

# LEDA

## A Library of Efficient Data Types and Algorithms

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### Abstract

LEDA is a library of efficient data types and algorithms. At present, its strength is graph algorithms and the data structures related to them. The computational geometry part is evolving. The main features of the library are

- 1) a clear separation of specification and implementation
- 2) parameterized data types
- 3) a comfortable data type graph, and
- 4) its ease of use.

## I. Introduction

There is no standard library of the data structures and algorithms of combinatorial computing. This is in sharp contrast to many other areas of computing. There are e.g. packages in statistics (SPSS), numerical analysis (LINPACK, EISPACK), symbolic computation (MACSYMA, SAC-2) and linear programming (MPSX).

In fact the situation is worse, since even within small groups, say the algorithms group at our home institution, software is frequently not shared. Rather, each researcher starts from scratch and e.g. develops his own version of a balanced tree. Of course, this continuous "reimplementation of the wheel" slows down progress, within research and even more so outside. This is due to the fact that outside research the investment for implementing an efficient solution is frequently not made, because it is doubtful whether the implementation can be reused, and therefore methods which are known to be less efficient are used instead. Thus the scientific discoveries migrate only slowly into practice.

One of the major differences between combinatorial computing and other areas of computing such as statistics, numerical analysis and linear programming is the usage of complex data types. Whilst the built-in types, such as integers, reals, vectors, matrices

and functions, usually suffice in the other areas, combinatorial computing heavily relies on types like stacks, queues, dictionaries, sequences, sorted sequences, priority queues, graphs, points, planes, . . .

One year ago, we started a project (called LEDA for Library of Efficient Data types and Algorithms) to build a small, but growing library of data types and algorithms in a form which allows them to be used by non-experts. We hope that the system will narrow the gap between algorithms research, teaching, and implementation. In this paper we report on our difficulties and achievements. The main features of the library are:

- 1) A clear separation between (abstract) data types and the data structures used to implement them, cf. section II. This distinction is frequently not made in the combinatorial algorithms literature, but it is crucial for a library. Note that we stated above that each researcher implemented its own version of a balanced tree, i.e. a data structure, and not its own version of a dictionary, i.e. a data type. The data types currently available are stack, queue, list, set, dictionary, ordered sequence, priority queue, directed graph and undirected graph. The most difficult decision, which we faced, was the treatment of positions and pointers. For the efficiency of many data structures it is crucial that some operations on the data structure take positions in (= pointers into) the data structure as arguments. We have chosen an item concept to cast the notion of position into an abstract form, cf. section II.
- 2) Type parameters, cf. section II. Most of our data types have type parameters. For example, a dictionary has a key type  $K$  and an information type  $I$  and a specific dictionary type is obtained by setting, say,  $K$  to `int` and  $I$  to `real`.
- 3) A comfortable data type graph. It offers the standard iterations such as “for all nodes  $v$  of a graph  $G$  do” (written *forall\_nodes*( $v, G$ )) or “for all neighbors  $w$  of  $v$  do” (written *forall\_adj\_nodes*( $w, v$ )), it allows to add and delete vertices and edges and it offers arrays and matrices indexed by nodes and edges, . . ., cf. section III for details. The data type graph permits to write programs for graph problems in a form close to the way the algorithms are usually presented in text books. Section VII contains a list of the algorithms which are currently in LEDA.
- 4) Ease of use: LEDA is written in C++ (cfront version 1.2.1). All data types and algorithms are precompiled modules which can be linked with application programs. All examples given in this paper show executable code.

This paper is organized as follows. In section II we discuss data types and data structures, in section III the data type graph and in section IV we discuss the interaction of graphs and data types. In section V we briefly discuss the internal structure of LEDA and in section VI we report about our experiences with designing, implementing and using LEDA.

The design of LEDA is joint work by the two authors, the implementation was mostly done by the second author. LEDA is available from the authors for a handling charge of DM 100.

## II. Data types

One of the major differences between combinatorial computing and other areas of computing such as statistics, numerical analysis and linear programming is the usage of complex data types. Whilst the built-in types, such as integers, reals, vectors, matrices and functions, usually suffice in the other areas, combinatorial computing heavily relies on types like stacks, queues, dictionaries, sequences, sorted sequences, priority queues, graphs, points, planes, . . . . Even modern programming languages do not provide these types and hence they have to be defined and implemented by the individual user. A significant improvement would result, if these data types were made available as “packages” by some users and could then be reused by others. Of course, this requires to specify the properties of the data type independently of the implementation. We next discuss the LEDA specification of the data types dictionary, priority queue and partition.

