



GCPR-VMV 2024

September 10-13, Munich, Germany

Joint Tutorial on Virtual Humans and Quantum-enhanced Computer Vision (QeCV)

10.09.2024



Vladislav Golyanik

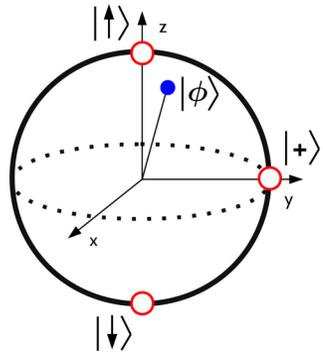
golyanik@mpi-inf.mpg.de

4D and Quantum
Vision Group $\langle \mathcal{A} | \psi \rangle$

Visual Computing and
AI Department

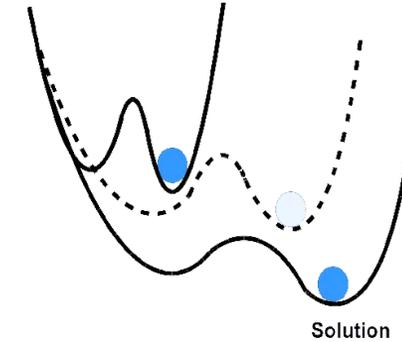
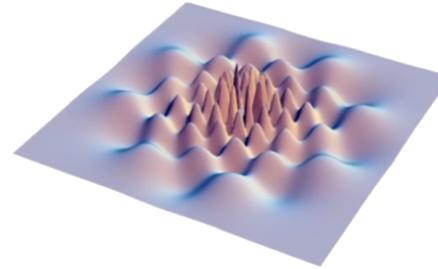
 max planck institut
informatik

Today's Schedule

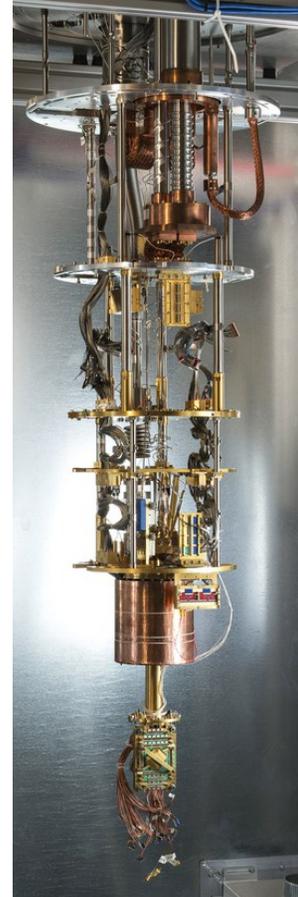
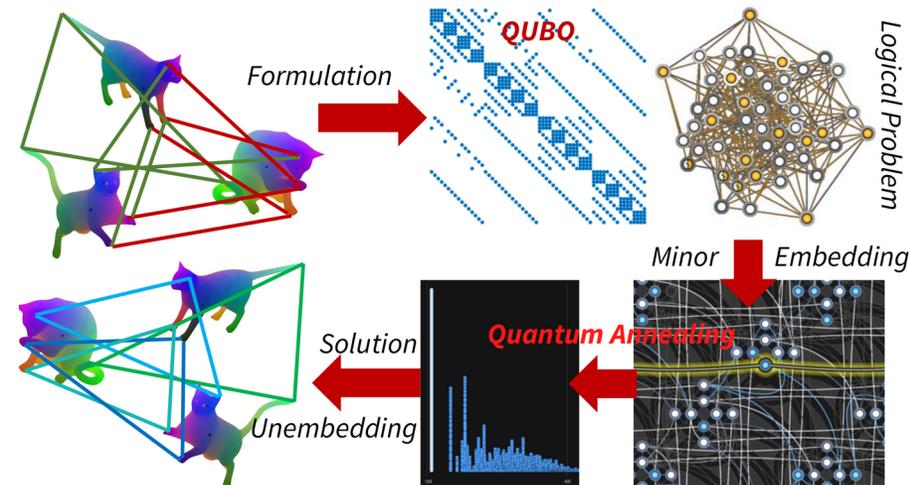


$$|0\rangle \otimes |1\rangle = |01\rangle$$

$$\min_{s \in \{-1,1\}^n} s^T J s + b^T s.$$



- Quantum-enhanced Computer Vision (QeCV)
- Foundations of Quantum Computing
 - Gate-based Quantum Computers
 - Quantum Annealers
- QeCV Methods
 - Multi-Model Fitting (CVPR 2023)
 - Mesh Alignment (ICCV 2021)

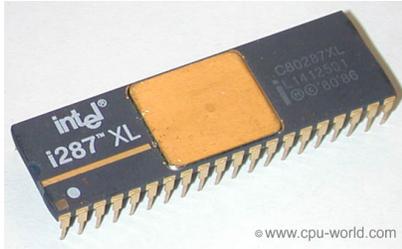


Quantum-enhanced Computer Vision (QeCV)

Reasons for Quantum Computers



CPU (1971)



FPU (1980s)



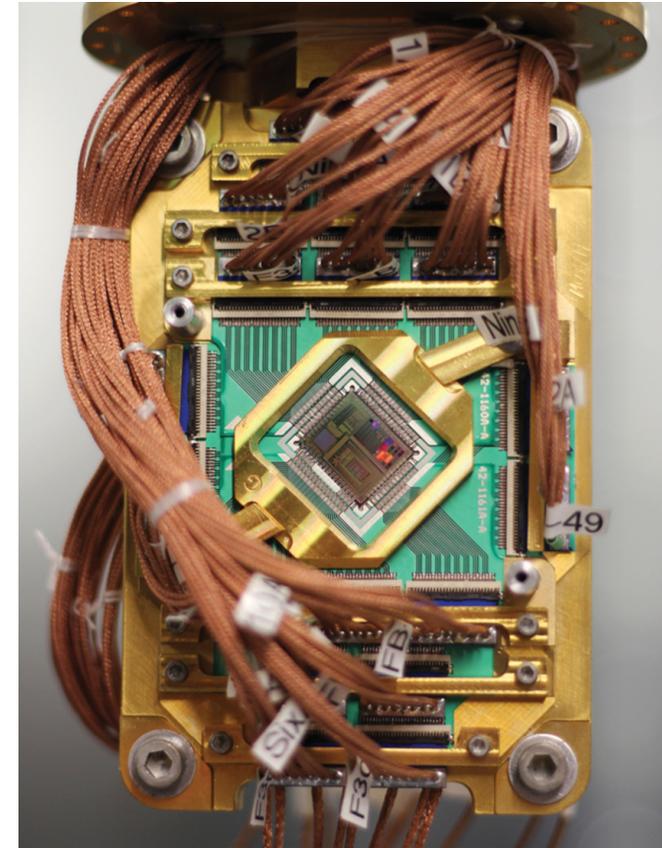
GPU (1999)



NVIDIA DGX A100 (2020)

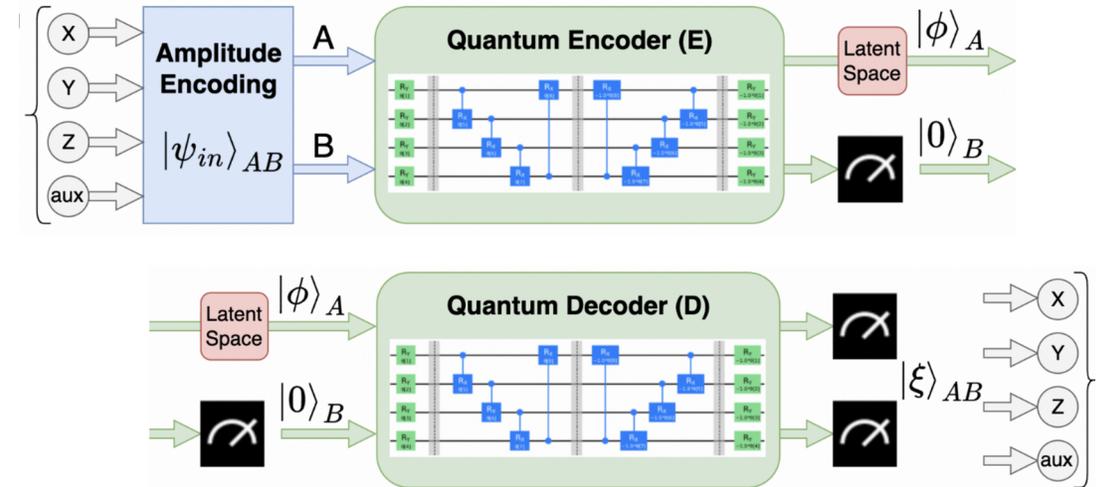
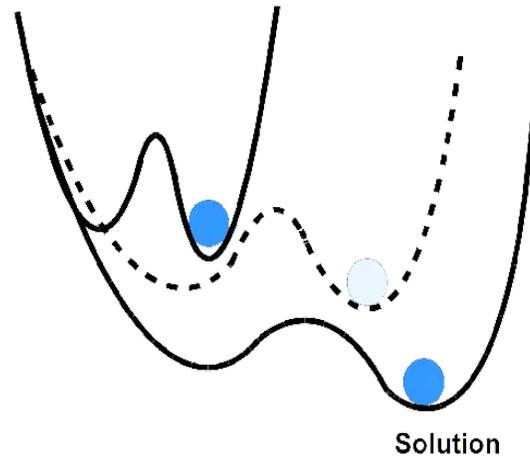
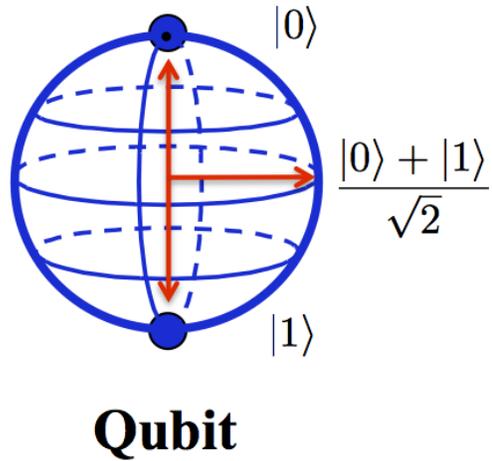
NPU

VPU...



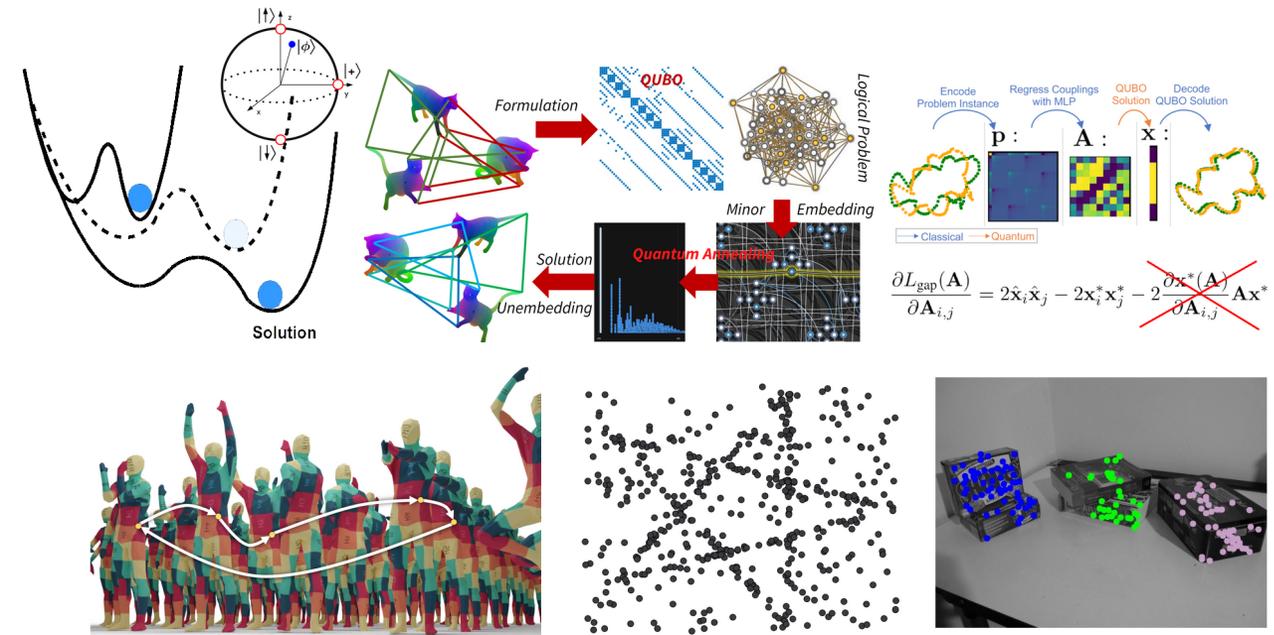
QPU (2020)

Quantum-enhanced Computer Vision

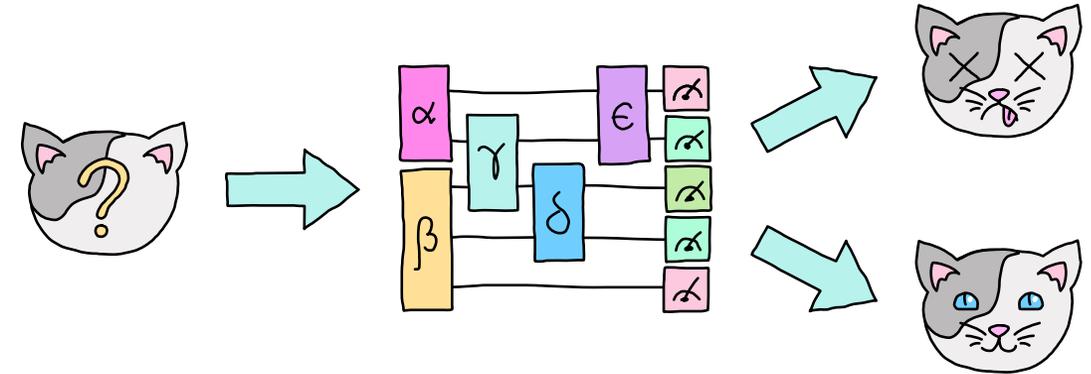


The goal of quantum-enhanced computer vision is the development of innovative computer vision techniques (improved or fundamentally new ones) **leveraging quantum computational paradigms** and surpassing classical methods **in terms of processing speed, required resources, accuracy or the ability to learn patterns from complex visual data.**

Quantum Computing Paradigms



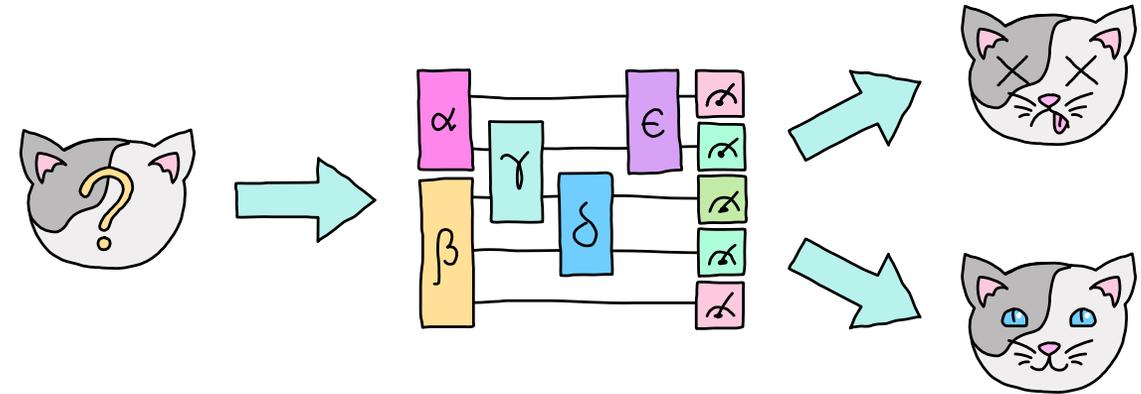
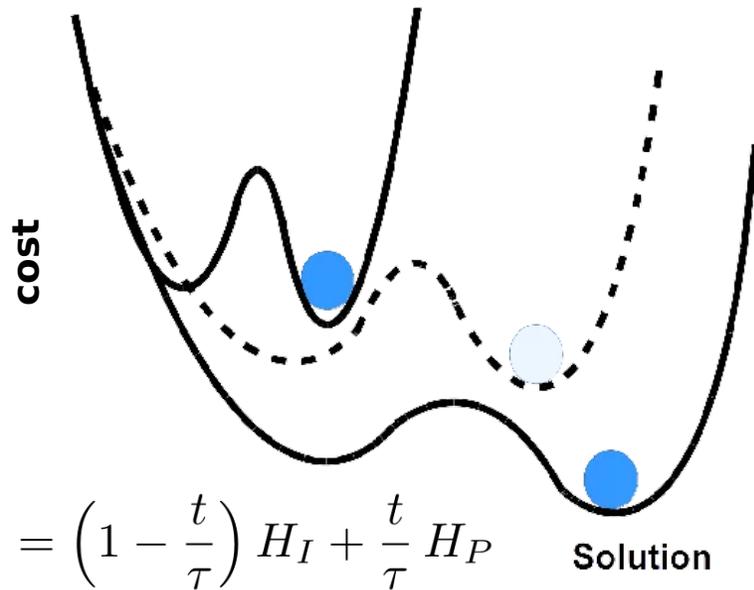
QUBO Formulations for Computer Vision



Quantum Machine Learning (QML) for Computer Vision

- QUBO formulations (predominantly for quantum annealers)
- Quantum Machine Learning (QML) techniques (for gate-based QCs)

