Exercise 13.1: (3 P)
To prove the completeness of Knuth-Bendix completion, a cost is associated with every proof step. This cost is a triple. In which subcase of the proof of Lemma 3.44 is the last component of this triple used? How does this subcase work?

Exercise 13.2: (2 P)
Use superposition to show that the following set of (implicity universally quantified) clauses is unsatisfiable:

\[ f(b, a) \approx a \]
\[ f(x, a) \approx x \]
\[ f(c, b) \not\approx c \lor a \not\approx b \]

Use the LPO with precedence \( f > a > b > c \) and left-to-right lexicographic status for \( f \) as term ordering; compute only inferences that are required according to the ordering restrictions of the superposition calculus.

Exercise 13.3: (2 P)
Give an example of a signature \( \Sigma \), a reduction ordering \( \succ \) that is total on ground \( \Sigma \)-terms, and a set \( N \) of two ground clauses such that

- \( R_\infty \neq \emptyset \),
- all clauses in \( N \) are true in \( R_\infty \), and
- \( N \) is not saturated up to redundancy.
Exercise 13.4: (2 P)
Prove: If $N$ contains (not necessarily ground) clauses

\[ C = C' \lor s[t\sigma] \not\models s' \]
\[ D = t \approx t' \]
\[ C_0 = C' \lor s[t'\sigma] \not\models s' \]

and $t\sigma \not\supset t'\sigma$, then $C$ is redundant w. r. t. $N$.

Put your solution into the mail box at the door of room 607 in the MPI building (46.1) before Friday, July 23, 11:00. Don’t forget to write your name and the name of your tutorial group (A, B, C) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed. Joint solutions should be submitted only once, and all the authors should be indicated.