**Example 1, Dictionary:** A dictionary is a partial function with finite domain from some universe  $K$  of keys into some set  $I$  of informations associated with the keys. An access operation looks up the function value at an argument, an insert extends the domain of definition, . . . . Thus a clean definition of the data type dictionary is available and hence a dictionary module should cause no principal difficulties. In fact, the programming language Comskee [BKMRS84] includes the dictionary among its built-in data types. A (minor) problem lies in the fact that most programming languages force the user to edit source text when he wants to change the key or information type. This problem is resolved in languages such as Clu or C++. Program 1 shows a small illustration of LEDA’s dictionary modul.

---

```

(1)  # include <LEDA/dictionary.h>
(2)  declare2(dictionary,int,int);      //K = I =int
(3)  main()
(4)  {  dictionary(int,int) D;
(5)      int k;
(6)      while (cin >> k)
(7)      {  if (D.defined(k))
(8)          D.change_inf(k,D.lookup(k)+1);
(9)          else D.insert(k,1);
(10)     }
(11)     forall_keys(k,D) cout << k << “ ” << D.lookup(k) << “\ n”;
(12) }

```

**Program 1:** Counting the number of occurrences of each element of a sequence of integers

---

### Program 1

---

The program reads a sequence of integers and counts the number of occurrences of each integer in the sequence. The number of occurrences is then listed for each integer in the sequence. The details are as follows. Line (1) includes LEDA’s dictionary modul.

In line (2) the dictionary type `dictionary(int,int)` is defined. Note that the dictionary module provides dictionaries from  $K$  to  $I$  where  $K$  and  $I$  are type variables. A specific dictionary type is introduced as shown in line (2). Line (4) introduces  $D$  as the name of an object of type `dictionary(int,int)`;  $D$  is initialized with the empty function. Line (6) to (10) step through the input sequence. In line (7) we test whether  $k$  belongs to the domain of  $D$  and then either increase the value at argument  $k$  by one or insert the pair  $(k,1)$  into the dictionary. Finally, line (11) steps through all keys  $k$  in the domain of  $D$  and prints  $k$  and the value of  $D$  at  $k$ .

LEDA is written in C++ which is an extension of the C programming language. In addition to the facilities of C, C++ provides data abstraction, data hiding, object based programming, operator overloading and many other features suitable for implementing abstract data types. For details of the C++ language see “The C++ Programming Language” [St86]. Every data type in LEDA is implemented as a C++ class which is a user defined type similar to a record type in PASCAL or a struct type in C. The members of a class in C++ are not only data fields (as in C or PASCAL) they can also be functions. We use these member functions to realize the operations on the corresponding data type. Member functions are invoked using the known syntax for record field or structure member access: Let  $D$  be an object (variable) of a class with a member functions  $f()$  then  $D.f()$  calls  $f$  for  $D$ .

**Example 2, Priority Queues:** Priority queues are frequently used in network algorithms; cf. section III for an example. It is tempting to define a priority queue as a dictionary with the additional operations *findmin* and *decrease\_inf* (We require a linear order on  $I$ ). The operation  $PQ.findmin()$  returns the element  $v$  in the domain of  $PQ$  with minimal function value and  $PQ.decrease\_inf(v, i)$  decreases the function value at argument  $v$  to  $i$ . The operation raises an error if  $i$  is larger than the old value at argument  $v$ . It is clear that a *decrease\_inf* operation involves a lookup and hence must be at least as costly as a lookup operation in a dictionary. However, several recent papers, e.g. [FT84] and [AMOT88] show that the *decrease\_inf* operation can be realized in time  $O(1)$  if its argument is the *position* of the pair  $(v, PQ(v))$  in the data structure realizing the priority queue  $PQ$ . This led to improvements in several network algorithms.

We conclude that the argument of a *decrease\_inf* operation cannot be the key  $v$ . But what should it be? It cannot be the position in a data structure since we want to clearly separate data type and data structure and therefore cannot use a notion of the particular implementation in the definition of the data type. What we need is an abstraction of position.

In LEDA, we call an abstract position an *item*. For most data types  $XYZ$  there is also a type  $XYZ.item$ . This type represents the set of positions in objects of data type  $XYZ$ . So, together with the data type *priority\_queue* there is also a type *priority\_queue.item* or briefly *pq.item*. Each *pq.item* contains or has associated with it a pair  $(v, i)$  with  $v \in K$  and  $i \in I$ . We use  $\langle v, i \rangle$  to denote the item which contains the pair  $(v, i)$ . A priority queue  $PQ$  over  $K$  and  $I$  is then nothing but a collection of *pq.items* where each item contains a pair  $(v, i)$ .

**Remark 1:** In mathematical terms a priority queue in LEDA is simply a partial function from the set of *pq.items* to the cartesian product  $K \times I$ . In computer science

terminology, we may view a *pq\_item* as the name ( $\hat{=}$  address) of a container holding an argument-value pair  $(v, i)$ .

