Optimising for Scale in Globally Multiply-Linked Gravitational Point Set Registration Leads to Singularities
—Supplementary Material—

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In this supplementary material, we provide more details about the experiments and visualise the results. The primary purpose of the experiments is to confirm that the proofs for the geometric primitives (*a unit circle, a unit sphere, a unit disk and a unit ball*) presented in the main matter are generalisable to 1) other geometric primitives and 2) real data. Many geometric primitives such as cubes, cones, cylinders, pyramids, prisms, octahedrons, icosahedrons, dodecahedrons and ellipsoids are topologically isomorphic to a sphere. We believe that our proof is generalisable to all geometric primitives which are topologically isomorphic to a sphere. Our experiments witness that our reasoning is also generalisable beyond geometric primitives, with large topological deviations from a sphere. Nevertheless, we do not have a rigorous proof for that yet. Moreover, the next relevant question is, whether a shape exists at all, so that at optimal alignment with a copy of itself, GPE is locally minimal. Search for such a shape is the matter of future work.

I. Datasets

As mentioned in the main matter, we use different types of shapes in the experimental validation, including further geometric primitives, humans, animals and equipment. The geometric primitives are generated with *MeshLab* [4]. For the mesh upsampling, we convert the surfaces into signed distance field representation [5] and run marching cubes over the volume [7]. The animals and equipment shapes are taken from the *ply* files repository [2]. We also use a 3D model of the comet 67P/Churyumov-Gerasimenko acquired during the *Rosetta* space mission [3], and own human scans reconstructed from a multi-view system with AgiSoft [1]. The selected samples cover a wide variety of shapes arising in many practical applications.

II. Visualisation of the Results

Visualisation of selected point set alignments with scaling are provided in Fig. I. We use a 7 DoF version of the Barnes-Hut Rigid Gravitational Approach (BH-RGA) [6] with the varying octree threshold γ, though this parameter barely influences the outcomes. Fig. I provides an overview of the performed tests on the datasets described in Sec. I. The demonstrated combinations are *cube and cone*, *cone and cube*, *comet and icosahedron*, *icosahedron and comet*, *a human in different poses*, *cow and spider*, *spider and cow*, *helicopter and tommy gun* and *tommy gun and helicopter* (the first sample is the reference, and the second one is the template). In all experiments, the samples cannot be exactly matched with 7 DoF, due to substantial differences in the shapes. Thus, the templates have always large amounts of clustered outliers. Nevertheless, we can imagine how these shapes look like when optimally aligned, in terms of scale.

All alignments with 7 DoF result in singularities. Depending on the shape, a singularity does not mean collapsing to a single point but rather approaching a single-point singularity with $s{\sim}0$. We see that templates always converge towards the centres of gravity of the references. Aligned templates can be seen in Fig. I with zoom.

References

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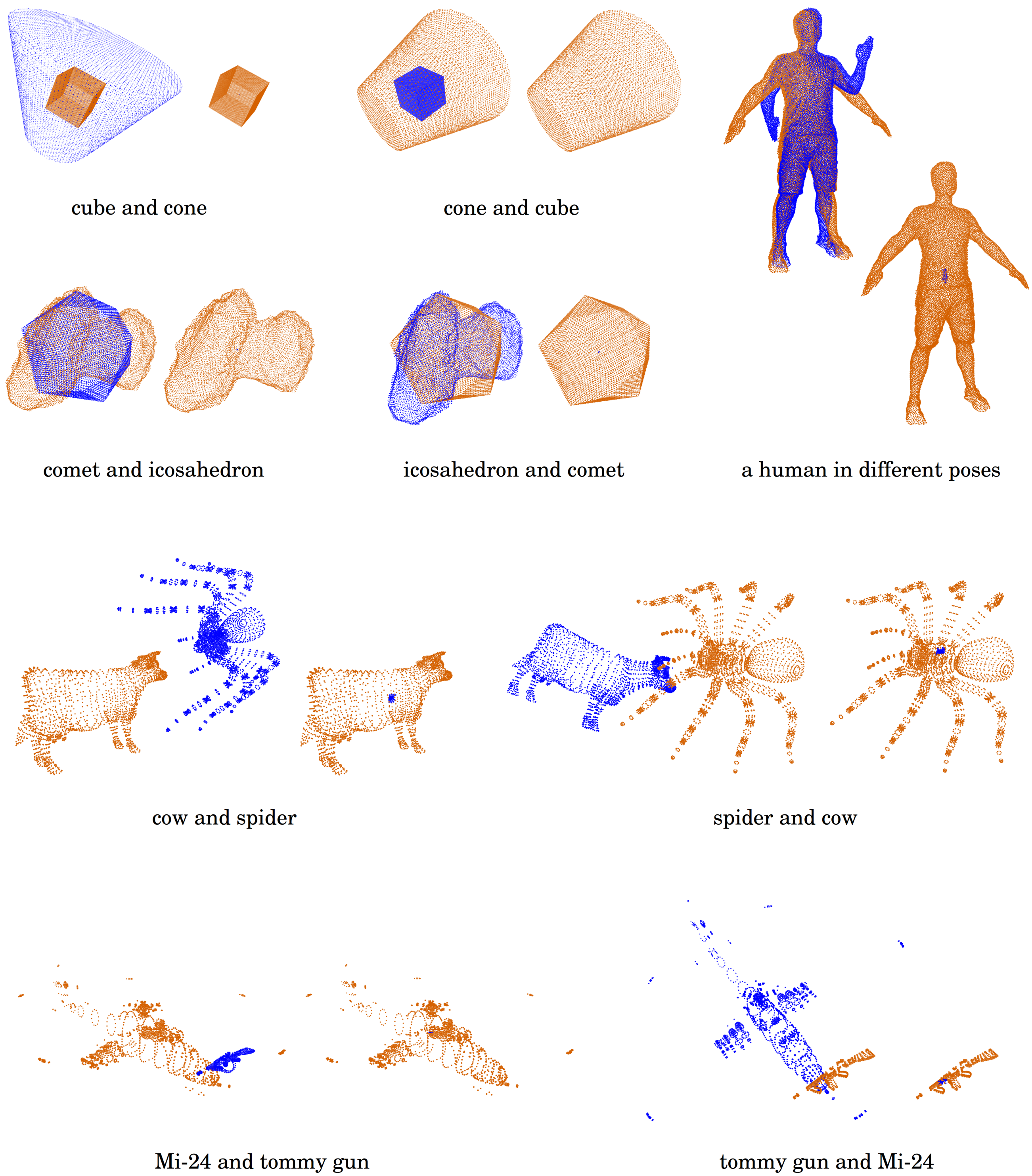

Figure I: Selected registration results by a 7 DoF version of BH-RGA [6] with different shape combinations. For every pair of shapes, initialisation is on the left, and the alignment is on the right. All registrations produce singularities. Note that some templates do not collapse to a single point but converge very close to a single-point singularity. This happens for highly differing shapes due to the inhomogeneous gravitational force field at the centre of gravity of the reference, *i.e.,* every point of the template has its local singularity.