Expected Advantages of Quantum-enhanced Methods w.r.t. Classical Techniques



Schematic QML architecture for classification

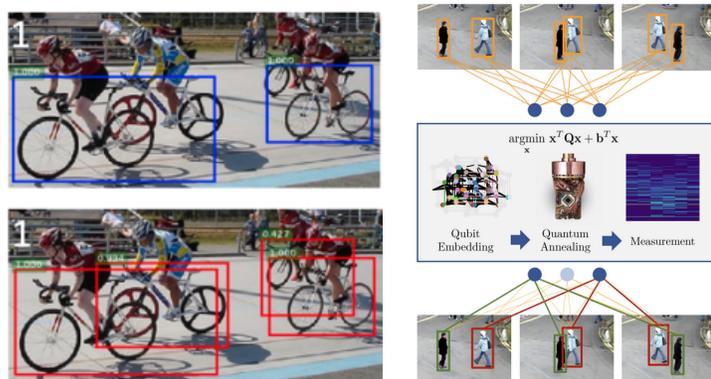
Methods relying on **Quantum Annealing**:

- Solutions without relaxations/approximations
- More accurate solutions w.r.t. classical methods
- Sampling of possible solutions (incl. sub-optimal)
- *Method characteristics of a new kind*

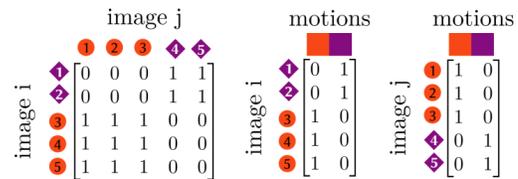
Quantum Machine Learning (QML) models:

- Faster training/convergence
- Smaller number of parameters
- Better generalisation
- *Model characteristics of a new kind*

Quantum-enhanced CV Methods



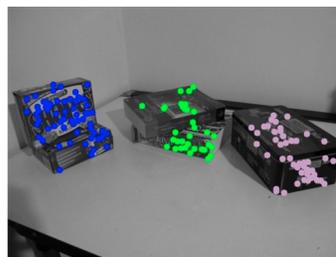
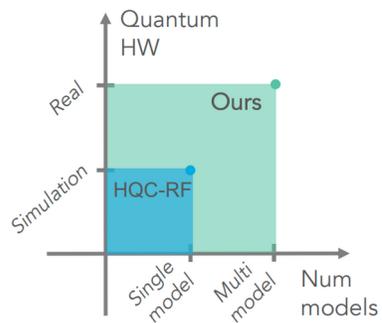
Object Tracking [1,2]



Motion Segmentation [3]



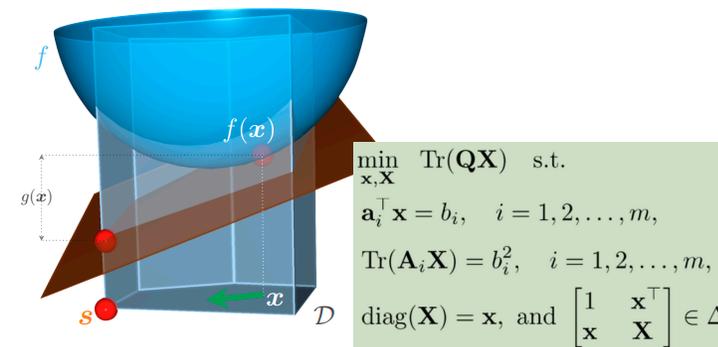
Mesh Alignment [4]



Robust Model Fitting [5]



Transformation Estimation [6]



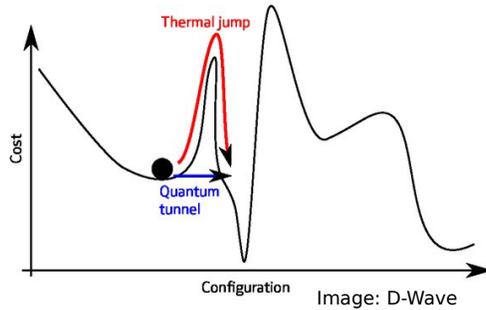
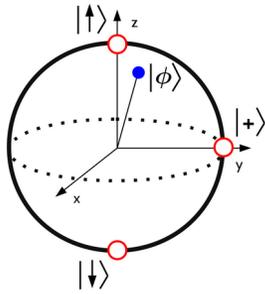
Constrained Optimisation [7]

[1] Li and Ghosh. Quantum-Soft QUBO Suppression for Accurate Object Detection. ECCV, 2020.
 [2] Zaech et al. Adiabatic Quantum Computing for Multi Object Tracking. CVPR, 2022.
 [3] Arrigoni et al. Quantum Motion Segmentation. ECCV, 2022.
 [4] Bhatia et al. CCuantuMM: Cycle-Consistent Quantum-Hybrid Matching of Multiple Shapes. CVPR, 2023.

[5] Farina et al. Quantum Multi-Model Fitting. CVPR, 2023.
 [6] Meli et al. An Iterative Quantum Approach for Transformation Estimation From Point Sets. CVPR, 2022.
 [7] Yurtsever et al. Q-FW: A Hybrid Classical-Quantum Frank-Wolfe for Quadratic Binary Optimization. ECCV, 2022.

Foundations of Quantum Computing

Preface: Terminology and the Definitions



quantum notion

qubit

Hamiltonian

eigenstate

ground state

classical counterpart

bit

energy function

an energy state

globally optimal energy state

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle 0| = [1 \quad 0]$$

$$\langle 1| = [0 \quad 1]$$

Exemplary definitions:

Graph minor

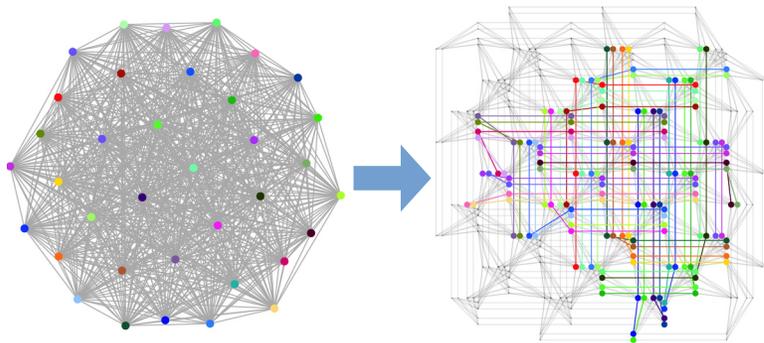
In graph theory, an undirected graph H is called a minor of the graph G if H can be formed from G by deleting edges, vertices and by contracting edges.

Minor Embedding

Embedding a graph minor to another graph (checking if H can be a minor of G).

Bra-Ket (Dirac) notation

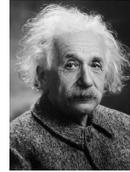
Compact notation for linear algebra and linear operations used in quantum mechanics.



Preface: Secrets of Learning QeCV

If [quantum theory] is correct, it signifies the end of physics as a science.

A. Einstein



I think I can safely say that nobody really understands quantum mechanics.

R. Feynman



Quantum mechanics isn't weird, we're just to big.

P. Ball



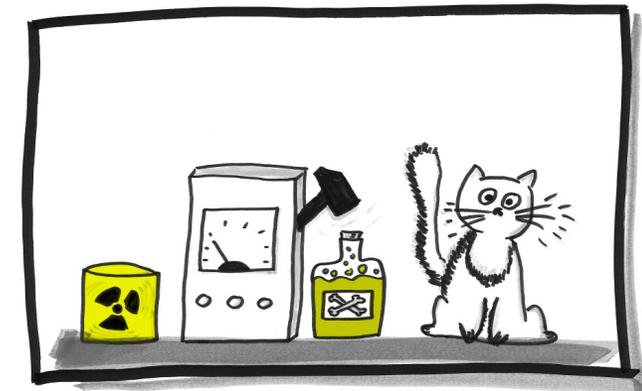
Due to the impossibility to decompose the system into individual elements, the state of a quantum automata can be seen as multiple states of different virtual classical automata simultaneously.

Y. Manin

Can you do it* with a new kind of computer – a quantum computer?”
* simulate quantum-mechanical effects

R. Feynman

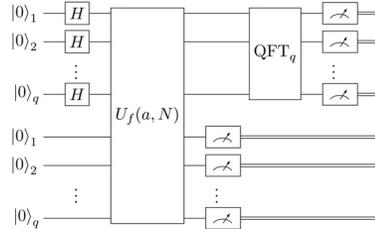
- **#1: *Be familiar with the QeCV notations!***
 - *In many cases, they express familiar mathematical notions and operators!*
- **#2: *Do not try to deeply interpret (e.g. geometrically or intuitively) what happens, as no satisfactory interpretation can be found.***



<https://www.leifiphysik.de/atomphysik/quantenmech-atommodell/versuche/schroedingers-katze-ein-gedankenexperiment>

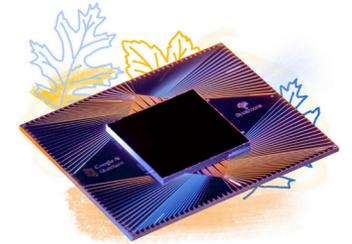
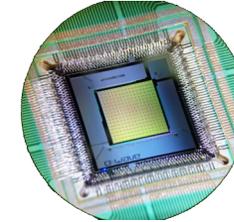
Timeline of Quantum Computing

Simulating Physics with Computers
Richard P. Feynman



A Quantum Adiabatic Evolution
Algorithm Applied to Random
Instances of an NP-Complete
Problem

Edward Farhi,^{1*} Jeffrey Goldstone,¹ Sam Gutmann,²
Joshua Lapan,³ Andrew Lundgren,³ Daniel Preda³



...

1980/1981: Idea of quantum computing

1992: Deutsch–Jozsa algorithm

1994: Shor’s algorithm

1996: Grover’s algorithm

1997: Bernstein–Vazirani algorithm

1998: First 2- and 3-qubits computers

1999: Superconducting circuit as qubit

2001: Quantum adiabatic evolution algorithm

2019: IBM Q System One; 53-qubit system (IBM);
Google’s Sycamore (53 qubits)

2015: 1k-qubit Quantum Annealer (D-Wave)

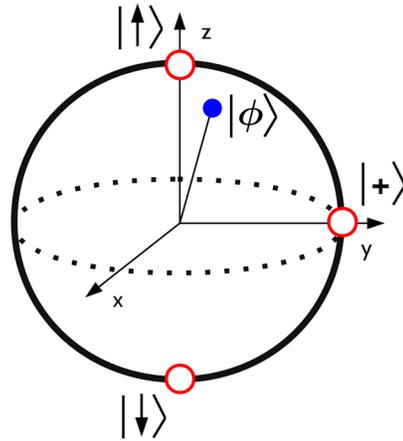
2014: Quantum Approximate Optimisation Algorithm

2008: The HHL algorithm for solving systems of
linear equations

Qubits and Their Properties / Bra-Kets

complex numbers

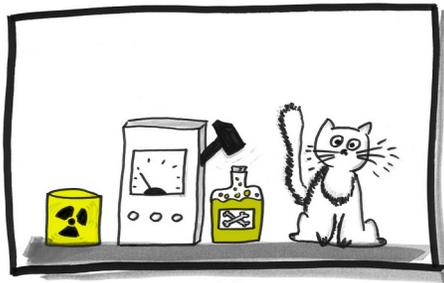
$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$



Born's rule:

$|\alpha|^2$ to obtain $|0\rangle$

$|\beta|^2$ to obtain $|1\rangle$



Qubit measurement

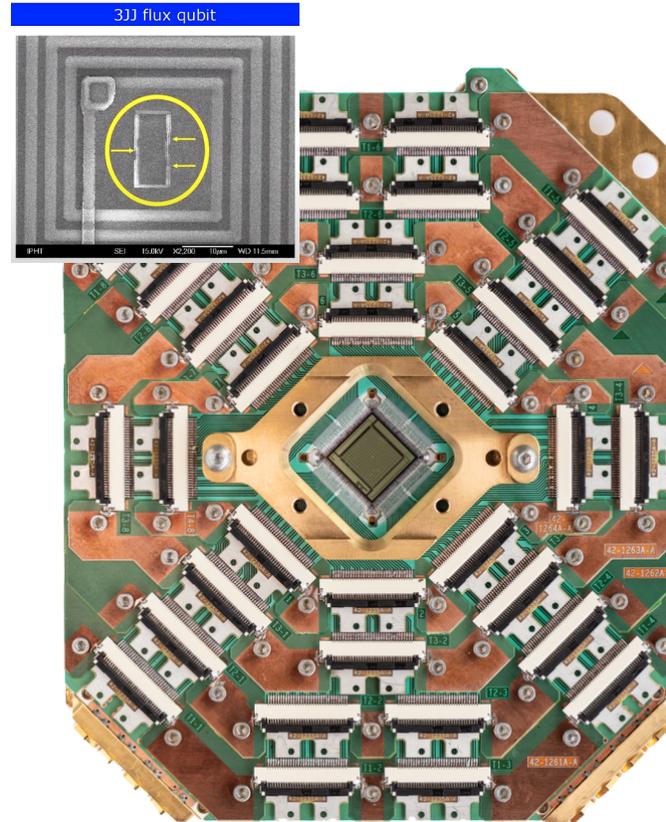


Image of Advantage sys. 1.1
(a qubit and a QPU); D-Wave Systems

Bra-Ket (Dirac) notation:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

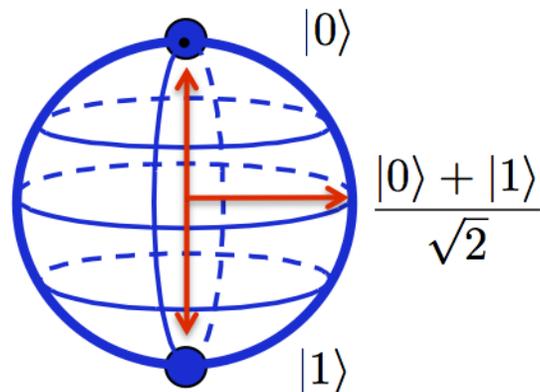
$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle 0| = [1 \ 0]$$

$$\langle 1| = [0 \ 1]$$

$$\langle q_1 | q_2 \rangle = [\alpha \ \beta] \begin{bmatrix} \eta \\ \zeta \end{bmatrix} = \alpha\eta + \beta\zeta$$

Qubit Measurement



Qubit

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$



Measurement (Born's rule):

$|\alpha|^2$ to obtain $|0\rangle$

$|\beta|^2$ to obtain $|1\rangle$

either ● 0

or ● 1

Classical Bit

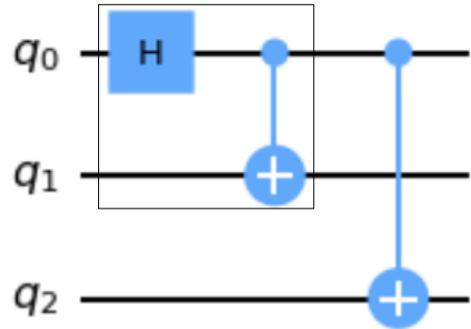
Two-Qubit Systems

Hilbert spaces (2^n -dimensional vector spaces):

$$|0\rangle \otimes |1\rangle = |01\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad |00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$|\phi\psi\rangle = \alpha |00\rangle + \beta |01\rangle + \eta |10\rangle + \zeta |11\rangle$$

Qubit entanglement:



$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Bell state

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$CNOT = CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|\text{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

* acts on a single qubit

* acts on two qubits

A three-qubit GHZ state

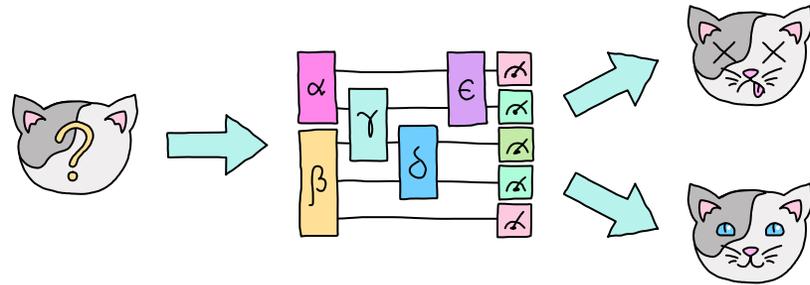
Data Encoding in Quantum-enhanced CV

data processing device



C - classical, Q - quantum

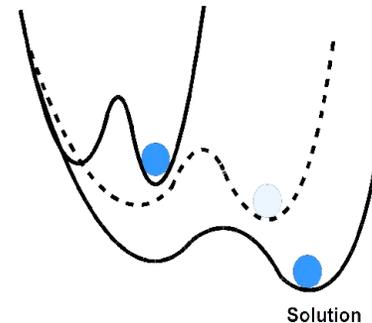
Approaches to combine quantum computing and machine learning [1]



Quantum ML for CV (gate-based QC)

- Encoding (basis, angular, amplitude, higher-order) **encodes the data**;
- The algorithm is expressed in the **converged Parametrised Quantum Circuits (PQCs)** after training.

$$\min_{x \in \mathbb{B}^N} x^T Q x + s^T x$$



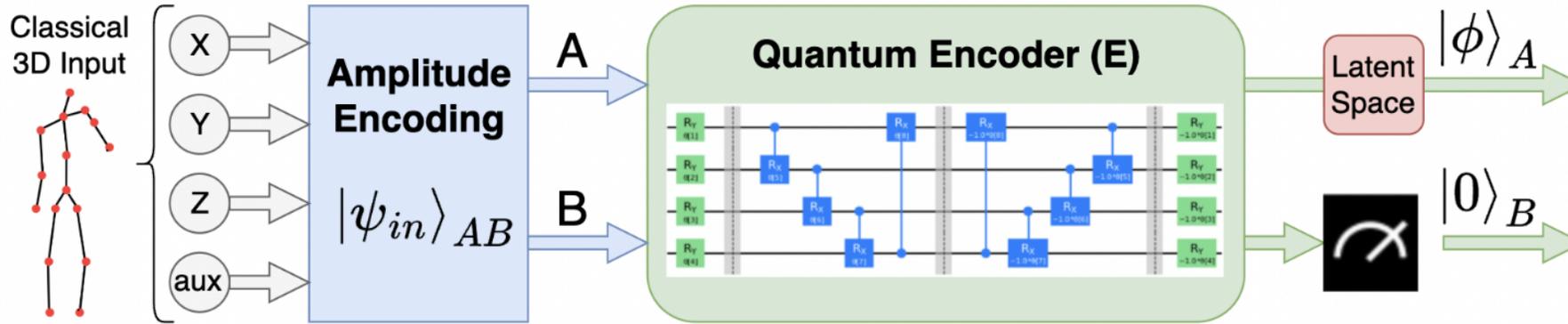
Quantum-enhanced CV (QA paradigm)

- Ising form **encodes the problem and the data**;
- The algorithm/meta-heuristic is the **same (quantum annealing)**.