**Remark 2:** Access by position can now be achieved as follows. When we insert a pair  $(v, i)$  into a priority queue *PQ*, the operation *PQ.insert(v, i)* returns an item, say *it*, i.e. the name of a container which holds the pair  $(v, i)$ . The user of the priority queue remembers this name and can now access the pair  $(v, i)$  in two different ways. He can either access it through the item *it* which was returned by the insert (this is access by position) or through the key *v* (this is access by key). There is a crucial difference between the two modes of access. In the latter case, the key *v* identifying the pair  $(v, i)$  was provided by the user and hence access involves a dictionary look-up, in the former case, the name *it* identifying the pair  $(v, i)$  was produced by the machine and therefore can give direct access. The *decrease\_inf* operation uses access by position, i.e. whenever the user wants to decrease the value associated with *v*, he calls *PQ.decrease\_inf* and gives it the name of the container as an argument. In this way, the item concept captures the notion of access by position but is nevertheless independent of implementation.

The complete LEDA specification of the data type priority queue follows.

A **priority queue** *Q* is collection of items (predefined type “pq\_item”). Every item contains a key from a type *K* and an information from a type *I*. *K* is called the key type of *Q* and *I* is called the information type of *Q*. The number of items in *Q* is called the size of *Q*. If *Q* has size zero it is called the empty priority queue. We use  $\langle k, i \rangle$  to denote the pq\_item with key *k* and information *i*. There must exist a total ordering “ $\leq$ ” on *I*.

### 1. Declaration of a priority queue type

```
declare2(priority_queue,K,I)
```

introduces a new data type with name “priority\_queue(K,I)” consisting of all priority queues with key type *K* and information type *I*.

### 2. Declaration of a priority queue variable

```
priority_queue(K,I) Q;
```

declares a variable *Q* of type “priority\_queue(K,I)” and initializes it with the empty priority queue.

### 3. Operations on a priority queue

<i>K</i>	<i>Q.key(pq_item it)</i>	returns the key of item <i>it</i>
<i>I</i>	<i>Q.info(pq_item it)</i>	returns the information of item <i>it</i>
<i>pq_item</i>	<i>Q.find_min()</i>	returns the item with minimal information
<i>pq_item</i>	<i>Q.insert(K k, I i)</i>	adds a new item $\langle k, i \rangle$ to <i>Q</i> and returns it
<i>pq_item</i>	<i>Q.delete_item(pq_item it)</i>	removes and returns the item <i>it</i> from <i>Q</i>
<i>pq_item</i>	<i>Q.delete_min()</i>	removes the item with minimal information from <i>Q</i> and returns it

<i>pq_item</i>	<i>Q.decrease_inf(pq_item it, I i)</i>	makes <i>i</i> the new information of item <i>it</i> ( $i \leq info(it)$ )
<i>void</i>	<i>Q.clear()</i>	makes <i>Q</i> the empty priority queue
<i>int</i>	<i>Q.empty()</i>	tests whether <i>Q</i> is the empty priority queue

Note that the access in the *delete\_item* and the *decrease\_inf* operation is by position. An application of priority queues is shown in program 2 of section III.

#### 4. Implementations

There are currently two implementations of priority queues. The first implementation is based on Fibonacci-heaps ([FT84]) and achieves amortized time  $O(1)$  for all operations except *delete\_item* and *delete\_min*. The latter two operations take amortized time  $O(\log n)$  where  $n$  is the current size of the priority-queue. The second implementation is based on restricted heaps ([AMOT88]). It requires the keys to be non-negative integers and achieves amortized time  $O(1)$  for all operations except *insert*, *delete\_item* and *delete\_min* which take amortized time  $O(\log C)$  where  $C$  is the largest key ever in the priority queue. ■

**Example 3, Partitions or Disjoint Sets:** In this example we discuss partitions of finite sets. An application of partitions is shown in program 4 of section IV. Partitions are also used to discuss some implementation details of LEDA in section V.

An object *P* of type *partition* consists of a finite set of *partition\_items* and a partition of this set into blocks. The declaration *partition P* introduces *P* as the name of a partition and initializes it to the empty partition. There are three operations.

<i>partition_item</i>	<i>P.make_block()</i>	returns a new <i>partition_item it</i> and adds the block $\{it\}$ to the partition <i>P</i> .
<i>int</i>	<i>P.same_block(partition_item p, q)</i>	returns true if <i>p</i> and <i>q</i> belong to the same block of the partition <i>P</i> .
<i>void</i>	<i>P.union_blocks(partition_item p, q)</i>	unites the blocks of <i>P</i> containing items <i>p</i> and <i>q</i> .

