Optimal assignment of periods and deadlines to EDF-scheduled tasks

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February 28th, 2012
The task model

- A set \( \{\tau_1, \ldots, \tau_n\} \) of \( n \) periodic arbitrary deadline tasks. Each task \( \tau_i \) has:
  - execution \( C_i \);
  - period \( T_i \);
  - relative (to the job release) deadline \( D_i \) (in this talk it is assumed \( D_i \leq T_i \), in the paper \( D_i \) arbitrary).

- Each task releases an infinite sequence of job, one every period. \( j \)-th job of \( \tau_i \):
  - is released at \( r_{ij} = (j - 1)T_i \)
  - has absolute deadline \( d_{ij} = r_{ij} + D_i \)
  - must complete not later than \( d_{ij} \)
Earliest Deadline First (EDF)

- Jobs are scheduled preemptively according to their absolute deadline

\[
\begin{array}{ccc}
C_i & T_i & D_i \\
1 & 5 & 5 \\
1 & 4 & 5 \\
3 & 7 & 7 \\
\end{array}
\]
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<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$T_i$</th>
<th>$D_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
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<tr>
<td>1</td>
<td>4</td>
<td>5</td>
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<td>3</td>
<td>7</td>
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</tbody>
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Theorem (Baruah ’90)

The task set is schedulable by EDF if and only if:

$$\forall t \in \{d_{ij}\}, \quad \sum_{i=1}^{n} k_i C_i \leq t$$

with

$$k_i = \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor$$
An equivalent formulation 1

The EDF necessary and sufficient condition is

\[ \forall t \in \{d_{ij}\}, \quad \sum_{i=1}^{n} k_i C_i \leq t \quad \text{with} \quad k_i = \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor \]

What is the interpretation of the vector \((k_1, k_2, \ldots, k_n) \in \mathbb{N}^n\)?
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- \(k_i\) is the number of \(\tau_i\) jobs within the interval \([0, t]\)
- \(k_1 C_1 + k_2 + C_2 + \ldots + k_n C_n\) is the demand bound function
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Given a vector \((k_1, k_2, \ldots, k_n) \in \mathbb{N}^n\), what is the interval that must contain \(\sum_{i=1}^{n} k_i C_i\) to guarantee feasibility?
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\[ [0, \max_i \{d_{ik_i}\}] = [0, \max_i \{(k_i - 1)T_i + D_i\}] \]
An equivalent formulation 2

With a leap of faith, we have

Theorem

The task set is schedulable by EDF if and only if:

\[ \forall k \in \mathbb{N}^n \setminus \{0\}, \quad \sum_{j=1}^{n} k_j C_j \leq \max_i \{d_{ik_i}\} = \max_i \{(k_i - 1)T_i + D_i\} \]
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*or, equivalently,*

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\forall \mathbf{k} \in \mathbb{N}^n \setminus \{0\}, \quad \exists i = 1, \ldots, n \quad (T_i - C_i)k_i - \sum_{j \neq i} C_j k_j \geq T_i - D_i
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$$\text{Speedup } \frac{5}{4} = 1.25 \text{ for these values of period deadline.}$$
Space of $D_i$

Given the periods $T_i$ and the execution times $C_i$, what is the space of feasible deadlines?
Space of $D_i$

Given the periods $T_i$ and the execution times $C_i$, what is the space of feasible deadlines?

If $T_1 = 4$, $C_1 = 2$, $T_2 = 7$, $C_2 = 3.5$ (notice that $\frac{C_1}{T_1} + \frac{C_2}{T_2} = 1$), we have