Parallel Prefix Computation (PPC)

Given:
- associative operator $\oplus: D \times D \rightarrow D$
- circuit $C$ implementing $\oplus$
- inputs $d_i \in D$, $i \in \{1, \ldots, n\}$

Compute:
- outputs $\pi_i = \bigoplus_{j=1}^{i} d_i$, $i \in \{1, \ldots, n\}$

Goals:
- minimize size of circuit
- minimize delay of circuit

Simplifications:
- assign unit size to $C$, ignore buffers
- assign unit depth to $C$, assume buffers have smaller depth
- minimize depth and fan-out in lieu of delay

State of the Art and Our Results

Kogge and Stone:
- depth $\lceil \log n \rceil$ (optimal)
- size $n \log n$ (bad)
- fan-out 2 (optimal)

Ladner and Fischer (LF):
- depth $\lceil \log n \rceil$ (optimal)
- size $\approx 4n$ (asymptotically optimal)
- fan-out $\Theta(n)$ (bad)

Our Circuit:
- adds redundant gates to the LF construction to reduce fan-out
- achieves fan-out $F \geq 3$ by increasing size
- optimal depth and asymptotically optimal size
- can increase depth for smaller size (as for LF)

Applications: Adders, metastability-containing Gray code comparators

References

