Exercise 5.1: (2 P)
Give an example of a signature $\Sigma$, a $\Sigma$-algebra $A$, and a closed prenex $\Sigma$-formula $F$ which contains existential quantifiers such that $A \models F$, but $A \not\models F$, where $A$ is the $\Sigma$-algebra having as universe the set of all term-generated elements of $U$ (cf. the definition given in Exercise 4.5).

Exercise 5.2: (2 P)
Two ordered sets $(M_1, \triangleright_1)$ and $(M_2, \triangleright_2)$ are called order-isomorphic, if there exists a bijection $h : M_1 \rightarrow M_2$ such that for all $x, y \in M_1 : x \triangleright_1 y$ if and only if $h(x) \triangleright_2 h(y)$.

Let the ordering $\triangleright$ over $\{A, B\}$ be defined by $A \triangleright B$. Is the set of finite multisets over $\{A, B\}$ (ordered by the multiset extension of $\triangleright$) order-isomorphic to $(\mathbb{N}, \triangleright)$ or not? Prove your hypothesis.

Exercise 5.3: (2 P)
Find a total ordering $\triangleright$ on the ground atoms $A, B, C, D, E$, such that the associated clause ordering $\triangleright_C$ orders the following clauses like this:

$$B \lor C \triangleright_C A \lor A \lor \neg C \triangleright_C C \lor E \triangleright_C C \lor D \triangleright_C \neg A \lor D \triangleright_C \neg E$$

Exercise 5.4: (2 P)
Prove that the general binary resolution rule in $\text{Res}$:

$$\frac{C \lor A \ D \lor \neg B}{(C \lor D)\sigma} \text{ if } \sigma = \text{mgu}(A, B)$$

is sound.
Exercise 5.5: \((2 \, P)\)
Prove part (iv) of Proposition 2.22: Let \(D', D, \) and \(C\) be ground clauses such that \(D, D' \in N\) and \(D' \succ D \succ C\). Then
\[
I_D \models C \Rightarrow I_{D'} \models C \quad \text{and} \quad I_N \models C.
\]
If, in addition, \(C \in N\) or \(\max(D) \succ \max(C)\):
\[
I_D \not\models C \Rightarrow I_{D'} \not\models C \quad \text{and} \quad I_N \not\models C.
\]

Put your solution into the mail box at the door of room 607 in the MPI building (46.1) before Friday, May 28, 11:00. Don’t forget to write your name and the name of your tutorial group (A, B, C) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed. Joint solutions should be submitted only once, and all the authors should be indicated.