Data encoding schemes and models/algorithms

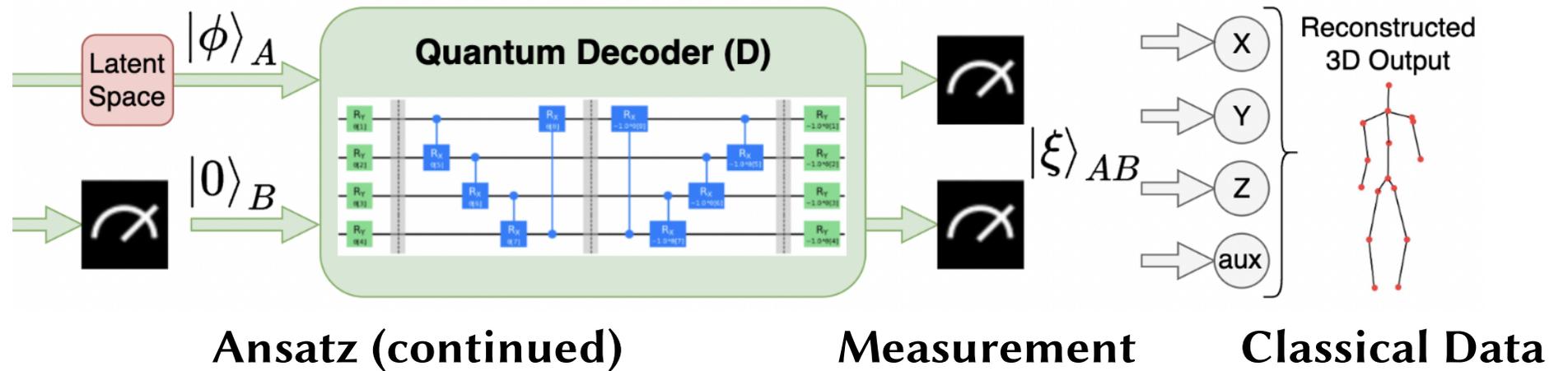
Gate-based Paradigm

Quantum Machine Learning

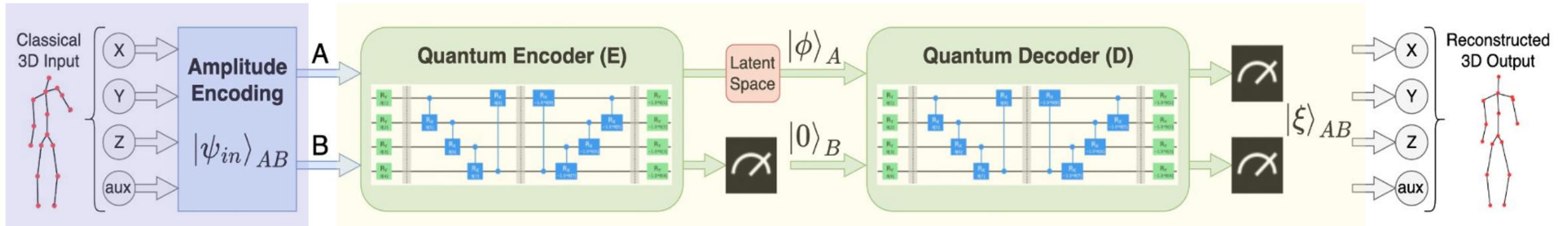


Classical Data Data Encoding

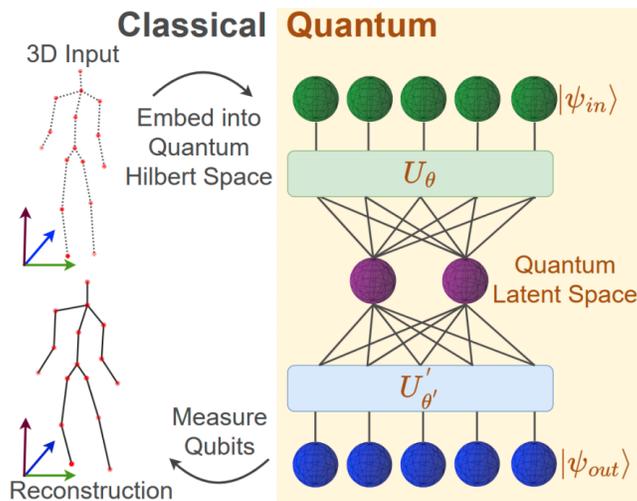
Ansatz



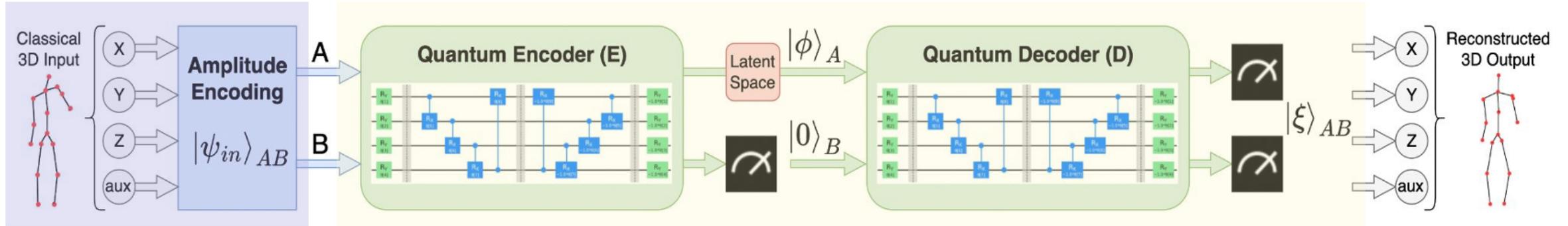
3D-QAE (Fully Quantum Auto-Encoding)



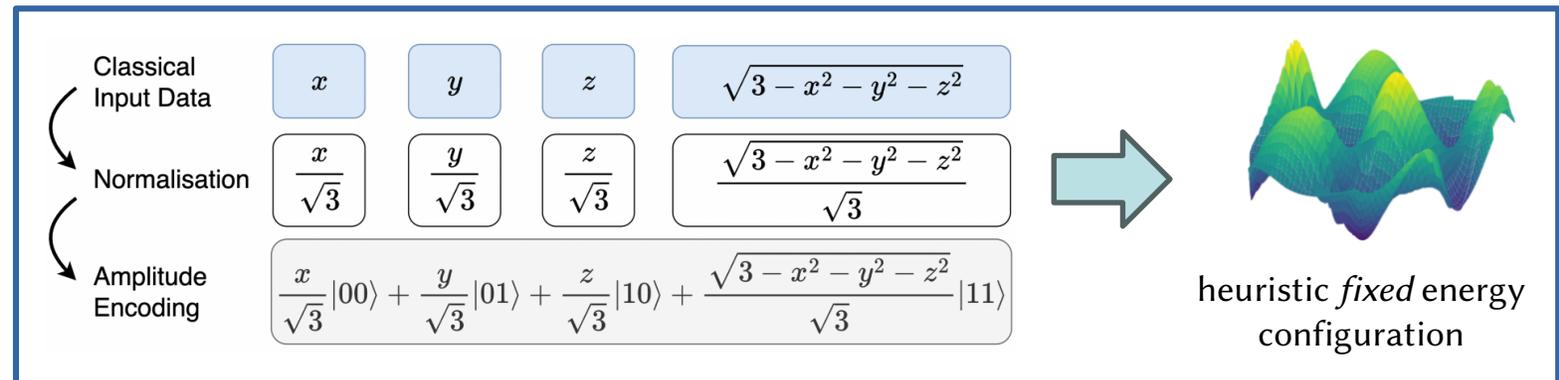
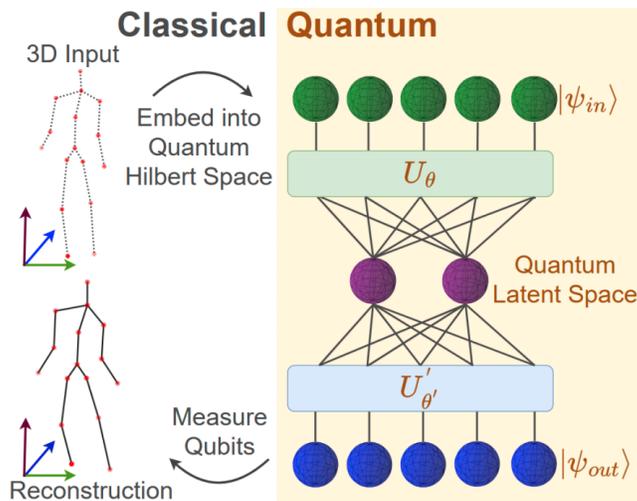
Scheme of the 3D-QAE approach for 3D point cloud auto-encoding.



3D-QAE (Fully Quantum Auto-Encoding)



Scheme of the 3D-QAE approach for 3D point cloud auto-encoding.



Amplitude encoding of a single 3D point.

Quantum Annealing Paradigm

Quantum annealing in the transverse Ising model

Tadashi Kadowaki and Hidetoshi Nishimori

Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan

(Received 30 April 1998)

We introduce quantum fluctuations into the simulated annealing process of optimization problems, aiming at faster convergence to the optimal state. Quantum fluctuations cause transitions between states and thus play the same role as thermal fluctuations in the conventional approach. The idea is tested by the transverse Ising model, in which the transverse field is a function of time similar to the temperature in the conventional method. The goal is to find the ground state of the diagonal part of the Hamiltonian with high accuracy as quickly as possible. We have solved the time-dependent Schrödinger equation numerically for small size systems with various exchange interactions. Comparison with the results of the corresponding classical (thermal) method reveals that the quantum annealing leads to the ground state with much larger probability in almost all cases if we use the same annealing schedule. [S1063-651X(98)02910-9]

Kadowaki and Nishimori, 1998 (Phys. Rev. E 58, 5355)

- Introduces quantum annealing
 - Quantum fluctuations instead of thermal fluctuation
- Application: Finding ground states of the Ising model
- Quantum tunnelling to escape local minima

A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem

Edward Farhi,^{1*} Jeffrey Goldstone,¹ Sam Gutmann,²
Joshua Lapan,³ Andrew Lundgren,³ Daniel Preda³

A quantum system will stay near its instantaneous ground state if the Hamiltonian that governs its evolution varies slowly enough. This quantum adiabatic behavior is the basis of a new class of algorithms for quantum computing. We tested one such algorithm by applying it to randomly generated hard instances of an NP-complete problem. For the small examples that we could simulate, the quantum adiabatic algorithm worked well, providing evidence that quantum computers (if large ones can be built) may be able to outperform ordinary computers on hard sets of instances of NP-complete problems.

Although a large quantum computer has yet to be built, the rules for programming such a device, which are derived from the laws of quantum mechanics, are well established. It is already known that quantum computers could solve problems believed to be intractable on

classical (i.e., nonquantum) computers. An intractable problem is one that necessarily takes too long to solve when the input gets too big. More precisely, a classically intractable problem is one that cannot be solved using any classical algorithm whose running time grows only polynomially as a function of the length of the input. For example, all known classical factoring algorithms require a time that grows faster than any polynomial as a function of the number of digits in the integer to be factored. Shor's quantum algorithm for the factoring problem (I) can factor an integer in a time that grows (roughly) as the square of the number of digits. This raises the question of whether quantum computers could solve other classically difficult prob-

¹Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA. ²Department of Mathematics, Northeastern University, Boston, MA 02115, USA. ³Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

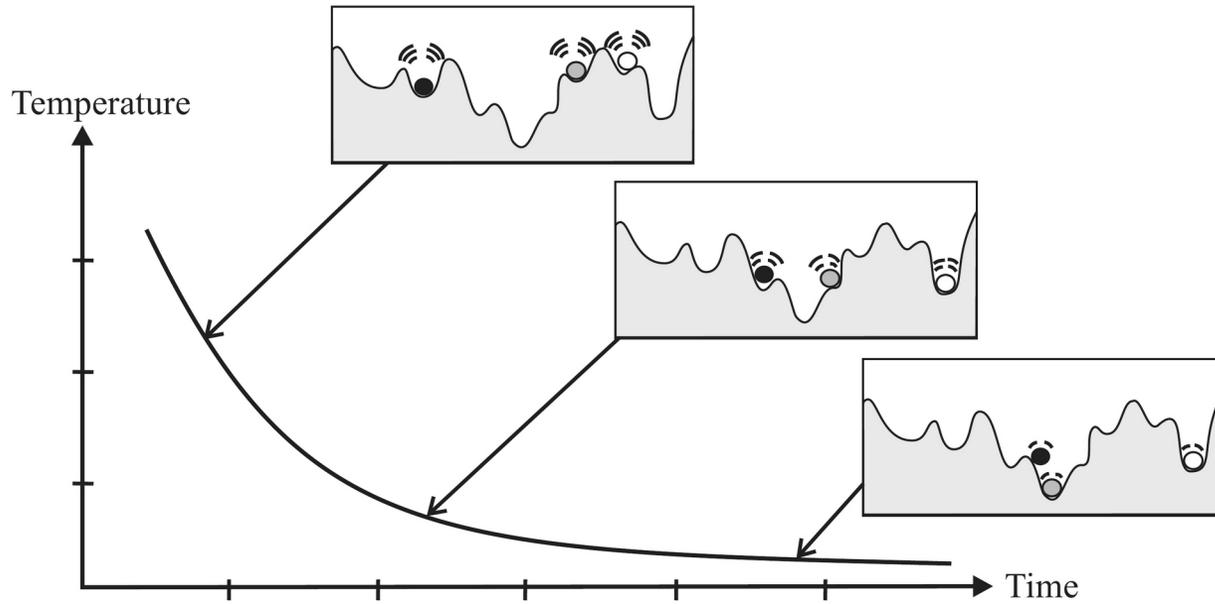
*To whom correspondence should be addressed. E-mail: farhi@mit.edu

Farhi et al., 2001 (arXiv:0104129)

- Introduces adiabatic quantum computation (universal model)
- Relies on the adiabatic theorem of quantum mechanics
- Application: Combinatorial optimisation problems (NP-hard)

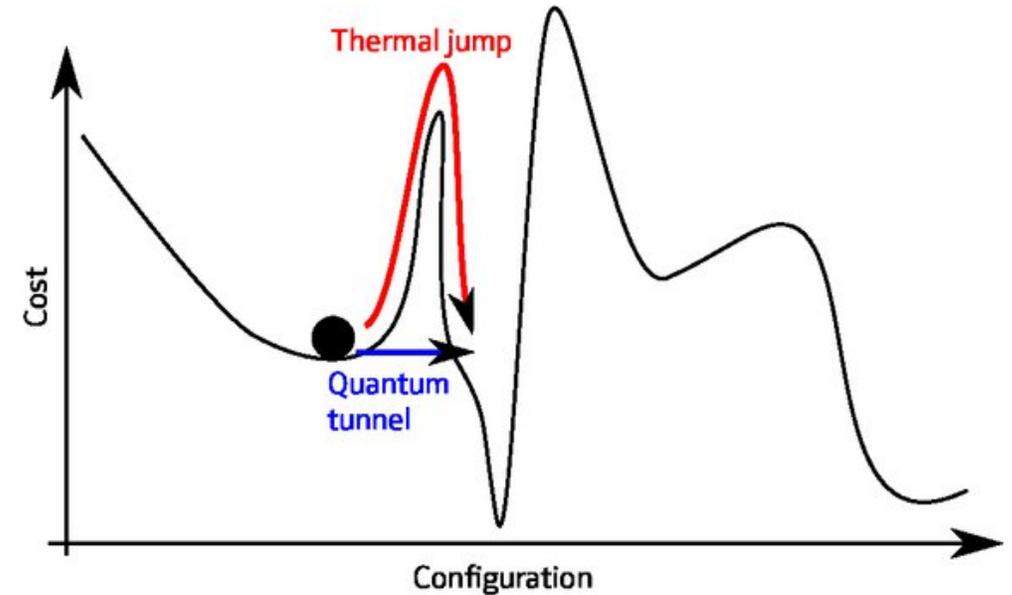
Adiabatic means that no heat is transferred to or from the system. Possibilities:
1) Process takes place in an insulated container; 2) Process happens very quickly.