We want to stress that the *make\_block*-operation has no parameter. It is not given an object which it is supposed to add to the partition *P* (this would imply access by key lateron), but the operation itself chooses an item *it*, returns it and adds the block  $\{it\}$  to the partition *P*. The user has no idea what *it* is and he does not need to know. The only thing, that is important to him, is the partition of the items into blocks. This usage of items is similar to the usage of atoms in SETL. ■

Access by position instead of key is abundant in the design of efficient data structures; e.g. lists are accessed by position, all operations in the union-find problem use access by position, the *decrease-inf* operation in priority queues is based on it and fingers in finger trees are positions. Thus the position concept is crucial for the design of efficient data structures. In LEDA we use items as an abstraction of positions. The examples above show that the item concept leads to natural specifications of data types.

### III. Graphs

Graph algorithms are a prime example of combinatorial computing. We use the shortest path and the minimum spanning tree problem to illustrate LEDA's comfortable data type `graph` and its interaction with other data types. Program 2 shows Dijkstra's algorithm (cf. [AHU83], [M84,section IV.7.2], [T83]) for the single source shortest path problem in digraphs with non-negative edge costs.

---

```

(1)  #include <LEDA/graph.h>
(2)  #include <LEDA/prio.h>
(3)  declare2(priority_queue,node,float)
(4)  declare(node_array,pq_item)
(5)  void DIJKSTRA(graph& G, node s, edge_array(float)& cost,
(6)          node_array(float)& dist, node_array(edge)& pred )
(7)  { priority_queue(node,float) PQ;
(8)    node_array(pq_item) I(G, nil);
(9)    pq_item it;
(10)   float c;
(11)   node u, v;
(12)   edge e;
(13)   forall_nodes(v, G)
(14)   { pred[v] = 0;
(15)     dist[v] = infinity;
(16)     I[v] = PQ.insert(v, dist[v]);
(17)   }
(18)   dist[s] = 0;
(19)   PQ.decrease_inf(I[s], 0);
(20)   while (!PQ.empty())
(21)   { it = PQ.delete_min();
(22)     u = PQ.key(it);
(23)     forall_adj_edges(e, u)
(24)     { v = target(e);
(25)       c = dist[u] + cost[e];
(26)       if ( c < dist[v] )
(27)       { dist[v] = c;
(28)         pred[v] = e;
(29)         PQ.decrease_inf(I[v], c);
(30)       }
(31)     }
(32)   } // while
(33) }
```

Program 2: Dijkstra's algorithm

The algorithm uses the data type graph and priority queue (lines (1) and (2)). The input to the algorithm is a graph  $G$ , a node  $s$  of  $G$  and a non-negative cost for each edge. It returns for each node  $v$  the length of a shortest path from  $s$  to  $v$  (array  $dist$ ) and the last edge on such a shortest path (array  $pred$ ). In LEDA we use edge- and node-arrays for the latter three parameters. A `node_array(XYZ)` is an array which is indexed by the nodes of some graph and whose entries have values of type XYZ. Thus a `node_array(edge)` is a mapping from node to edges.

The algorithm maintains for each node  $v$  a temporary distance label  $dist[v]$ . Initially,  $dist[s] = 0$  and  $dist[v] = \infty$  for  $v \neq s$ , cf. lines (13)–(19). In LEDA the loop `forall_nodes(v, G){...}` can be used to iterate over all nodes  $v$  of a graph  $G$ .

Dijkstra's algorithm uses a priority queue  $PQ$ . The priority queue contains pairs  $(v, dist[v])$  and hence has type `priority_queue(node, float)`; cf. lines (3) and (7). Each node  $v$  of the graph needs to know the position of the item  $\langle v, dist[v] \rangle$  in the priority queue. We therefore declare the data type `node_array(pq_item)` in line (4) and declare `node_array(pq_item) I(G, nil)` in line (8). In this declaration the parameter  $G$  tells LEDA that we want an array which is indexed by the nodes of  $G$  and the second parameter tells it that we want all entries initialized to the `pq_item nil`.

Initially, the items  $\langle s, 0 \rangle$  and  $\langle v, infinity \rangle$  for  $v \neq s$  are put into  $PQ$ , cf. line (16). Then in each iteration we select and delete an item  $it$  with minimal  $inf$  from  $PQ$ , cf. line (21). Let  $it = \langle u, dist[u] \rangle$ , cf. line (22). We now iterate through all edges  $e$  starting in edge  $u$ ; cf. line (23). Let  $e = (u, v)$  and let  $c = dist[u] + cost[e]$  be the cost of reaching  $v$  through edge  $e$ , cf. lines (24) and (25). If  $c$  is smaller than the temporary distance label  $dist[v]$  of  $v$  then we change  $dist[v]$  to  $c$  and record  $e$  as the new predecessor of  $v$  and decrease the information associated with  $v$  in the priority queue., cf. lines (26) to (29).

