2.16 Other Inference Systems

Instantiation-based methods for FOL:

- Partial instantiation;
- Resolution-based instance generation;
- Disconnection calculus.

Further (mainly propositional) proof systems:

- Hilbert calculus;
- Sequent calculus;
- Natural deduction.
Instantiation-Based Methods for FOL

Idea:

Overlaps of complementary literals produce instantiations (as in resolution);

However, contrary to resolution, clauses are not recombined.

Instead: treat remaining variables as constant and use efficient propositional proof methods, such as DPLL.

There are both saturation-based variants, such as partial instantiation [Hooker et al.] or resolution-based instance generation (Inst-Gen) [Ganzinger and Korovin], and tableau-style variants, such as the disconnection calculus [Billon; Letz and Stenz].
Hilbert Calculus

Hilbert calculus:

Direct proof method (proves a theorem from axioms, rather than refuting its negation)

Axiom schemes, e.g.,

\[ F \rightarrow (G \rightarrow F) \]
\[ (F \rightarrow (G \rightarrow H)) \rightarrow ((F \rightarrow G) \rightarrow (F \rightarrow H)) \]

plus Modus ponens:

\[ \begin{array}{c}
F \\
F \rightarrow G \\
\hline \\
G 
\end{array} \]

Unsuitable for both humans and machines.
Natural Deduction

Natural deduction (Prawitz):

Models the concept of proofs from assumptions as humans do it (cf. Fitting or Huth/Ryan).
Sequent Calculus

Sequent calculus (Gentzen):

Assumptions internalized into the data structure of sequents

\[ F_1, \ldots, F_m \rightarrow G_1, \ldots, G_k \]

meaning

\[ F_1 \land \cdots \land F_m \rightarrow G_1 \lor \cdots \lor G_k \]

A kind of mixture between natural deduction and semantic tableaux.

Perfect symmetry between the handling of assumptions and their consequences.

Can be used both backwards and forwards.