Simulated Annealing vs Quantum Annealing



Simulated Thermal Fluctuations

Main Parameter: Temperature



Quantum fluctuations (physical phenomenon)

Main Parameter: Transverse magnetic field

Quantum-mechanical effects: Tunnelling; qubit superposition and entanglement

Transition between Hamiltonians

Transition
(simplified):

$$H(t) = A(t)H_{\text{init}} + B(t)H_{\text{final}}$$

—————> gradual transformation (convex combination)

$$|\psi_{\text{init}}\rangle$$

known and easy obtainable

$$|\psi_{\text{final}}\rangle$$

unknown

$$|\psi(t=0)\rangle = \bigotimes_{i=1}^n \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

Initial state

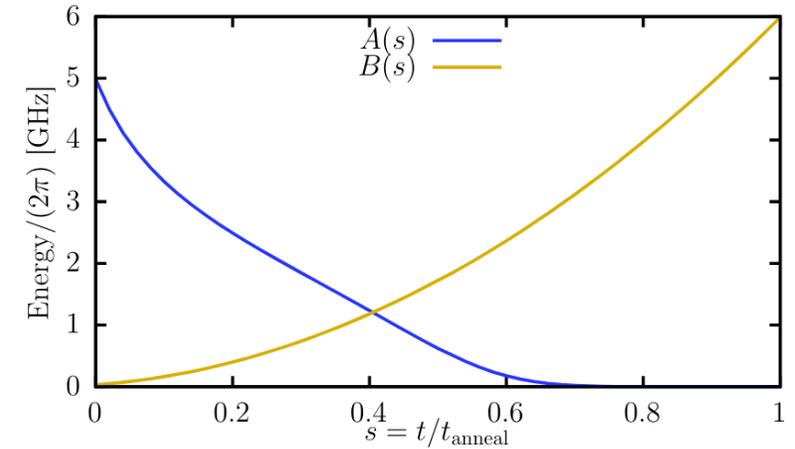
qubit couplings (interaction weights) qubit biases (individual weights)

$$\min_{s \in \{-1,1\}^n} s^T J s + b^T s$$

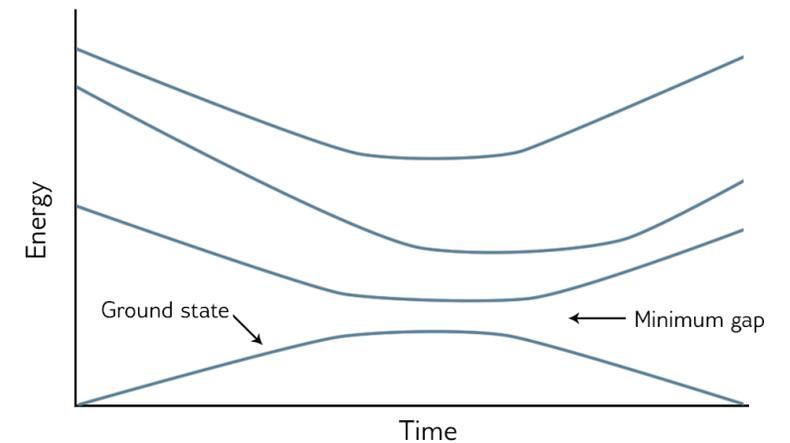
Ising/QUBO problem

Adiabatic theorem of quantum mechanics:

[Born and Fock, 1928]: *A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum.*

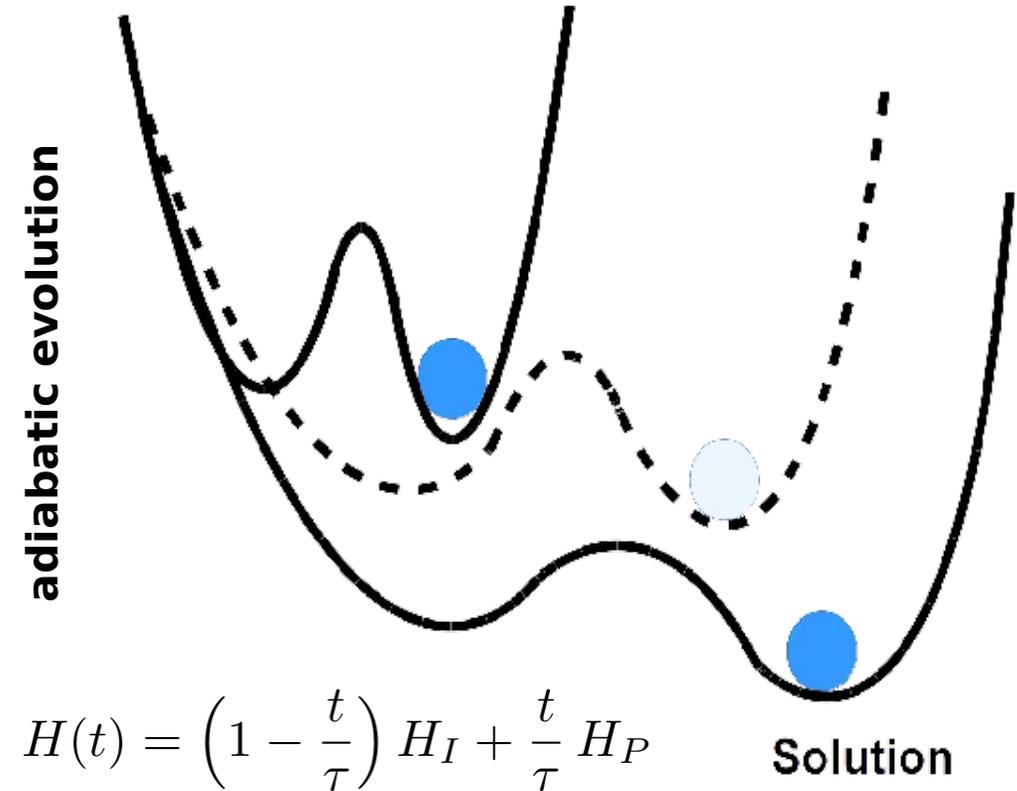
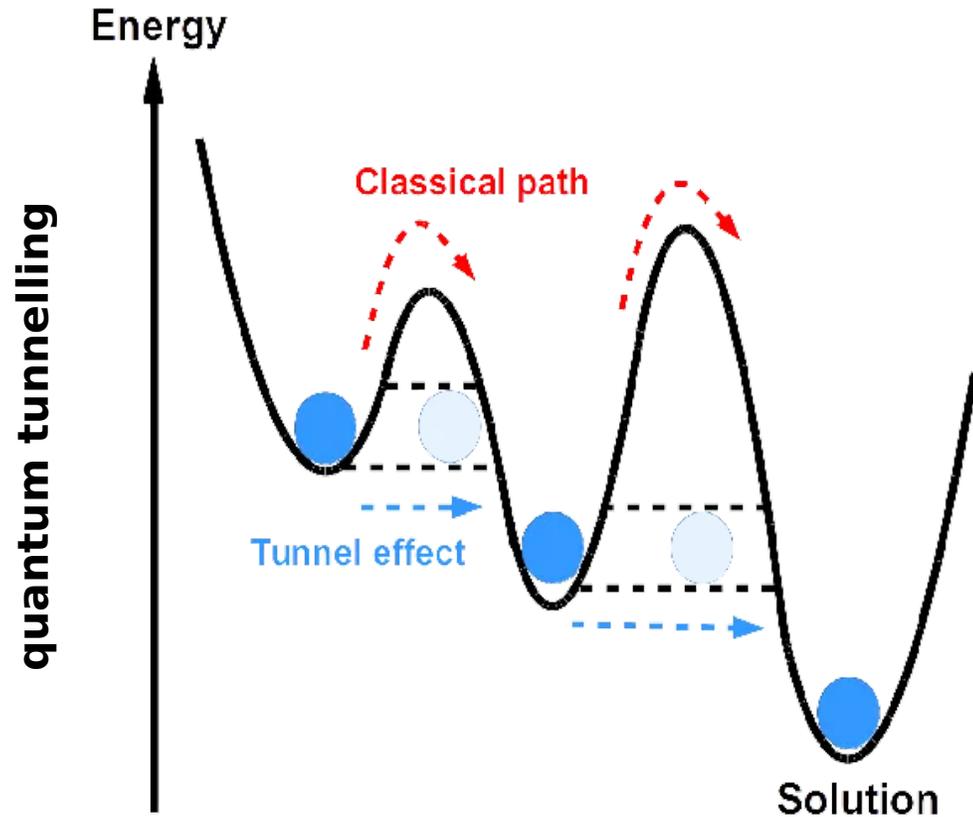


exemplary annealing schedule



instantaneous energy levels of $H(t)$

Adiabatic Quantum Annealing



- Adiabatic quantum computation (AQC): Encodes solution to a computational problem into **ground state of a time-dependent Hamiltonian** (this paradigm is equivalent to the gate-based model)
- Quantum annealing (QA): Quantum evolution towards the ground states of final Hamiltonians (no adiabaticity, no universality or equivalency to the gate-based model)

From Ising Problem to Ising Hamiltonian

$$Q(\mathbf{x}) = \sum_i Q_{i,i} x_i + \sum_{i<j} Q_{i,j} x_i x_j$$

- Q is invariant under symmetrisation
- The weight is added if both binary variables are equal to 1

$$\uparrow x_i = \frac{1 - s_i}{2} \quad s_i = 2x_i - 1$$

$$I(s) = \sum_i h_i s_i + \sum_{i<j} J_{i,j} s_i s_j$$

Ising Problem

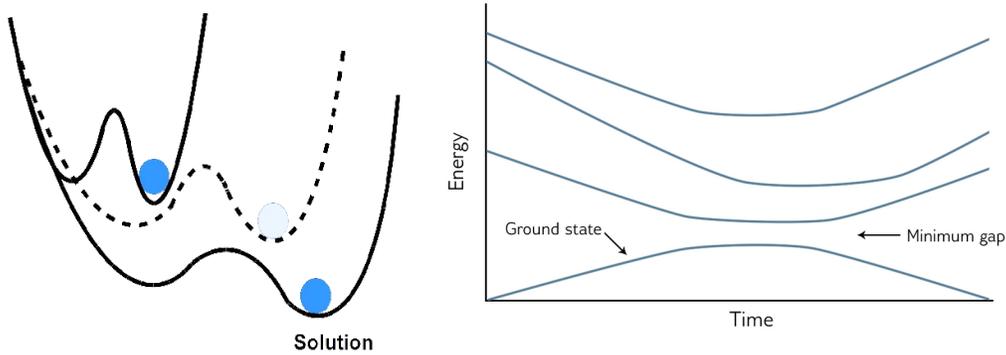
$$H_{\text{final}} = \sum_i h_i \sigma_i^z + \sum_{i<j} J_{i,j} \sigma_i^z \sigma_j^z, \quad \text{where}$$

$$\sigma_i^z = \underbrace{I \otimes I \dots \otimes I}_{i-1 \text{ many times}} \otimes \sigma_z \otimes \underbrace{I \otimes \dots \otimes I}_{n-i \text{ many times}},$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{and} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- H_{final} is a diagonal $2^n \times 2^n$ matrix
- Diagonal entries of H_{final} are obtained through enumeration of all costs obtained via $s^T J s + s^T b$
- H_{final} is often irreducible

Quantum Annealers as Samplers



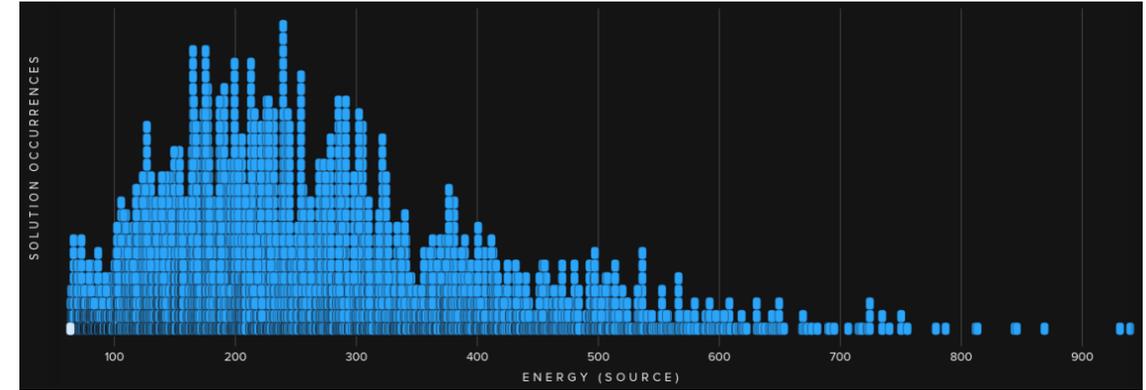
$$H(t) = A(t)H_{\text{init}} + B(t)H_{\text{final}}$$

gradual transformation (convex combination) →

$|\psi_{\text{init}}\rangle$ $|\psi_{\text{final}}\rangle$
 known and easy obtainable **unknown**

- QA always finds the ground state

Perfect adiabatic evolution



Probability of being in state σ :

$$p_{\sigma} = \frac{\exp \frac{-H_{\text{final}}(\sigma)}{kT}}{Z}$$

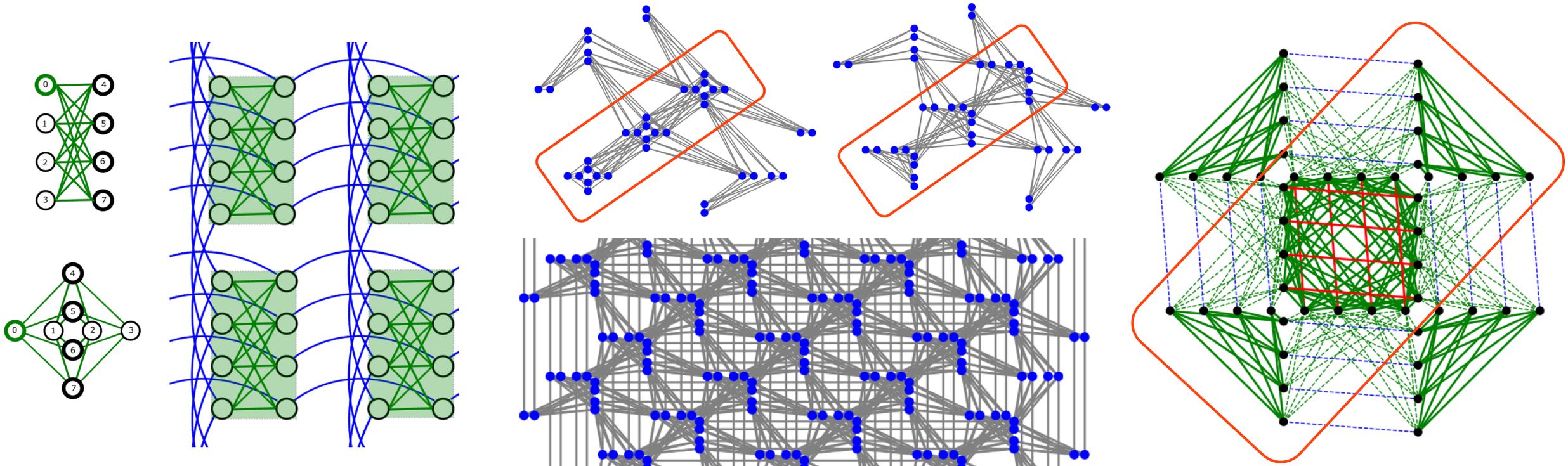
Partition function (normalisation over all states):

$$Z = \sum_i \exp \frac{-H_{\text{final}}(\sigma_i)}{kT}$$

- QA samples obey approximations to Boltzmann distributions (that are difficult to model)
- Low-energy states are more probable

Practical case (thermal excitations)

D-Wave Quantum Annealers



- 2048 qubits (16x16x8)
- Nominal length 4 (internal couplers)
- Degree 6 (+2 external qubits)
- Internal and external couplers

2000Q (2017)

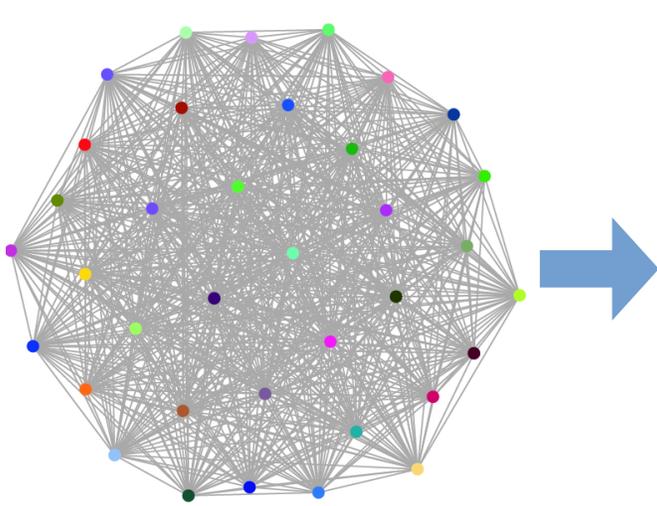
- 5640 qubits (~16x16x24)
- Nominal length 12 (internal couplers)
- Degree 15 (+3 external qubits)
- Internal, external and odd couplers

Advantage (2020)