The running time of this algorithm for a graph  $G$  with  $n$  nodes and  $m$  edges is  $O(n + m + T_{declare} + n(T_{insert} + T_{Deletemin} + T_{get\_inf}) + m \cdot T_{Decrease\_key})$  where  $T_{declare}$  is the cost of declaring a priority queue and  $T_{XYZ}$  is the cost of operation XYZ. With the time bounds stated in section II we obtain an  $O(m + n \log n)$  and  $O(m + n \log C)$  algorithm respectively.

Program 2 is very similar to the way Dijkstra's algorithm is presented in textbooks, cf. [AHU83], [M84], [T83]. The main difference is that **program 2 is executable code** whilst the textbooks still require the reader to fill in (non-trivial) details.

Dijkstra's algorithm is a useful subroutine for the solution of the all-pair shortest path problem in graphs with arbitrary edge costs, cf. [M84, section IV.7.4]. One uses the algorithm of Bellman-Ford to solve the single-source shortest path for some source  $s$ , then uses the solution of this computation to make all edge costs non-negative and then uses Dijkstra's algorithm to solve  $n - 1$  single-source problems with non-negative edge costs. In order for this approach to work it is important that all nodes of the graph are reachable from  $s$ . The easiest way to achieve this is to add a new node  $s$  and to add edges of high cost from  $s$  to all other nodes. The details are given in program 3.



---

```

(1)  #include <fopra/graph.h>
(2)  void all_pairs_shortest_paths(graph& G, edge_array(float)& cost,
(3)                                     node_matrix(float)& DIST)
(4)  {
(5)      // computes for every node pair (v, w) DIST[v][w] = cost of the least cost
(6)      // path from v to w, the single source shortest paths algorithms BELLMAN_FORD
(7)      // and DIJKSTRA are used as subroutines
(8)      edge e;
(9)      node v;
(10)     float C = 0;
(11)     forall_edges(e, G) C += cost[e];
(12)     node s = G.new_node();           // add s to G
(13)     forall_nodes(v, G) G.new_edge(s, v); // add edges (s, v) to G
(14)     node_array(float) dist1(G);
(15)     node_array(edge) pred(G);
(16)     edge_array(float) cost1(G);
(17)     forall_edges(e, G) cost1[e] = (source(e) == s)?C : cost[e];
(18)     BELLMAN_FORD(G, s, cost1, dist1, pred);
(19)     G.del_node(s);                   // delete s from G
(20)     edge_array(float) cost2(G);
(21)     forall_edges(e, G) cost2[e] = dist1[source(e)] + cost[e] - dist1[target(e)];
(22)     forall_nodes(v, G) DIJKSTRA(G, v, cost2, DIST[v], pred);
(23)     forall_nodes(v, G)
(24)         forall_nodes(w, G) DIST[v][w] = DIST[v][w] - dist1[v] + dist1[w];
(25) }

```

Program 3: All Pairs Shortest Path

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### Program 3

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## IV. Graphs and Data Types

We use the minimum spanning tree problem to further discuss the interaction between graphs and data types. Program 4 shows a minimum spanning tree algorithm. We do not discuss the details of the algorithm, cf. [M84, section IV.8] for the proof of correctness, but concentrate instead on the similarities of programs 2 and 4. In both cases a `node_array(item)` is used and in both cases the program starts by creating one item for each node of the graph. Similar statements occur in many graph algorithms.

A user of LEDA may want to incorporate all these statements into the declaration of either the partition or the priority queue. He can do (in fact we have done it already)

---

```

(1)  #include <LEDA/graph.h>
(2)  #include <LEDA/partition.h>
(3)  declare(node_array,partition_item);
(4)  int cmp(edge e1, edge e2, edge.float(array)& C) { return (C[e1] - (C[e2])); }
(5)  void MST(graph& G, edge_array(float)& cost, edgelist& EL)
(6)  // the input is an undirected graph G together with a cost function
(7)  // cost on the edges; the algorithm outputs the list of edges EL of
(8)  // a minimum spanning tree
(9)  {
(10)     node v, w;
(11)     edge e;
(12)     partition P;
(13)     node_array(partition_item) I(G);
(14)     forall_nodes(v, G) I[v] = P.make_block();
(15)     edgelist OEL = G.all_edges();
(16)     OEL.sort(cmp, cost);
(17)     // OEL is now the list of edges of G ordered by increasing cost
(18)     EL.clear();
(19)     forall(e, OEL)
(20)     { v = source(e);
(21)       w = target(e);
(22)       if (!(P.same_block(I[v], I[w])))
(23)       { P.union_blocks(I[v], I[w]);
(24)         EL.append(e);
(25)       }
(26)     }
(27) }

```

Program 4: Minimum Spanning Tree

---

#### Program 4

---

so by deriving a data type `node_partition` from the data type `partition` (and similarly for `priority_queue`). A `node_partition`  $Q$  consists of a `node_array(partition_item)`  $I$  and a `partition`  $P$ . The declaration

```
node_partition Q(G)
```

will then execute lines (3), (12), (13), and (14). The operations on `node_partitions` are also easily derived, e.g.  $Q.same\_block(v, w)$  just calls  $P.same\_block(I[v], I[w])$ . Altogether, this yields the simplified program 5.

