Exercise 1. (3 points)
Show that \( \vec{p} = \left( \frac{2}{5}, \frac{3}{5}, 0 \right) \) and \( \vec{q} = \left( \frac{13}{15}, 0, \frac{2}{15} \right) \) form a Nash Equilibrium of the following game:

\[
\begin{bmatrix}
B_1 & B_2 & B_3 \\
A_1 & (5, 1) & (-3, 10) & (7, 7) \\
A_2 & (3, 4) & (2, -8) & (20, 0) \\
A_3 & (0, 0) & (8, 6) & (14, 4)
\end{bmatrix}
\]

Exercise 2. (total: 5 points)
Alice and Bob play a game. Each player chooses a number between 1 to 3 (inclusive). Let \( w \) be the sum of the two chosen numbers.

- If \( w \) is odd, then Alice earns \( w \) and Bob earns \( w^2 \).
- If \( w \) is even, then Alice loses \( 2w \) and Bob earns \( 1 \).

(a) (2 points) Write down the bimatrix that represents this game.
(b) (3 points) You are assured that there exists a Nash Equilibrium \((\vec{p}, \vec{q})\) such that \( p_1, p_2, q_2, q_3 \) are strictly positive, while \( p_3 = q_1 = 0 \). Compute this Nash Equilibrium.

Exercise 3. (total: 4 points)
The following matrix represents a two-person zero-sum game:

\[
\begin{bmatrix}
B_1 & B_2 & B_3 \\
A_1 & 3 & 10 & -8 \\
A_2 & -1 & 2 & -4 \\
A_3 & 5 & 0 & 7
\end{bmatrix}
\]

(a) (2 points) Write down the linear program that computes the lower game value of Alice.
(b) (2 points) Compute the solution to the linear program.

Exercise 4. (2 points)
Given a two-person general-sum game, for Alice, a strategy \( i' \) is said to be strictly dominated if there exists another strategy \( i \) such that for any \( j \in [k] \), \( u_{ij} > u_{i'j} \).

Prove that if a strategy \( i' \) is strictly dominated, then in any Nash Equilibrium \((\vec{p}, \vec{q})\) of the game, \( p_{i'} = 0 \).

Exercise 5 (*). (3 points)
Let \( G \) be a two-person zero-sum game represented by the matrix \( A \), where each entry \( A_{ij} \) is a real number. Let \( v \) be the game value of \( G \).

Let \( G' \) be another two-person zero-sum game represented by the matrix \( A' \), such that \( A'_{ij} = c_1 \cdot A_{ij} + c_2 \), where \( c_1 \) is a positive real number, and \( c_2 \) is a real number.
Prove that the game value of $G'$ is exactly $c_1 \cdot v + c_2$.

**Exercise 6 (*).** (3 points)

First of all, given a two-person zero-sum game, we say the game favors Alice if the game value is strictly positive, and we say the game favors Bob if the game value is strictly negative.

Alice and Bob play a two-person zero-sum game, defined as follows. There are 8 items. Alice chooses a set of 4 items, denoted by $S_A$, while Bob chooses a set of 3 items, denoted by $S_B$. Alice earns $1 if the sets $S_A$ and $S_B$ intersect, and she loses $1 if $S_A$ and $S_B$ do not intersect.

Determine whether the game favors Alice or Bob. What about the general situation when the number 8 is replaced by $n$?