land \neg Q(u, x, x)) \lor (P(y, u) \land Q(u, y, z)) \right) \]

Computational complexity:
The satisfiability problem for GBSR / SF sentences induces a hierarchy of \( k \)-\( \text{NExpTime} \)-complete problems:

More properties:

(1) GBSR and GAF possess the finite model property:
- Satisfiable GAF sentences have models over finite domains.
- Every model of a GBSR sentence contains a finite substructure that is a model, too.

(2) GBSR and BSR are equivalent: Every GBSR sentence can be transformed into an equivalent BSR sentence.
In the worst case, this transformation incurs a non-elementary blowup.

(3) GBSR and SF without equality are closed under Craig-Lyndon interpolation:

\[ \varphi \models \psi \quad \exists x \quad \text{and} \quad \chi \models \psi \]

- \( \varphi, \psi \) are GBSR sentences without equality.
- \( \chi \) is a BSR sentence over \( \varphi \)'s and \( \psi \)'s joint voc.
- Every \( P \) with a positive (negative) occurrence in \( \chi \) occurs positively (negatively) in \( \varphi \) and in \( \psi \).

References
Sturm, Voigt, Weidenbach. Deciding Satisfiability when Universal and Existential Variables are Separated. LICS 2016.