The reader may ask at this point why we provide the elegant types `node_partition` and `node_priority_queue` in this roundabout way. Why do we first introduce items and then show how to hide them? The reason is that in the case of graphs the ground set of the partition or priority queue is static. In general, this is not the case

Consider for example, the standard plane sweep algorithm (cf. [M84, section VII.4.1,

---

```

(1)  #include <LEDA/graph.h>
(2)  #include <LEDA/partition.h>
(3)  int cmp(edge e1, edge e2, edge_array(float)& C) { return (C[e1] - (C[e2])); }
(4)  void MST(graph& G, edge_array(float)& cost, edgelist& EL)
(5)  { node v, w;
(6)    edge e;
(7)    node_partition Q(G);
(8)    edgelist OEL = G.all_edges();
(9)    OEL.sort(cmp, cost);
(10)   EL.clear();
(11)   forall(e, OEL)
(12)   { v = source(e);
(13)     w = target(e);
(14)     if (!(Q.same_block(v, w))
(15)     { Q.union_blocks(v, w);
(16)       EL.append(e);
(17)     }
(18)   }
(19) }

```

Program 5: Simplified MST Program

---

### Program 5

---

section VII.4.1]) for computing line segment intersections. It uses two information structures, usually called the *X*- and *Y*-structure. The *Y*-structure is an ordered sequence of intersections of the sweep line with the line segments and the *X*-structure is a priority queue. The priority queue contains an event for each line segment *l* of the *Y*-structure which intersects the succeeding line segment *lsuc* in front of the sweep line. The event occurs when the sweep line passes the intersection.

In the algorithm the sweep line is moved from left to right. The sweep line stops whenever it passes through a left or right endpoint of a line segment or through an intersection. In either case the *X*- and the *Y*-structure have to be updated appropriately. Consider for example the situation where a left endpoint of some line segment *l* is encountered at coordinate *x*. The following actions have to be taken: insert *l* into the *Y*-structure, say between *lpred* and *lsuc*, remove the event, if any, associated with *lpred* from the *X*-structure and add the events associated with *lpred* and *l*, if they exist.

The appropriate LEDA types are

```

sort_seq(line_segment, pq_item) Y_structure;
priority_queue(sortseq_item, float) X_structure;

```

The *Y*-structure is a sequence of sortseq\_items. Each item contains a line segment as its key and a pq\_item as its information. The ordering is induced by the intersection of the line segments with the sweep line. Similarly, the *X*-structure stores for each item of the

---

```

(1)  x_sweep = x;
(2)  sort_seq_item sit = Y_structure.insert(l, nil);
(3)  sort_seq_item sitpred = Y_structure.predecessor(sit);
(4)  sort_seq_item sitsuc = Y_structure.successor(sit);
(5)  pq_item pqit;
(6)  if (sitpred != nil)
(7)  { if ((pqit = Y_structure.info(sitpred)) != nil) X_structure.delete_item(pqit);
(8)    // removes the event, if any, associated with sitpred from the event queue
(9)    line_segment lpred = Y_structure.key(sitpred);
(10)   condpair inter = intersection(lpred, l);
(11)   if (inter.status && (inter.x > x_sweep))
(12)     Y_structure.change_inf(sitpred, X_structure.insert(sitpred, inter.x))
(13)   else
        Y_structure.change_inf(sitpred, nil)
(14) }
(15) if (sitsuc != nil)
(16) { line_segment lsuc = Y_structure.key(sitsuc);
(17)   condpair inter = intersection(l, lsuc);
(18) }
(19) if (inter.status && (inter.x > x_sweep))
(20)   Y_structure.change_inf(sit, X_structure.insert(sit, inter.x));

```

Program 6: This program fragment processes the left endpoint of a line segment in the plane sweep algorithm for line segment intersection.

---

### Program 6

---

*Y-structure* the  $x$ -coordinate of the corresponding event. Each item in the *Y-structure* has direct access (through *pq\_item*) to the associated event and each event in the *X-structure* has direct access (through *sortseq\_item*) to the affected position of the *Y-structure*. Program 6 shows the code which processes the left endpoint of a line segment  $l$  at  $x$ -coordinate  $x$ . It makes use of a function *intersection(line\_segment lpred, l)*, which given two line segments returns a pair (*status, xcoord*) (type *condpair*), where *status* indicates whether the two segments intersect and, if so, *xcoord* is the  $x$ -coordinate of the intersection. The variable *x\_sweep* denotes the current position of the sweep line.