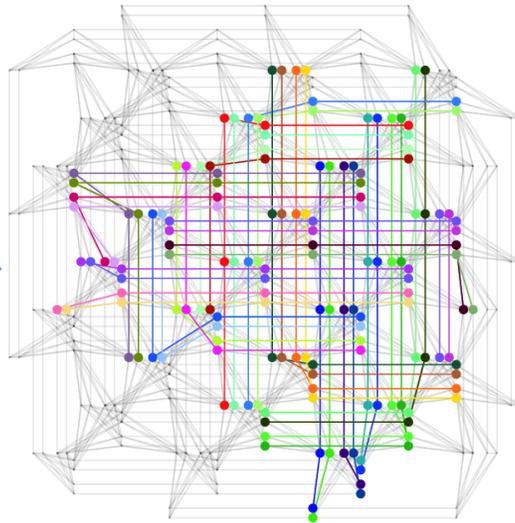
- 7440 qubits (~15x15x32)
- Nominal length 16 (internal couplers)
- Degree 20 (+4 external qubits)
- Internal, external and odd couplers

Advantage 2 (2024)

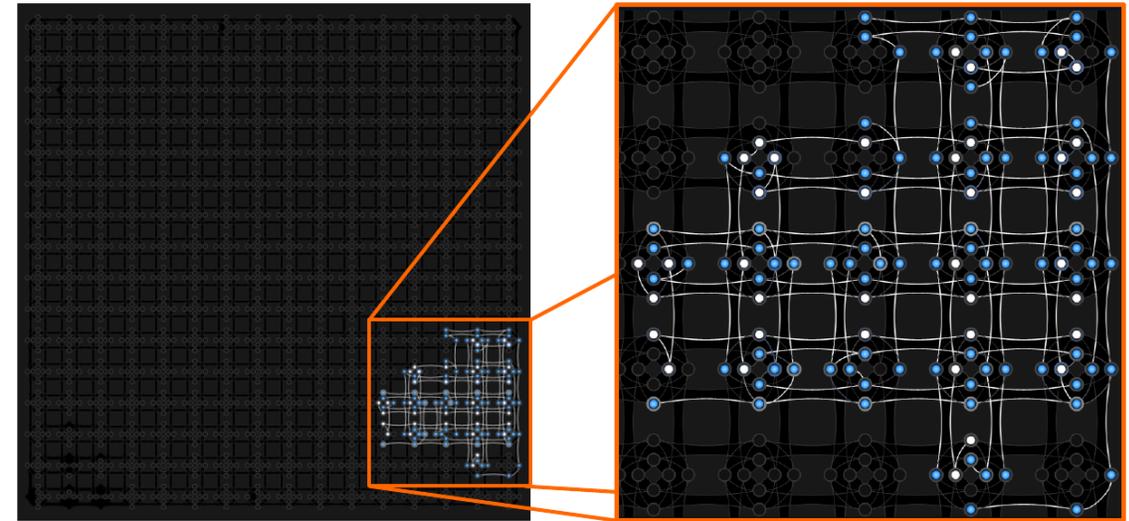
Minor Embedding (Form of Transpiling)



Fully connected graph with 36 logical qubits



Minor embedding



Successful minor embedding (2000Q)

$$\arg \min_{\mathbf{x} \in \mathcal{B}} \mathbf{x}^T \mathbf{Q} \mathbf{x}$$

Graph minor:

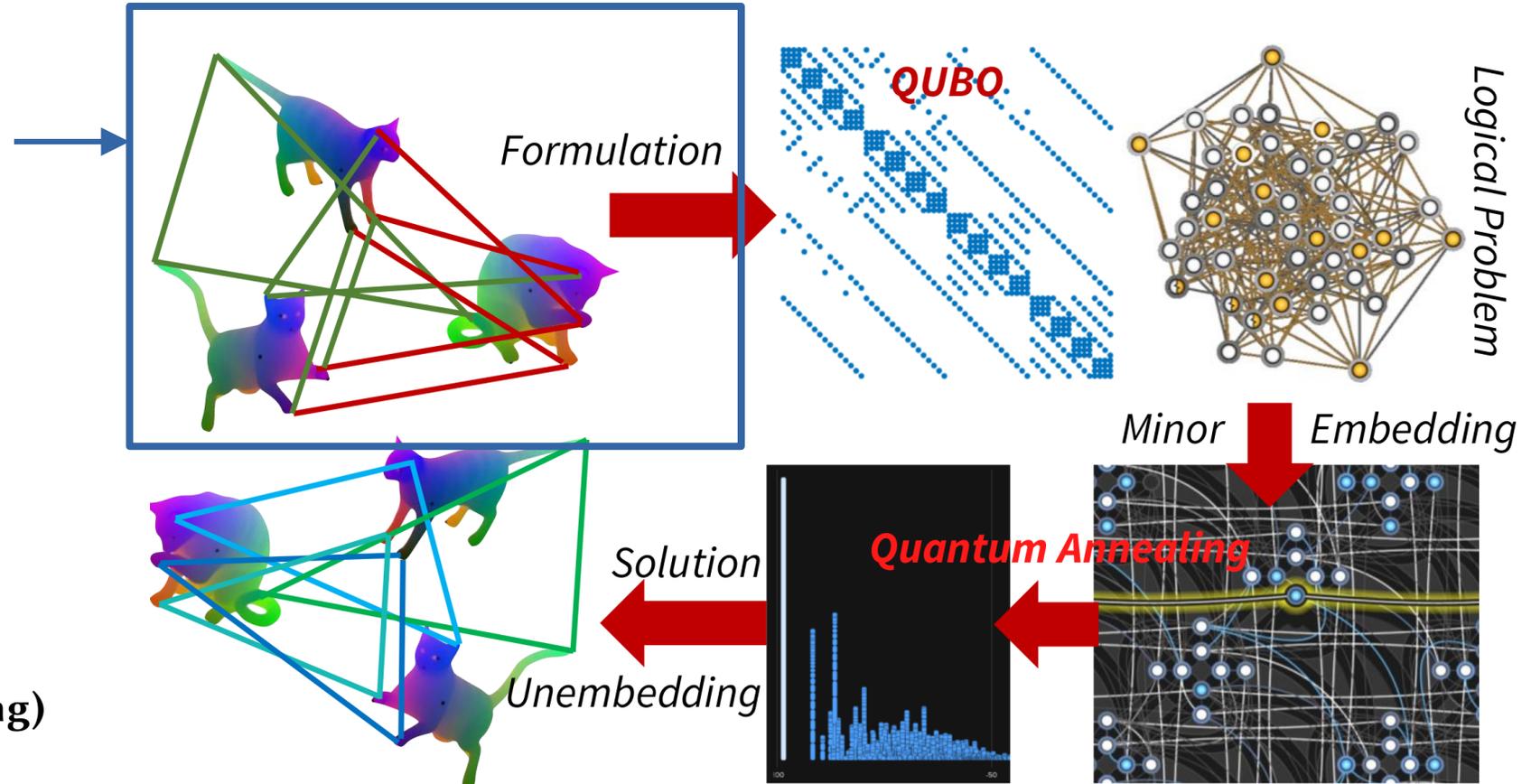
An undirected graph H is called a minor of the graph G if H can be formed from G by deleting edges, vertices and by contracting edges.

Minor Embedding:

Embedding a graph minor to another graph (checking if H can be a minor of G).

Six Steps of a QeCV Method (for QA)

Initial problem
(e.g., synchronisation of
permutations)

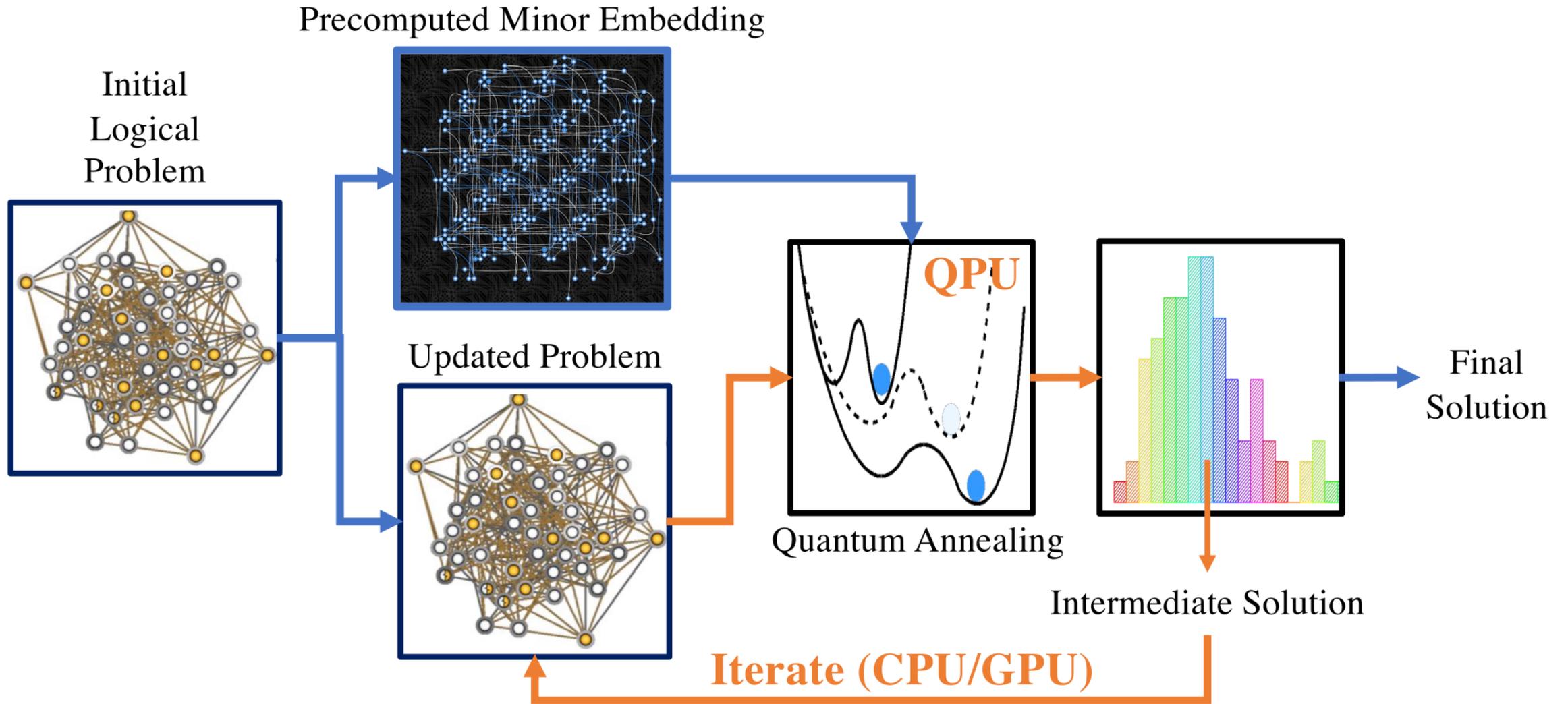


Six steps of every QeCV algorithm:

- 1) QUBO preparation
- 2) Minor embedding
- 3) Quantum annealing (sampling)
- 4) Unembedding
- 5) Bitstring selection
- 6) Solution interpretation

Birdal and Golyanik et al. CVPR, 2021.

Iterative QeCV Algorithms



Further Important Topics

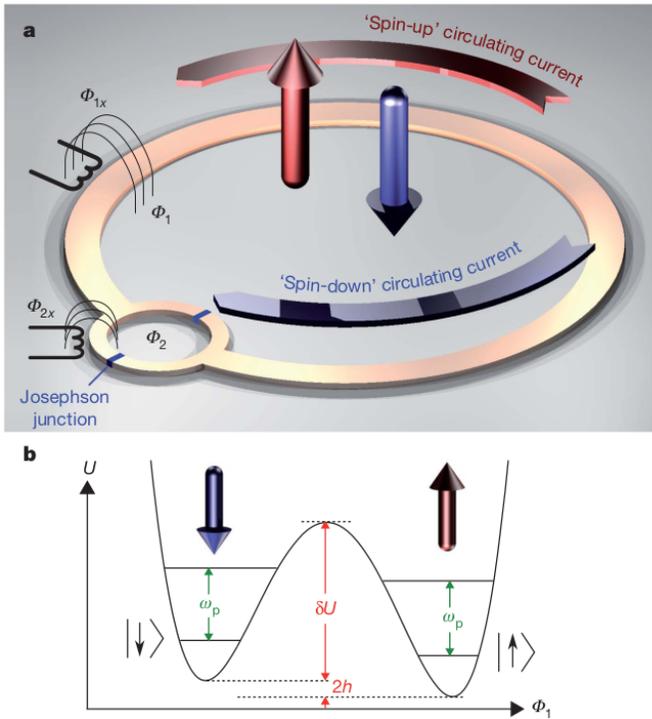
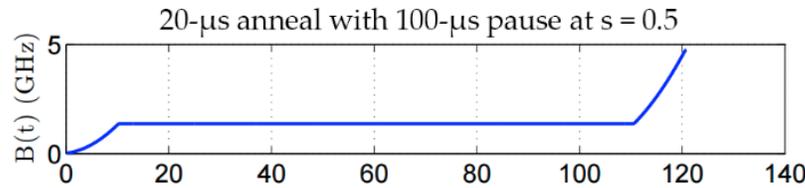
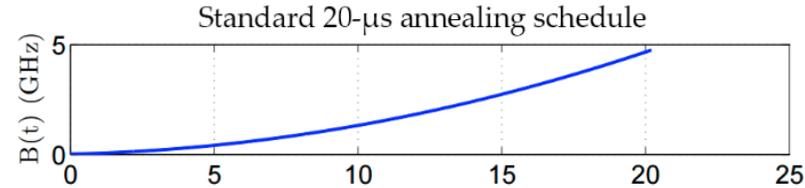


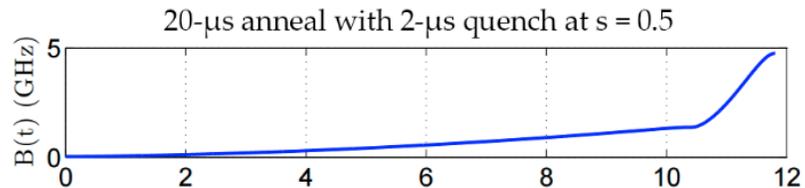
Figure 1 | Superconducting flux qubit. **a**, Simplified schematic of a superconducting flux qubit acting as a quantum mechanical spin. Circulating current in the qubit loop gives rise to a flux inside, encoding two distinct spin states that can exist in a superposition. **b**, Double-well potential energy diagram and the lowest quantum energy levels corresponding to the qubit. States $|\uparrow\rangle$ and $|\downarrow\rangle$ are the lowest two levels, respectively. The intra-well energy spacing is ω_p . The measurement detects magnetization, and does not distinguish between, say, $|\uparrow\rangle$ and excited states within the right-hand well. In practice, these excitations are exceedingly improbable at the time the state is measured.

Johnson et al., Nature, 2011.

qubit technology



Programming the D-Wave QPU: Parameters for Beginners



$$H(t) = A(t)H_{\text{init}} + B(t)H_{\text{final}}$$

docs.dwavesys.com/docs/latest/c_qpu_annealing.html

custom annealing schedules

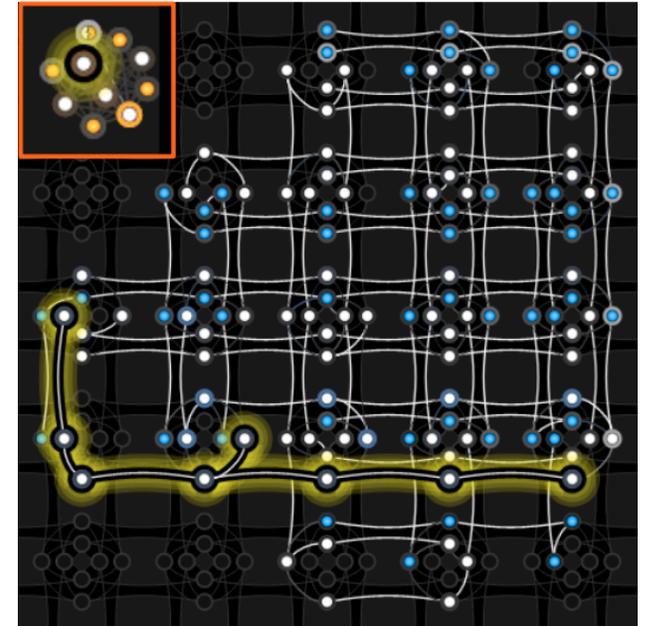
```
import dwave.inspector

dwave_sampler = FixedEmbeddingComposite(DWaveSampler(solver='qpu': True),
embedding)

bqm = dimod.BinaryQuadraticModel.from_qubo(Q, offset=offset)

sampleset = dwave_sampler.sample(bqm, num_reads=1000)

dwave.inspector.show(sampleset)
```

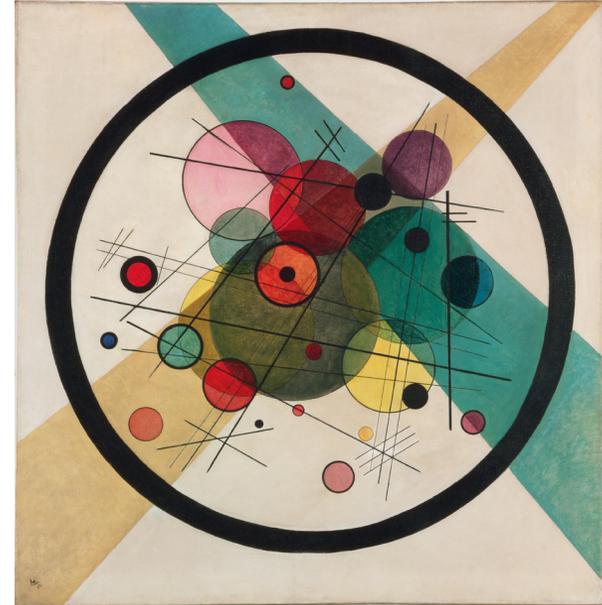


dwavesys.com/media/qvbjrzgg/guide-2.pdf

QA programming

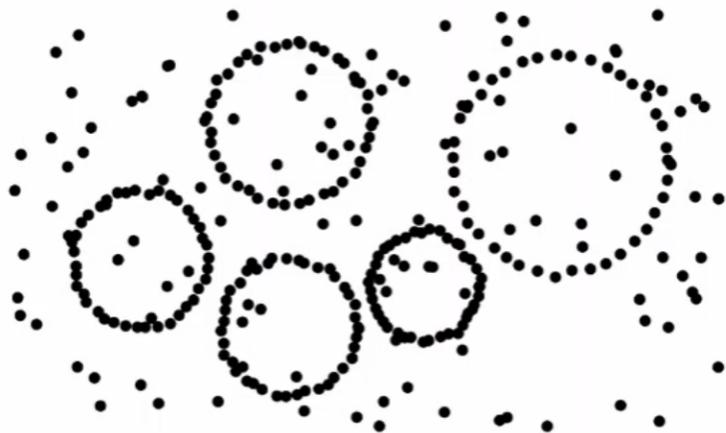
QeCV Methods

Multi-Model Fitting

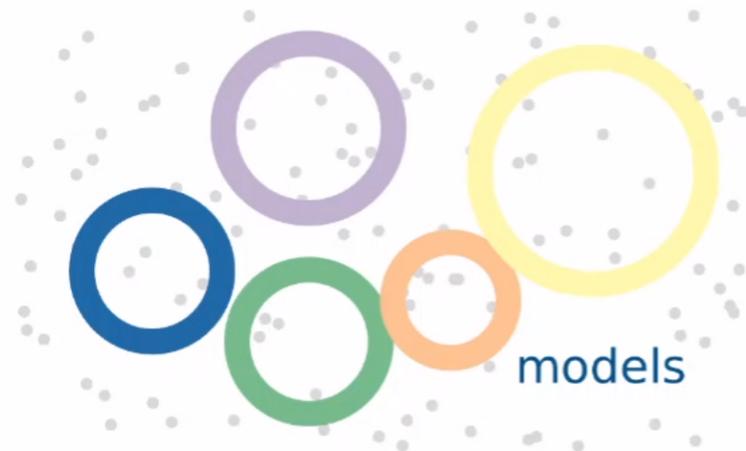


Kandinsky, 1923.

