Exercise 1 (10 points).
Given two metric spaces $A, B$ and a continuous map $\varphi : A \to B$. Prove that preimages under $\varphi$ of sets that are $\mathbb{C}$-closed in $B$ are $\mathbb{C}$-closed in $A$.

Exercise 2 (15 points).
Let $A = \mathbb{C}[X_1, \ldots, X_N]$ with the usual action of $\text{GL}_N$. Prove that this action preserves Waring rank and border Waring rank, i.e.,

$$\text{WR}(h) = \text{WR}(gh) \quad \text{and} \quad \text{WR}(h) = \text{WR}(gh)$$

for all $h \in A$, $g \in \text{GL}_N$.

Exercise 3 (15 points).
Let $0 \neq f \in \mathbb{C}[X_1, \ldots, X_n]$ and define the subset $D_f \subseteq \mathbb{C}^n$ via

$$D_f := \{x \in \mathbb{C}^n \mid f(x) \neq 0\}.$$ 

Prove that $\overline{D_f} = \mathbb{C}^n$. 