## V. Inside LEDA

This section gives some of the implementation details of LEDA. The reader should be familiar with the major features of C++.

### 1. Implementation of abstract data types

As mentioned before each data type in LEDA is realized by a C++ class. The operations and operators are either member- or friend-functions of the corresponding class. In C++ a class consists of two parts:

a) The *declaration* of the class describes the interfaces of its member functions (return and parameter types). This part of a class corresponds to the abstract specification of the data type. As an example we give the declaration of the class `partition` (used in programs 4 and 5):

```
// a partition is a forest of partition_nodes
class partition_node {
friend class partition;
// private:
partition_node* father; // parent node in the forest
partition_node* next; // to link all used nodes
int size;
public:
partition_node(partition_node* n) { father=0; size=1; next=n; }
}

// a partition item is a pointer to a partition node;
typedef partition_node* partition_item;

class partition {
// private:
partition_item used_items; // list of used partition items
public: // operations
partition_item make_block();
partition_item find_block(partition_item);
int same_block(partition_item, partition_item);
void union_blocks(partition_item, partition_item);
void clear();
partition() { used_items = 0; } // constructor
partition() { clear(); } // destructor
};
```

Only the public part of class `partition` appears in the LEDA manual.

b) The *implementation* of the class is the C++ code realizing the member functions declared in part a). The implementation of class `partition` follows:

```
// union find with weighted union rule and path compression
partition_item partition::make_block()
{ // create new item and insert it into list of used item
    used_items = new partition_node(used_items);
    return used_items;
}
```

```

partition_item partition::find_block(partition_item it)
{ // return the root of the tree that contains item it
  partition_item x, root = it;
  while (root->father) root = root->father;
  // path compression:
  while (it!=root)
  { x = it->father;
    it->father = root;
    it = x;
  }
  return root;
}

int partition::same_block(partition_item a, partition_item b)
{ return find_block(a)==find_block(b); }

void partition::union_blocks(partition_item a, partition_item b)
{ // weighted union
  a = find_block(a);
  b = find_block(b);
  if (a->size > b->size)
  { b->father = a;
    a->size += b->size; }
  else{ a->father = b;
        b->size += a->size; }
}

void partition::clear()
{ // delete all used items
  partition_item p = used_items;
  while (used_items)
  { p = used_items;
    used_items = used_items->next;
    delete p;
  }
}

```

Note that only member functions or member functions of friends are allowed to access the private data of a class. This guarantees that the user of a class can manipulate objects of this class only by using member functions, i.e. only by the operations defined in the specification of the data type. This data hiding feature of C++ supports complete separation of the specification and the implementation of data types.

For every data type XYZ there exists a so-called header file “XYZ.h” containing the declaration of class XYZ. Programs using XYZ have to include this file. For example, partitions can only be used after the line

```
#include <LEDA/partition.h>    (see program 4)
```

The implementation of all classes are precompiled and contained in a module library which can be used by the linker.

## 2. Parametrized Types

Most of the data types in LEDA have type parameters. In section II we defined a dictionary to be a mapping from a key type  $K$  to an information type  $I$ , here  $K$  and  $I$  are formal type parameters. The LEDA statement

```
“declare2(dictionary, t1, t2)”
```

declares a dictionary type with name “*dictionary(t<sub>1</sub>, t<sub>2</sub>)*” and actual type parameters  $K = t_1$  and  $I = t_2$ . How is this realized?

Note that the operations on a dictionary are independent of key type  $K$  and information type  $I$ . So it is possible to implement all dictionary operations (member functions) without knowing  $K$  and  $I$ . This is done by implementing a base class dictionary with  $K = I = void*$ . For example

```
class dictionary { // base class
  // private data
  :
public:
  :
  void insert(void* k, void* i);
  void* access(void* k);
  :
};
```

In C++ the type `void*` (pointer to void) is used for passing arguments to functions that are not allowed to make any assumptions about the type of their arguments and for returning untyped results from functions. To declare a concrete data type for given actual type parameters (e.g., `dictionary(int,int)`) a derived class of the corresponding base class (`dictionary`) has to be declared. This derived class inherits all operations and operators from its base class and performs in addition all necessary type conversion:

```
class dictionary(int,int): public dictionary {
  :
  void insert(int k,int i) { dictionary::insert((void*)k, (void*)i); }
  int access(int k) { return (int)dictionary::access((void*)k); }
  :
};
```

C++'s macro facility is used to fill in such declarations of derived classes. There are macros `declare`, `declare2`, ... to declare data types with one, two, ... type parameters. `declare2(dictionary,int,int)` for example just creates the above declaration of `dictionary(int,int)`.