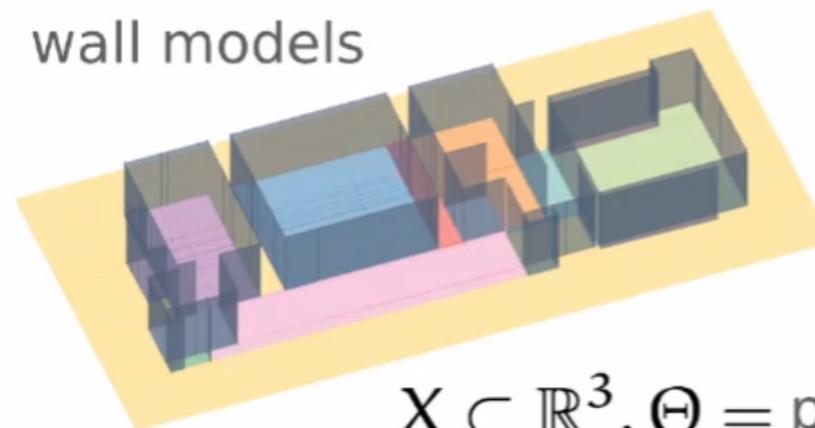
Multi-Model Fitting



scanned point cloud

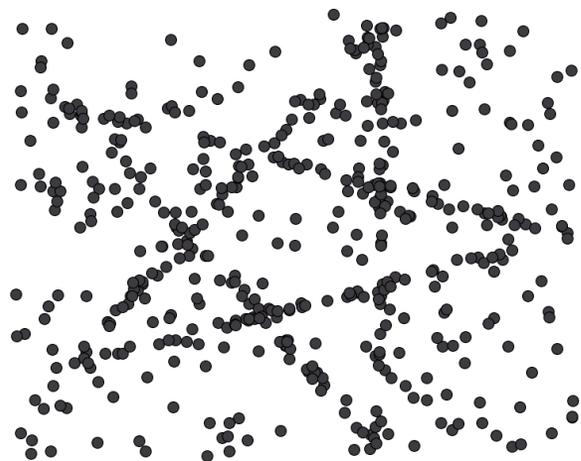


wall models



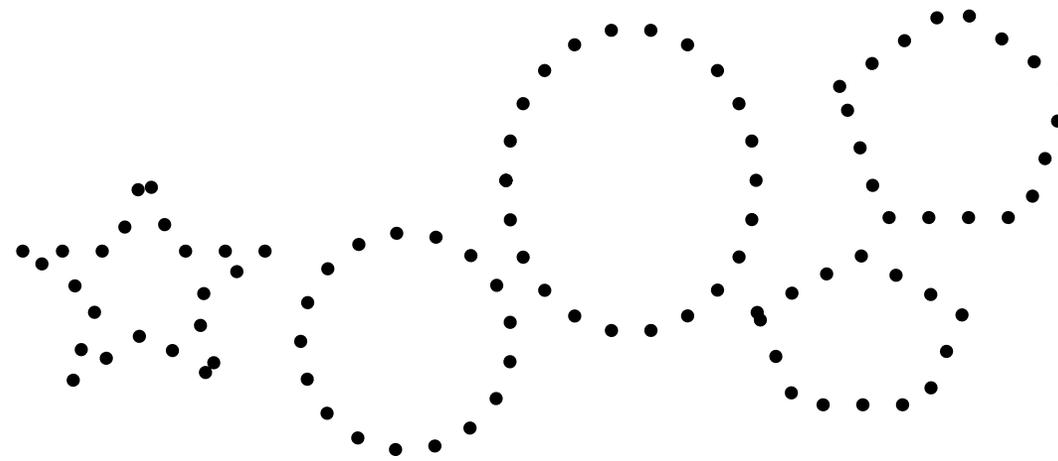
$$X \subset \mathbb{R}^3, \Theta = \text{planes}$$

Multi-Model Fitting



$$X = \{x_1, \dots, x_n\}$$

input data



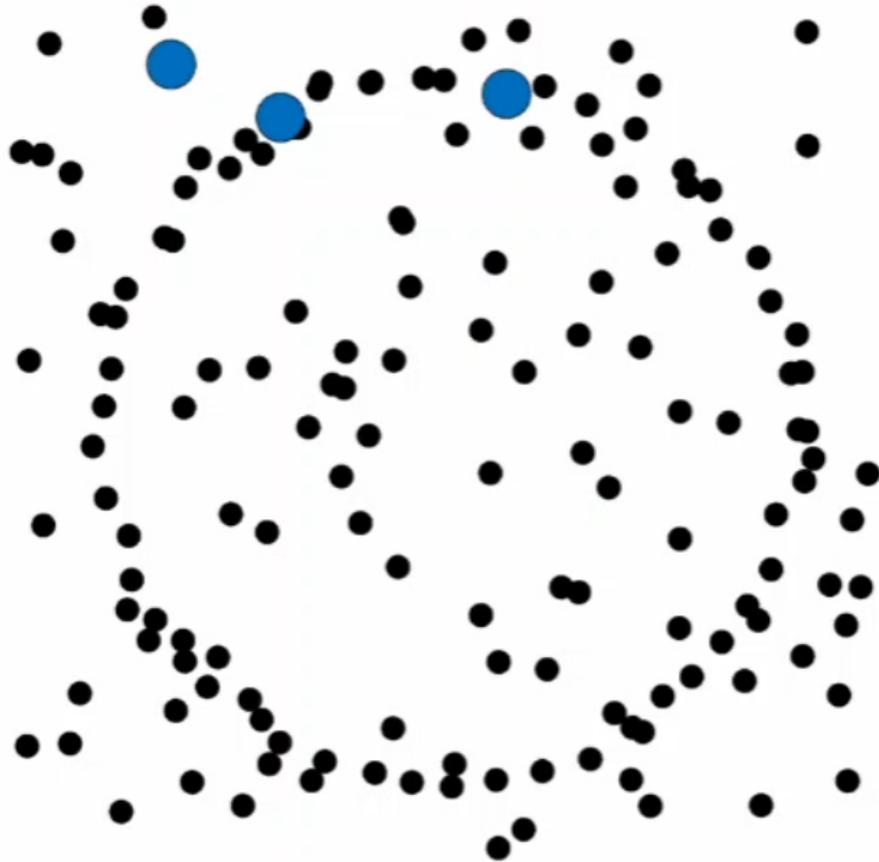
$$\Theta = \{\theta_1, \dots, \theta_m\}$$

example: multi-class multiple instance

Goal: To extract from Θ “best” models that describe X . Models are obtained by randomly selecting the minimum number of points to constrain model parameters:

- Two points for a line (2D)
- Eight points for a fundamental matrix

Random Sample Consensus (RANSAC)



Input: X data, ϵ inlier threshold, k_{\max} max iteration

Output: θ^* model estimate

$J^* = -\infty, k = 0;$

repeat

 Select randomly a minimal sample set $S \subset X;$

 Estimate parameters θ on $S;$

 Evaluate $J(\theta) = \sum_{x \in X} \hat{f}_\epsilon(r(x, \theta));$

if $J(\theta) > J^*$ **then**

$\theta^* = \theta;$

$J^* = J(\theta);$

end

$k = k + 1;$

until $k > k_{\max};$

Optimize θ^* on its inliers.

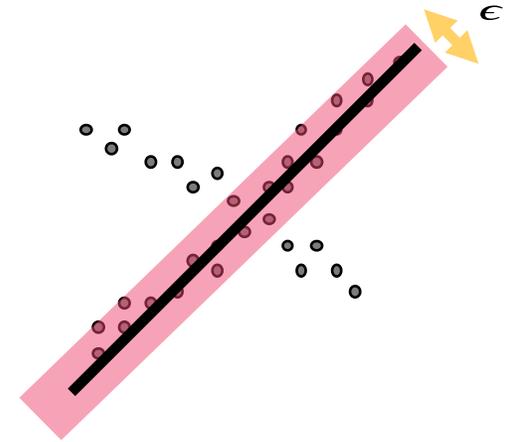
Preference-Consensus Matrix

$$P[i, j] = \begin{cases} 1 & \text{if } \boxed{\text{error}(x_i, \theta_j)} < \boxed{\epsilon} \\ 0 & \text{otherwise} \end{cases}$$

residual of point x_i w.r.t. model θ_j

Preference-consensus matrix (of size $n \times m$)

inlier threshold to assign a point to a model



Data:

$$X = \{x_1, \dots, x_n\}$$

Models:

$$\Theta = \{\theta_1, \dots, \theta_m\}$$

Preference-Consensus Matrix

$$P[i, j] = \begin{cases} 1 & \text{if } \text{error}(x_i, \theta_j) < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

residual of point x_i w.r.t. model θ_j

inlier threshold to assign a point to a model

Preference-consensus matrix (of size $n \times m$)

Data:

$$X = \{x_1, \dots, x_n\}$$

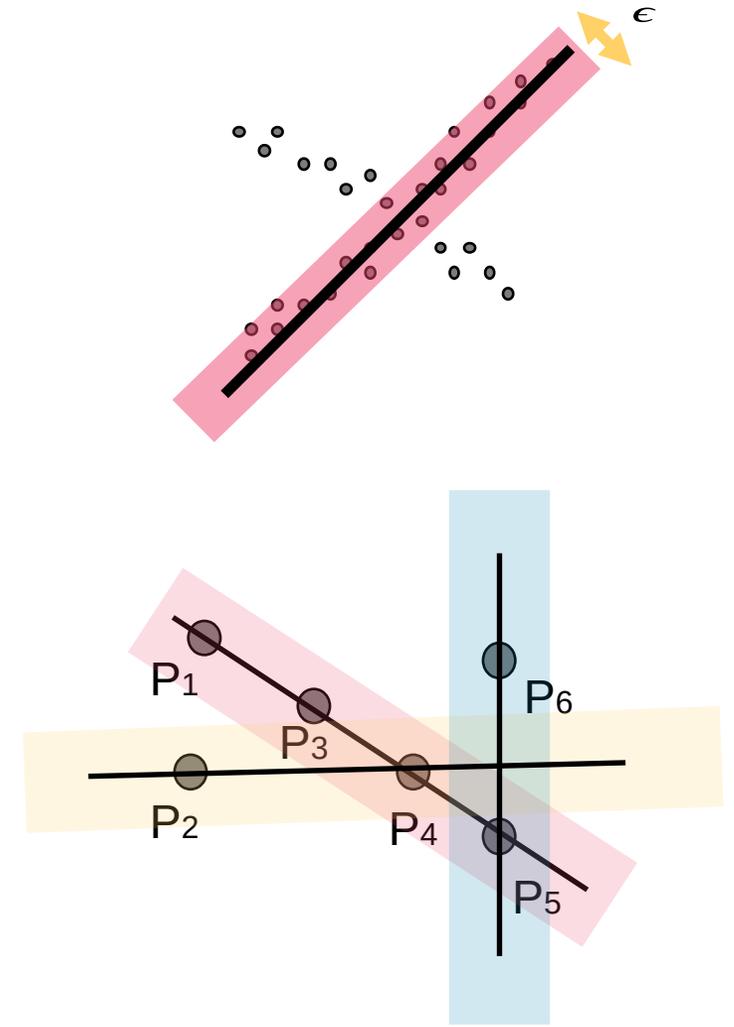
Models:

$$\Theta = \{\theta_1, \dots, \theta_m\}$$

	θ_1	θ_2	θ_3
P ₁	1	0	0
P ₂	0	1	0
P ₃	1	0	0
P ₄	1	1	0
P ₅	1	0	1
P ₆	0	0	1

preference set (per point)

consensus set of a model



From Set Cover to Disjoint Set Cover

Select the minimum number of columns of P explaining all the rows:

$$\min_{\mathbf{z} \in \mathbb{B}^m} \mathbb{1}_m^\top \mathbf{z} \quad \text{s.t. } P\mathbf{z} \geq \mathbb{1}_n$$

a vector of ones

model selection indication
(binary vector)

- The true number of models k is known
- **Outlier-free** data
- **Disjoint** models

integer linear
program (*NP-hard*)

$$\min_{\mathbf{z} \in \mathbb{B}^m} \mathbb{1}_m^\top \mathbf{z} \quad \text{s.t. } P\mathbf{z} = \mathbb{1}_n$$

Set cover: Minimise the number of selected models while ensuring that each point is explained by at least one of them.

Disjoint set cover: Ensure that each point is explained by exactly one model (the consensus sets of the selected models are disjoint).

From Disjoint Set Cover to a QUBO

Disjoint set cover:

$$\min_{\mathbf{z} \in \mathbb{B}^m} \mathbb{1}_m^\top \mathbf{z} \quad \text{s.t. } P\mathbf{z} = \mathbb{1}_n$$

↓ convert to a QUBO

$$\tilde{Q} = \lambda P^\top P, \quad \tilde{\mathbf{s}} = \mathbb{1}_m - 2\lambda P^\top \mathbb{1}_n$$

$$Q = 0, \quad \mathbf{s} = \mathbb{1}_m, \quad A = P, \quad \mathbf{b} = \mathbb{1}_n$$



Rectified QUBO:

$$\min_{\mathbf{y} \in \mathbb{B}^d} \mathbf{y}^\top Q \mathbf{y} + \mathbf{s}^\top \mathbf{y} + \lambda \|A\mathbf{y} - \mathbf{b}\|_2^2$$

$$\min_{\mathbf{y} \in B^d} \mathbf{y}^\top \tilde{Q} \mathbf{y} + \tilde{\mathbf{s}}^\top \mathbf{y}$$

$$\tilde{Q} = Q + \lambda A^\top A, \quad \tilde{\mathbf{s}} = \mathbf{s} - 2\lambda A^\top \mathbf{b}$$

From Disjoint Set Cover to a QUBO

Disjoint set cover:

$$\min_{\mathbf{z} \in \mathbb{B}^m} \mathbb{1}_m^\top \mathbf{z} \quad \text{s.t. } P\mathbf{z} = \mathbb{1}_n$$

↓ convert to a QUBO

$$\tilde{Q} = \lambda P^\top P, \quad \tilde{\mathbf{s}} = \mathbb{1}_m - 2\lambda P^\top \mathbb{1}_n$$

$$Q = 0, \quad \mathbf{s} = \mathbb{1}_m, \quad A = P, \quad \mathbf{b} = \mathbb{1}_n$$

$$\min_{\mathbf{z} \in \mathbb{B}^m} \lambda \mathbf{z}^\top (P^\top P) \mathbf{z} + (\mathbb{1}_m - 2\lambda P^\top \mathbb{1}_n)^\top \mathbf{z}$$

the final QUBO of the QMMF approach



Rectified QUBO:

$$\min_{\mathbf{y} \in \mathbb{B}^d} \mathbf{y}^\top Q \mathbf{y} + \mathbf{s}^\top \mathbf{y} + \lambda \|A\mathbf{y} - \mathbf{b}\|_2^2$$

