**Introduction**

Adiabatic Quantum Computers (AQC)s excel in solving Quadratic Unconstrained Binary Optimization Problems:

\[
\arg \min_{x \in \mathbb{B}} x^T Q x \quad \text{(QUBO)}
\]

In most vision problems, constraints are required:

\[
\min_{x \in \mathbb{E}} x^T Q x \quad \text{s.t.} \quad a_i^T x = b_i, \quad i = 1, \ldots, m
\]

This can be turned into a QUBO with regularization. Yet, constraints are not exactly satisfied.

We introduce Q-FW, a true quantum-classical hybrid solver tailored for binary optimization problems subject to linear equality and inequality constraints.

Such problems occur frequently in computer vision. Our solver enables the use of quantum hardware for computer vision, paving the way to quantum computer vision.

**Copositive Programming**

Copositive Programming concerns optimization over the set of completely positive matrices:

\[
\Delta := \text{conv}\{w w^T : w \in \mathbb{E}\}
\]

The standard form (CP) is:

\[
\min_{w \in \Delta} Tr(CW) \quad \text{s.t.} \quad AW = v \quad \text{(CP)}
\]

(CP) has the modeling power of non-convex optimization with the interpretability of convex optimization.

We reformulate (QUBO) with constraints as a (CP):

\[
\min_{x} Tr(QX) \quad \text{s.t.} \quad a_i^T x = b_i, \quad i = 1, 2, \ldots, m, \\
\text{Tr}(A,X) = b_i^2, \quad i = 1, 2, \ldots, m, \\
\text{diag}(X) = x, \text{ and } [1 \ x^T \ x] \in \Delta
\]

This reformulation is tight.

**Quantum Frank Wolfe**

**Challenge**: Solving (CP) is NP-Hard.

**Opportunity**: Linear Minimization over \(\Delta\) is a (QUBO).

\[
u = \arg \min_{w \in \mathbb{E}} Tr(w^T CW) \iff uu^T = \arg \min_{w \in \Delta} Tr(CW)
\]

**Idea**: Design a Frank-Wolfe-type algorithm and solve (LM) step with an AQC.

Consider the Augmented Lagrangian:

\[
L_\beta(W,y) = Tr(CW) + y^T(AW - v) + \frac{\beta}{2} ||AW - v||^2 \text{ for } W \in \Delta.
\]

**Algorithm 1 Q FW for Quadratic Binary Optimization**

Initialization: \(W \leftarrow 0, y \leftarrow 0, g \leftarrow 0, \beta_0 > 0 \)

Main loop:

\[
\text{for } t = 1, 2, \ldots, \text{ do}
\]

\[
\text{Primal step, inspired by FW}
\]

\[
W_{t+1} = (1-\eta)W_t + \eta \bar{W}_t
\]

\[
g_{t+1} = \text{arg } \min (w^T CW + g_t^T w) \text{ w } \in \mathbb{E}
\]

\[
\text{Dual step, gradient ascent}
\]

\[
y_{t+1} = y_t + \eta \gamma g_{t+1}
\]

Rounding: Extract \(X\) by removing the first row and first column of \(W\). Compute \(x\) as the top singular vector of \(X\) and project \(x\) onto \(\mathbb{E}\).

Output: Solution \(W \in \Delta\) for (CP), and \(x \in \mathbb{E}\) for (QUBO).

**Theoretical guarantees**:

\[
\text{Tr}(CW_t) - \text{Tr}(CW_*), \quad |AW_t - v| \leq O(1/\sqrt{t})
\]

Q-FW: A Hybrid Classical-Quantum Frank-Wolfe for Quadratic Binary Optimization

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**Evaluations**

**Graph Matching** on Random Problems

\[
\max_{\Pi} \text{vec}(\Pi)^T Q_{QUBO} \text{ vec}(\Pi) \quad \text{subject to} \quad \Pi \in \mathbb{P}
\]

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Table reports mean normalized energies over ten instances.

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Q-FWAL (ours) \(0.94 \pm 0.076\)