Exercise 10.1: (2 P)
Let \( E \) be a set of equations, and \( t, t' \) be terms in a given signature \( \Sigma \). Let \( \sigma : X \rightarrow T_\Sigma(X) \) be a substitution. Prove that if \( E \vdash t \approx t' \) then \( E \vdash t\sigma \approx t'\sigma \).

Exercise 10.2: (2 P)
Given the rewrite system

\[
R: \begin{align*}
x + 0 &\rightarrow x \\
-x &\rightarrow 0 \\
-x + x &\rightarrow 0 \\
-x + y &\rightarrow (y + (-x)) \\
x + (y + z) &\rightarrow (x + y + z)
\end{align*}
\]

compute the rewrite successors of \( s = -((x + y) + x) \) and \( t = ((-x) + (-y)) + x \).

Exercise 10.3: (4 P)
Compute all critical pairs for each of the following systems:

(a) \( f(g(f(x))) \rightarrow x, \quad f(g(x)) \rightarrow g(f(x)) \);
(b) \( f(x, x) \rightarrow a, \quad f(x, g(x)) \rightarrow b \);
(c) \( f(f(x, y), z) \rightarrow f(x, f(y, z)), \quad f(x, 1) \rightarrow x \);
(d) \( f(f(x, y), z) \rightarrow f(x, f(y, z)), \quad f(1, x) \rightarrow x \);

Which systems are locally confluent?
Exercise 10.4: (2 P)
Show that the following system is locally confluent.

\[ f(f(x)) \rightarrow f(x), \quad f(g(x)) \rightarrow g(x) \]
\[ g(g(x)) \rightarrow f(x), \quad g(f(x)) \rightarrow g(x). \]

Can you determine the normal form of a term as a function of the numbers of \( fs \) and \( gs \) in it?

Challenge Problem (3 extra points)

Give example of

- a signature \( \Sigma \), containing at least one constant symbol,
- a set \( E \) of equations over \( \Sigma \),
- two terms \( s, t \) over \( \Sigma \),

with the property that \( T_\Sigma(\{x_1\}) / E \models \forall \bar{x}(s \approx t) \), but \( T_\Sigma(\{x_1, x_2\}) / E \not\models \forall \bar{x}(s \approx t) \) (where \( \bar{x} \) consists of all the variables occurring in \( s \) and \( t \)).

Put your solution into the mail box at the door of room 607 in the MPI building (46.1) before Friday, July 1, 11:00. Don’t forget to write your name and the name of your tutorial group (A, B, C) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed. Joint solutions should be submitted only once, and all the authors should be indicated.