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Tutorials for “Automated Reasoning”
Exercise sheet 1

Exercise 1.1: (2 P)
Determine which of the following formulae are valid / satisfiable / unsatisfiable:

1. \((P \land Q) \rightarrow (P \lor Q)\)
2. \((P \lor Q) \rightarrow (P \land Q)\)
3. \(\neg(P \land \neg\neg P)\)
4. \(Q \rightarrow \neg Q\)
5. \(\neg\neg Q \rightarrow Q\)
6. \(\neg(P \lor \neg\neg P) \lor P\)
7. \([P \rightarrow Q] \land (\neg P \rightarrow R) \rightarrow (Q \lor R)\)

Exercise 1.2: (3 P)
Let \(F\) be a propositional formula which contains no occurrence of \(\rightarrow\) or \(\leftrightarrow\). The dual of \(F\), which we denote here by \(F^*\), is the propositional formula obtained by replacing every occurrence of \(\top\) by \(\bot\), every occurrence of \(\bot\) by \(\top\), every occurrence of \(\lor\) by \(\land\) and every occurrence of \(\land\) by \(\lor\).

(1) Write the dual of the following formulae:
   (a) \(\neg(P \lor Q)\);
   (b) \(\neg(P \land \neg\neg(Q \lor R))\);
   (c) \(\neg P \land (\neg\neg Q \lor R)\);
   (d) \(\neg P \lor Q\);
   (e) \((\neg P \lor Q) \land (\neg Q \lor P)\)
(2) Prove that for every formulae $F, G$ which contain no occurrences of $\to$ or $\leftrightarrow$, $\models F \leftrightarrow G$ if and only if $\models F^* \leftrightarrow G^*$. (*The duality principle*)

**Exercise 1.3: (3 P)**
Consider the following boolean formula $F := (P \land ((Q \land \neg R) \lor (\neg Q \land R))) \lor (\neg P \land \neg R)$.

(a) Construct a binary decision tree for $F$ such that the root is an $P$-node followed by $Q$- and then $R$-nodes.

(b) Construct another binary decision tree for $F$, but now let its root be a $R$-node followed by $Q$- and then $P$-nodes.

(c) Compute the reduced OBDD for $F$ with respect to the following ordering of variables: $P < Q < R, R < Q < P$.

**Exercise 1.4: (3 P)**
Give an example of a Boolean function $f$ with three variables $P, Q, R$, such that the minimal OBDD for $f$ has 5 interior nodes for the variable ordering $P < Q < R$ and 4 interior nodes for some other variable ordering.

**Challenge Problem: (3 extra points)**
Let $P_1 < \cdots < P_n$ be an ordering on propositional variables. Prove that a reduced OBDD for this ordering can contain at most twelve nodes labelled with $P_{n-1}$ (that is, nodes on the second lowest level).

Put your solution into the mail box at the door of room 607 in the MPI building (46.1) before Friday, April 30, 11:00. Please write your name and the name of your tutorial group (A, B, C, D) on your solution.

The distribution in tutorial groups will be available on Monday, April 26, at:
http://www.mpi-sb.mpg.de/~sofronic/courses/automated-reasoning.html

The tutorials start next week (April 26th-30th). In the first week the tutors will be available for discussions. The first exercise sheet will be discussed one week later, after the submitted solutions are corrected.

**Note:** Joint solutions, prepared by up to three persons together, are allowed. Joint solutions should be submitted only once, and all the authors should be indicated.