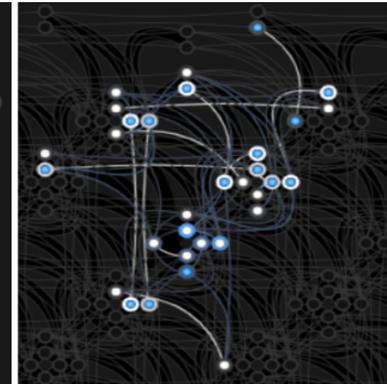
$$\min_{\mathbf{y} \in B^d} \mathbf{y}^\top \tilde{Q} \mathbf{y} + \tilde{\mathbf{s}}^\top \mathbf{y}$$

$$\tilde{Q} = Q + \lambda A^\top A, \quad \tilde{\mathbf{s}} = \mathbf{s} - 2\lambda A^\top \mathbf{b}$$

QA →



logical problem



minor embedding

... →

Iterative Decomposed Set Cover (DeQuMF)

Algorithm 1 DEQUMF

Require: P, s

```
while  $|P.\text{columns}| > s$  do  
  subproblems = ColumnPartition( $P, s$ )  
   $i \leftarrow 0$   
  while  $i < |\text{subproblems}|$  do  
     $\mathcal{J}_i \leftarrow$  models in the  $i$ -th subproblem  
     $P_{\mathcal{J}_i} \leftarrow P$  retaining only the  $\mathcal{J}_i$  columns  
     $\mathbf{z}_i = \text{QUMF}(P_{\mathcal{J}_i})$   
    remove from  $P$  columns  $P_{\mathcal{J}_i}[:, 1 - \mathbf{z}_i]$   
     $i \leftarrow i + 1$   
  end while  
end while  
 $\mathbf{z} \leftarrow \text{QUMF}(P)$   
return  $P[:, \mathbf{z}]$ 
```

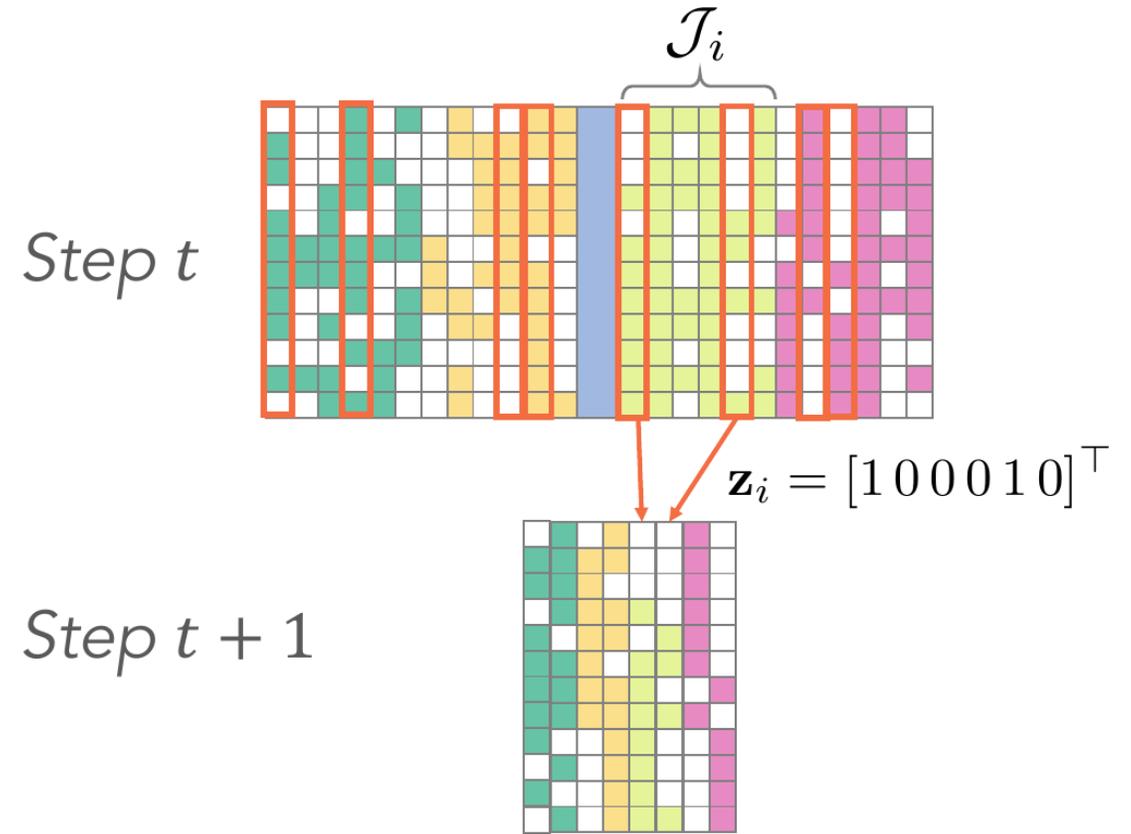


Illustration of the iterative pruning technique applied to P in DeQuMF over consecutive iterations.

Quantum Multi-Model Fitting

	RANSACOV [29]	QUMF (SA)	DEQUMF	DEQUMF (SA)
<i>mean</i>	9.79	3.85	16.22	0.77
<i>median</i>	7.97	3.54	11.0	0.18

Table 2. Misclassification Error [%] for several methods on the 15 **multi-model** sequences of the AdelaideRMF dataset [49].

Algorithm	Outlier ratio		
	10%	20%	Full sequences
QUMF (SA)	7.22	11.34	13.23
DEQUMF	2.41	10.53	16.17
DEQUMF (SA)	6.26	8.28	10.83
HQC-RF [17]	3.71	37.0	45.84

Table 3. Misclassification Error [%] for quantum methods on the **single-model** sequences of the AdelaideRMF dataset [49].

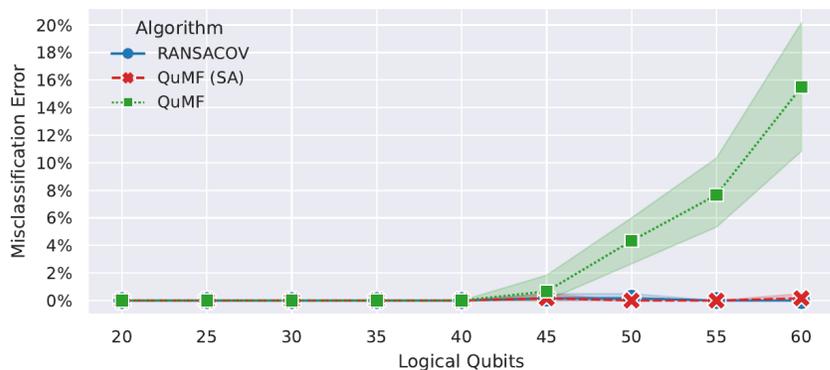


Figure 3. Misclassification Error for several methods on the *Star5* dataset [44]. The number of points n is fixed to $n = 30$, the number of ground-truth structures is fixed to $k = 5$, and the number of sampled models m is arranged on the x-axis.

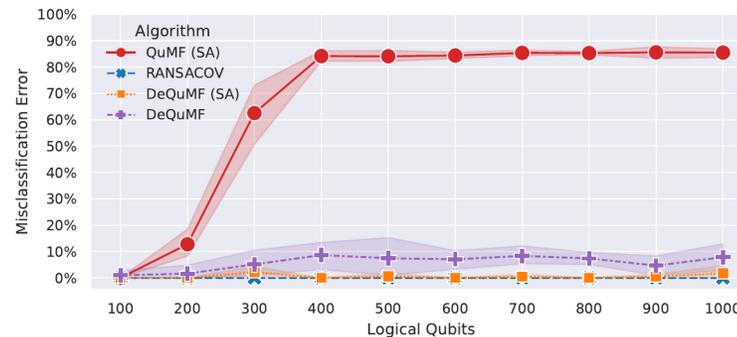


Figure 5. Misclassification Error for several methods on the *Star5* dataset [44]. The number of points n is fixed to $n = 250$, the number of ground-truth structures is fixed to $k = 5$, and the number of sampled models m (corresponding to the dimension of the search space) is arranged on the x-axis.

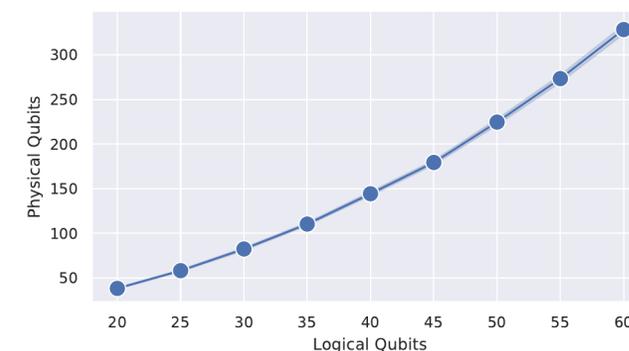
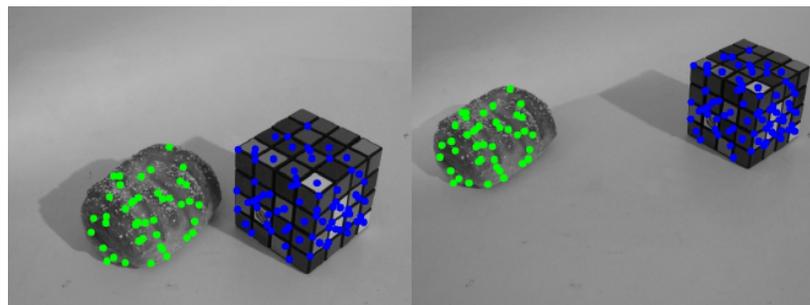
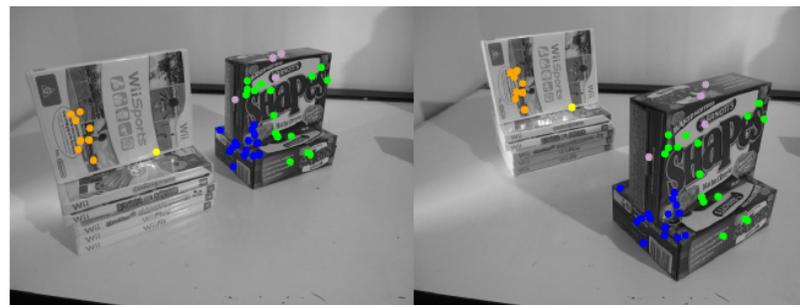


Figure 4. Relationship between *physical qubits* and *logical qubits* in embeddings produced with small-scale preference matrices from the *Star5* dataset [44].

Quantum Multi-Model Fitting



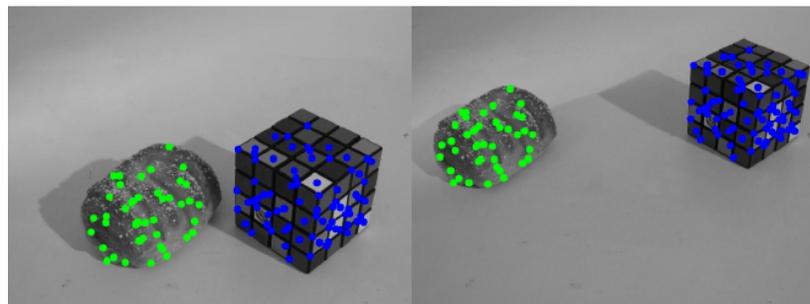
(a) DEQUMF outcome, misclassification error = 1.7%.



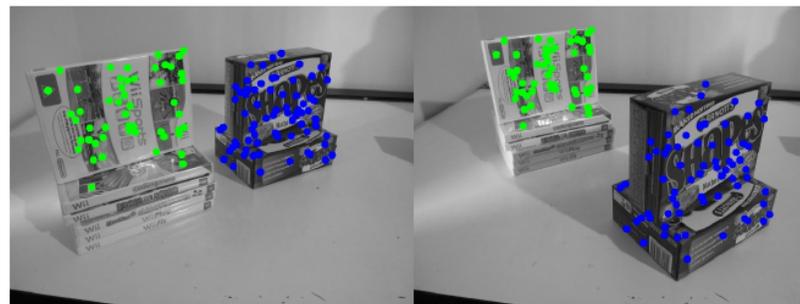
(a) DEQUMF outcome, misclassification error = 47.9%.



(a) DEQUMF outcome, misclassification error = 0.2%.



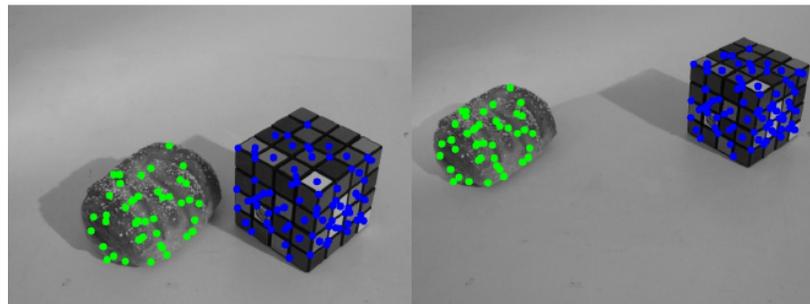
(b) DEQUMF (SA) outcome, misclassification error = 0%.



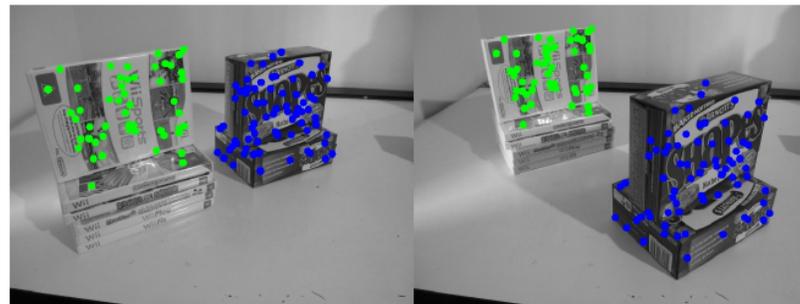
(b) DEQUMF (SA) outcome, misclassification error = 1.7%.



(b) DEQUMF (SA) outcome, misclassification error = 0%.



(c) Ground-truth segmentation.



(c) Ground-truth segmentation.



(c) Ground-truth segmentation.

Extension for Outlier Robustness (WiP)



(a) QuMF, $E_{mis} = 93.00\%$



(b) DeQuMF, $E_{mis} = 41.69\%$



(c) RQuMF, $E_{mis} = 2.9\%$



(d) De-RQuMF, $E_{mis} = 0.53\%$

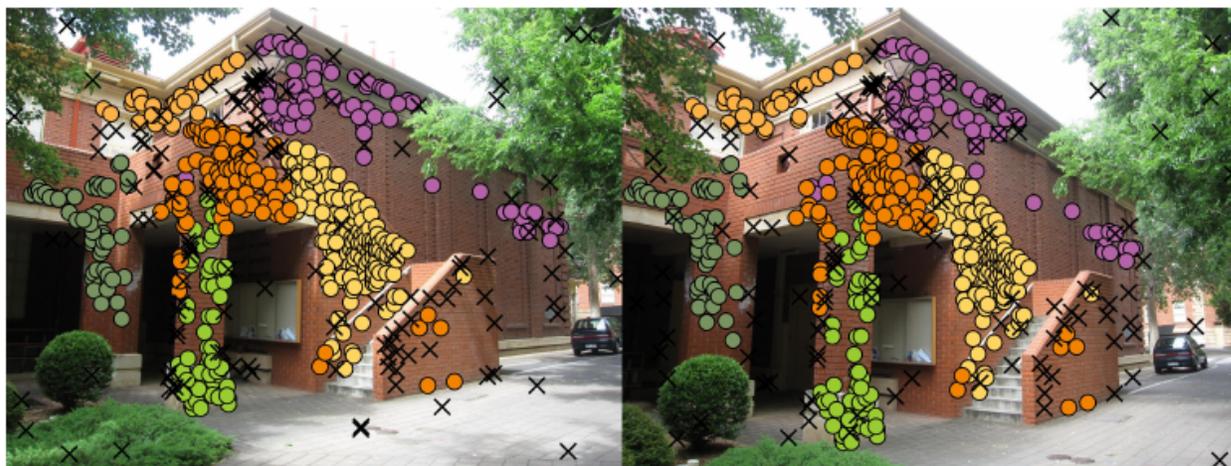
Extension for Outlier Robustness (WiP)