### 3. Iteration

LEDA provides various kinds of iteration statements. Examples are

for lists:

```
forall( $x,L$ ) { the elements of  $L$  are successively assigned to  $x$  }
```

for graphs:

```
forall_nodes( $v,G$ ) { the nodes of  $G$  are successively assigned to  $v$  }
```

```
forall_adj_nodes( $w,v$ ) { the neighbor nodes of  $v$  are successively assigned to  $w$  }
```

All these statements are macros that are expanded to more complicated for-statements. The list iteration macro `forall` is defined as follows

```
#define forall( $x,L$ ) for( $L$ .init_cursor();  $x = L$ .current_element();  $L$ .move_cursor(); )
```

Here `init_cursor()`, `move_cursor()` and `current_element()` are member functions of the class `list` that manipulate an internal cursor.

The other iteration statements are implemented similarly.

## VI. Experiences

We report on our experiences in designing, implementing and using LEDA.

We found the task of specifying a data type surprisingly difficult. The data types `dictionary` and `priority queue` were the first two examples which we tried. The `dictionary` was readily specified; we had, however, lengthy discussions whether a `dictionary` is a function from keys to variables of type  $I$  or to objects of type  $I$ . The former alternative allows array notation for dictionaries, e.g. line 8 in program 1 could be written `D[k]++`, but also allows the user to store pointers to variables in our modules. The latter alternative makes notation more cumbersome but seems to be safer. We did not resolve the conflict but now have both alternatives to gain further insight by experiments. The `priority queue` took us a long time. We wanted to support access by position and we wanted a complete separation of data type and data structure. We found neither the combinatorial algorithms nor the abstract data type literature very helpful. In the algorithms literature the position concept is usually only discussed in the context of concrete implementations and is then equated with an index in an array or a pointer to a node of a data structure. In this way, no abstract definition of the data type is given and the data structures are intimately tied with the applications; e.g. `priority queues` are tied to shortest path calculations and `partitions` are tied to graph algorithms. In the latter part of the literature the position concept is only discussed in simple examples, e.g. iterators in linear lists [TRE88].



We use items as an abstraction of positions. Items are similar to the atoms of SETL. We found the item approach very flexible and, once we used it for priority queues, the specification of data types like sequences, partitions and lists became easy.

The implementation of LEDA was mostly done by the second author, in particular, lists, graphs, and sorted sequences were implemented by him. Once the standards were set, we asked students to join in and to either realize additional data types or to give alternative realizations. Implementations of various kinds of dictionaries (BB[ $\alpha$ ]-trees, red-black-trees, (a,b)-trees, dynamic perfect hashing) and priority-queues (Fibonacci-heaps, C-heaps) were provided by Dirk Basenach, Jürgen Dedorath, Evelyn Haak, Michael Muth, Michael Wenzel and Walter Zimmer.

LEDA was used to write graph and geometry algorithms. Some examples are shortest paths, components of various kinds, unweighted and weighted matchings, network flows, embeddings of planar graphs, visibility graphs of line segments, Voronoi diagrams and intersection of half spaces. The graph users liked LEDA because all the required data types such as graphs, node- and edge-arrays, lists, dictionaries, ... were available and hence LEDA increased their productivity enormously. This has led to more experimental work, one of the goals of the project. The first geometry users of LEDA were much less enthusiastic because almost none of the required types such as points, lines, ... were available. Stefan Meiser implemented some of them and we are now hearing the first positive reactions from the geometry users.

## VII. Conclusions

LEDA is a library of efficient data types and algorithms. At present, its strength is graph algorithms and the data structures related to them. The computational geometry part is evolving.

There are several other projects which aim for similar goals as LEDA, e.g. [B88, So89, L89]. We believe, that LEDA compares well with these systems because of the

- clear separations between specification and implementation,
- the natural syntax, and
- the inclusion of many of the most recent and most efficient data structures and algorithms.

We close this section with a list of algorithms that we implemented using LEDA data types. All graph algorithms are part of the library. A LEDA function  $F(\text{graph} \& G, \dots)$  accepts any user-defined graph type ( $\text{graph}(\text{node\_type}, \text{edge\_type})$ ) as argument.

### 1. Graph Algorithms

#### 1.1. Basic Graph Algorithms

- depth first search
- breadth first search

- connected components
- transitive closure

### 1.2. Shortest Path Algorithms (see section III)

- Dijkstra's algorithm
- Bellman/Ford algorithm
- all pairs shortest paths

### 1.3. Matchings

- maximum cardinality bipartite matching
- maximum weight bipartite matching

### 1.4. Network Flow

- maximum flow algorithm of Galil/Namaad
- maximum flow algorithm of Tarjan/Goldberg

### 1.5. Planar Graphs

- triangulation
- straight line embedding

## 2. Computational Geometry

- intersection of half spaces
- convex hull of point sets
- construction of Voronoi diagrams
- construction of visibility graphs

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