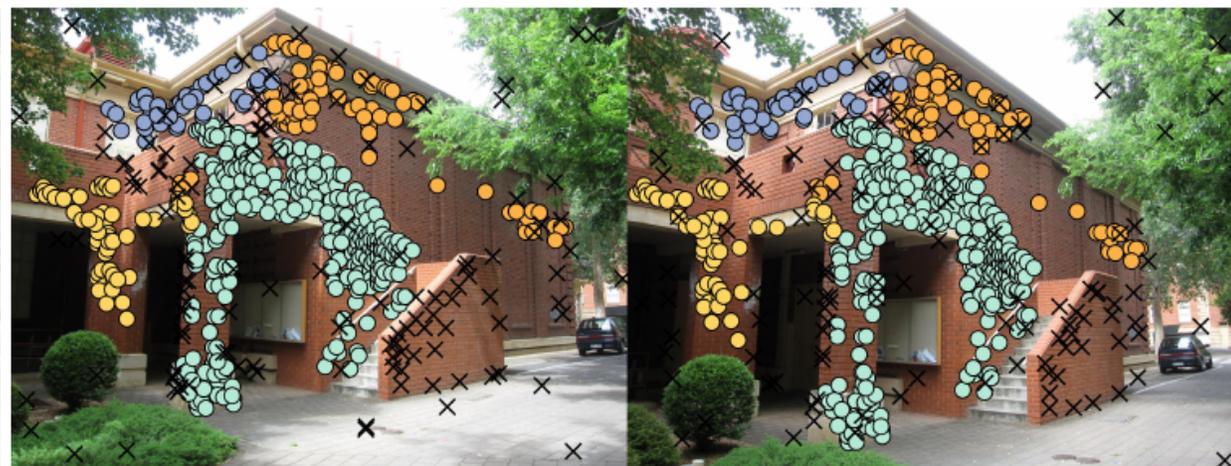
QuMF, $E_{mis} = 80.27\%$



DeQuMF, $E_{mis} = 27.40\%$

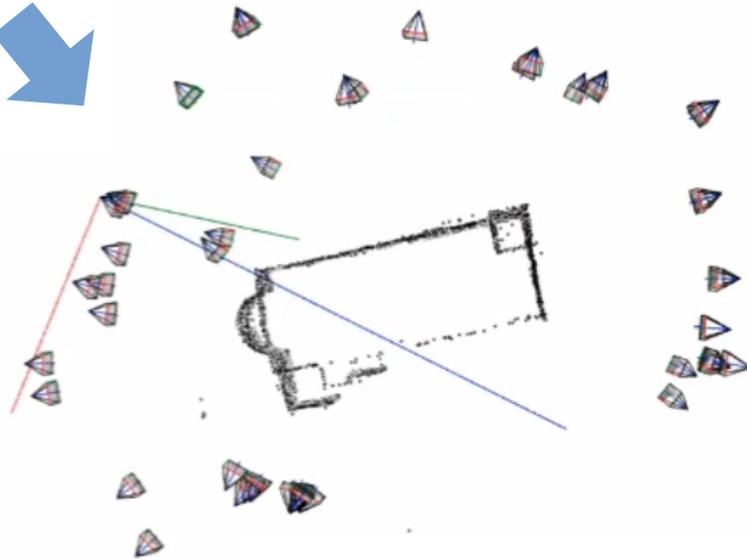


RQuMF, $E_{mis} = 35.67\%$

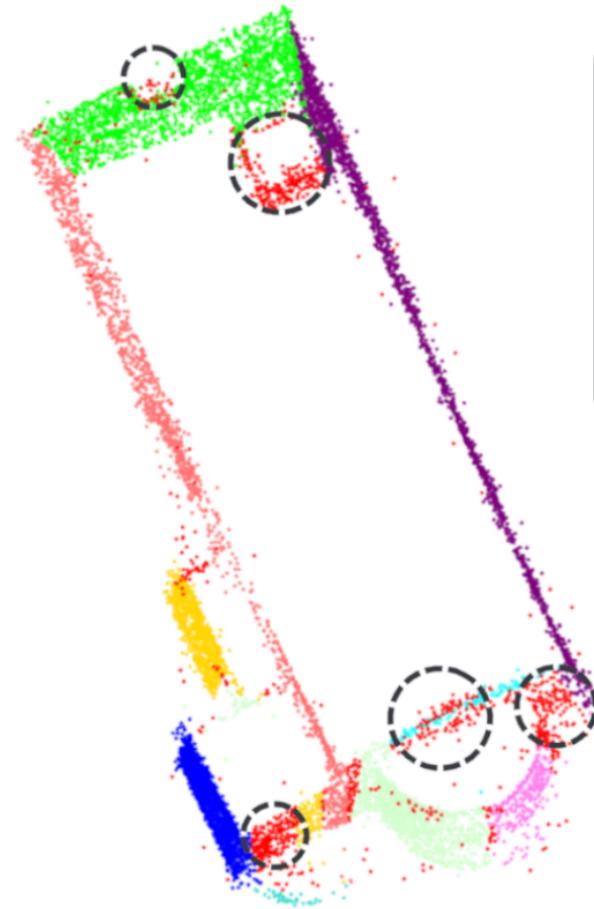


De-RQuMF, $E_{mis} = 22.49\%$

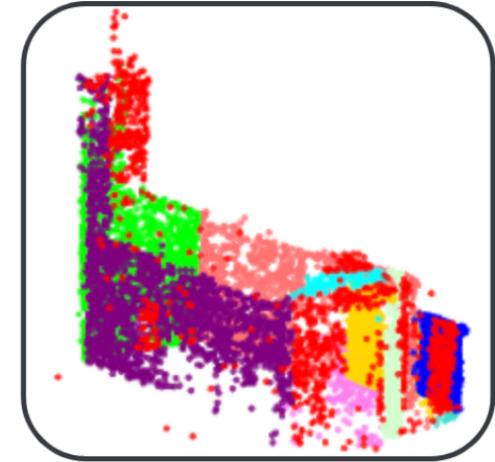
Robust Multi-model Fitting (3D Example)



Point cloud obtained by image-based 3D reconstruction.



The point cloud segmented with Robust Multi-Model Fitting Approach.





Kang Sung Hoon, Wind Lion-1, 2015

Mesh Alignment

Mesh Alignment as a Quadratic Assignment Problem

Generic shape matching can be formulated as QAP:

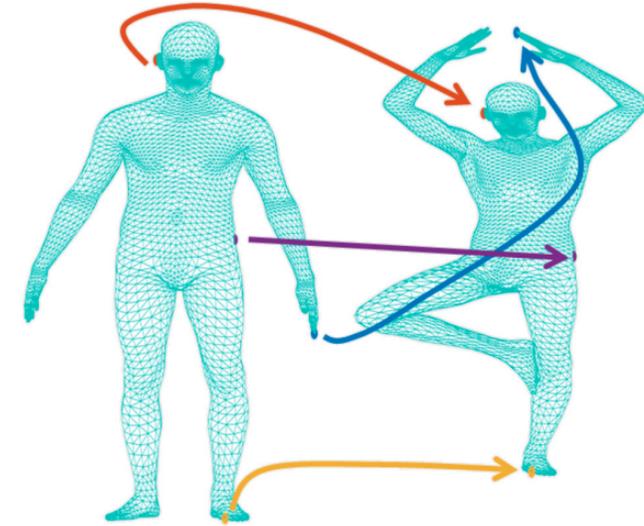
$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^T W \mathbf{x}$$

$$\mathbf{x} = \text{vec}(X)$$

$$\mathbb{P} \subset \{0, 1\}^{n \times n}$$

$$W \in \mathbb{R}^{n^2 \times n^2}$$

- * The solution space is **exponential in n**
- * **NP-hard problem**; finding global optima for large inputs is unfeasible
- * Allows quadratic costs for matching point pairs and regards point neighbourhoods
- * Existing methods either **do not guarantee globally-optimal solutions** or have prohibitive runtime complexity



- Use a QPU to solve QAP without relaxations, while providing theoretical global optimality guarantees

Mesh Alignment

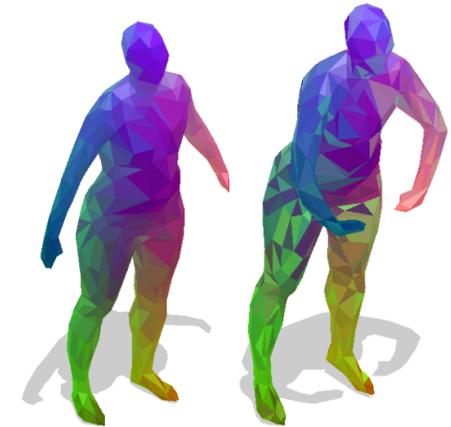
Given: 3D shapes M and N , both discretised with n vertices.

$$W_{i \cdot n + k, j \cdot n + l} = |d_M^g(i, j) - d_N^g(k, l)|$$

Geometric meaning of $d^g(a, b)$ influences the structure of QAP

Find: optimal P

$$\mathbb{P}_n = \{X \in \{0, 1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j\}.$$



M N

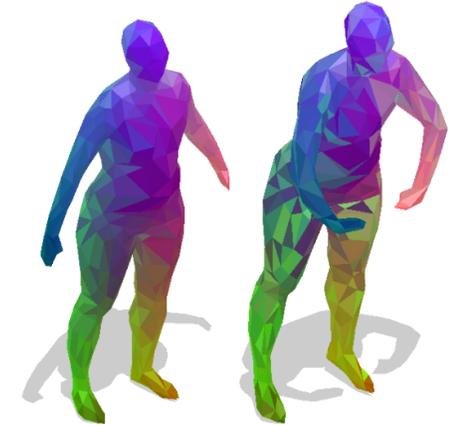
Mesh Alignment

Given: 3D shapes M and N , both discretised with n vertices.

$$W_{i \cdot n + k, j \cdot n + l} = |d_M^g(i, j) - d_N^g(k, l)|$$

Geometric meaning of $d^g(a, b)$ influences the structure of QAP

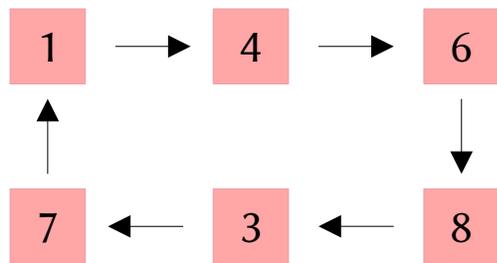
Find: optimal P



M N

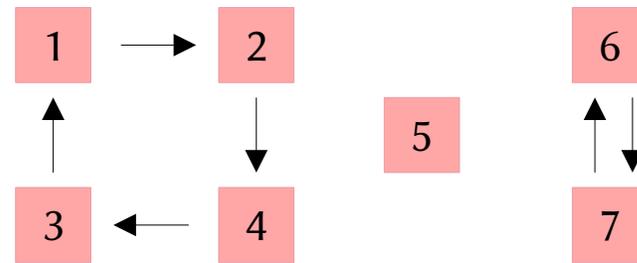
$$\mathbb{P}_n = \{X \in \{0, 1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j\}.$$

k-cycles:



six-cycle

Disjoint permutations commute:



four-cycle fixed point two-cycle

Any X can be written as

$$X = \prod_{i=0}^N c_i, \text{ i.e.,}$$

a product of 2-cycles.

Mesh Alignment

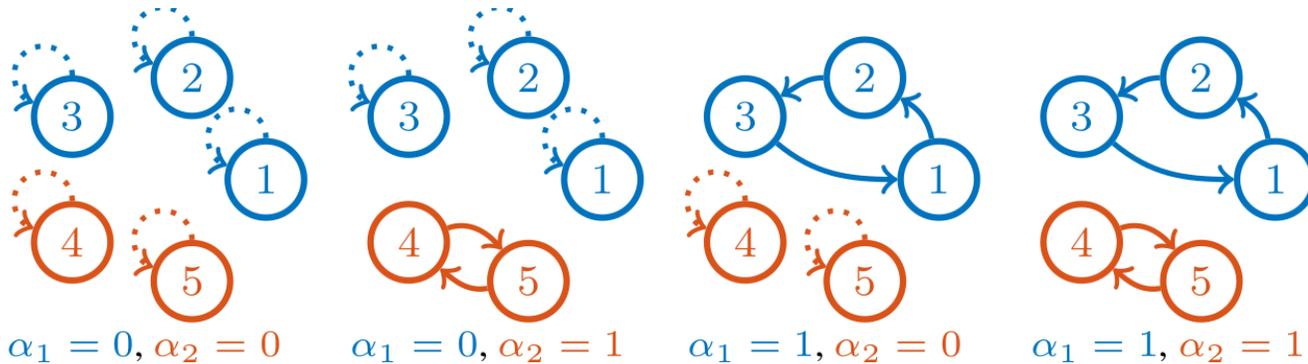
Assume $C = \{c_1, \dots, c_m\}$ is a set of disjoint cycles.

Consider

$$\arg \min_{\{P \in \mathbb{P}_n \mid \exists \alpha \in \{0,1\}^m : P = (\prod_i c_i^{\alpha_i}) P_0\}} E(P)$$

↓
↓
 binary vector parametrising P initial permutation

- 1) Decides if to apply each cycle;
- 2) The complexity depends on the number of cycles (not n).



$$\begin{pmatrix} 1-\alpha_1 & 0 & \alpha_1 & 0 & 0 \\ \alpha_1 & 1-\alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & 1-\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha_2 & \alpha_2 \\ 0 & 0 & 0 & \alpha_2 & 1-\alpha_2 \end{pmatrix}$$

Computationally expensive to solve:

$$\min_{X \in \mathbb{P}_n} E(X) := \mathbf{x}^T W \mathbf{x} \quad W_{i \cdot n + k, j \cdot n + l} = |d_M^g(i, j) - d_N^g(k, l)|$$

Solve instead

$$\arg \min_{\{P \in \mathbb{P}_n \mid \exists \alpha \in \{0, 1\}^m : P = (\prod_i c_i^{\alpha_i}) P_0\}} E(P) \quad (\text{cyclic alpha-expansion})$$

... leading to

$$\min_{\alpha \in \{0, 1\}^m} \alpha^T \tilde{W} \alpha \quad \tilde{W}_{ij} = \begin{cases} E(C_i, C_j) & \text{if } i \neq j, \\ E(C_i, C_i) + E(C_i, P_0) + E(P_0, C_j) & \text{otherwise.} \end{cases} \quad (\text{not submodular})$$

$$E(Q, R) = \text{vec}(Q)^T W \text{vec}(R)$$

$$P(\alpha) = P_0 + \sum_{i=1}^m \alpha_i \frac{C_i - I}{C_i} P_0$$

Q-Match

Initialise P_0 via descriptor-based similarity

repeat until converged

obtain I_M and I_N and choose from them a set of k random and disjoint 2-cycles

construct a submatrix of worst matches W_s

repeat until every 2-cycle occurred

choose a random set of 2-cycles

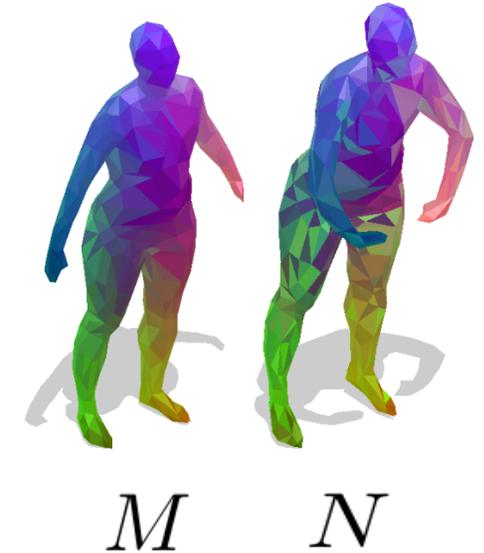
calculate \tilde{W}_s and solve $\min_{\alpha \in \{0,1\}^m} \alpha^T \tilde{W} \alpha$ on a QPU

$$P_i = \left(\prod_j c_j^{\alpha_j} \right) P_{i-1}$$

apply the obtained permutation to worst matches

sets of m vertices with the highest mismatch scores

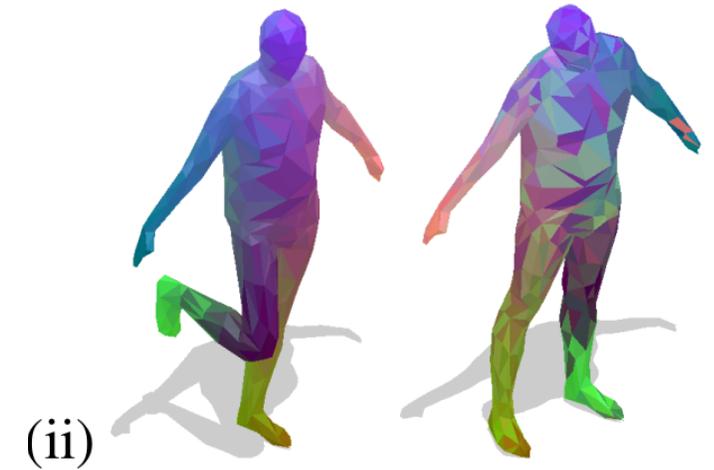
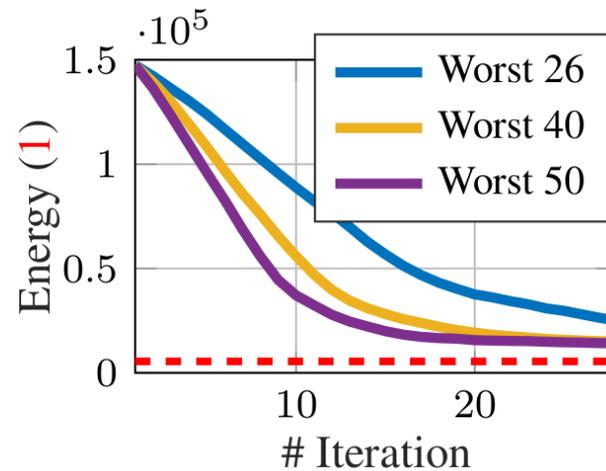
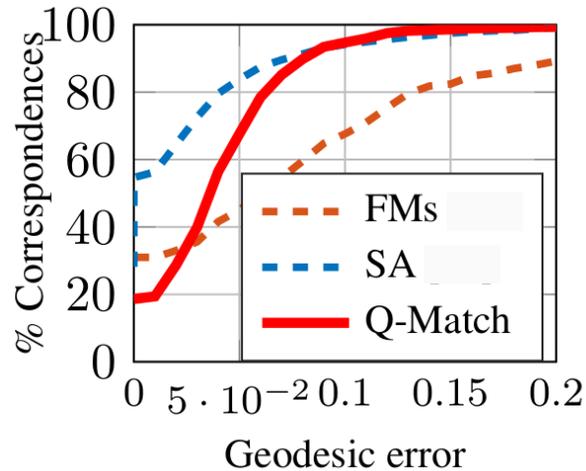
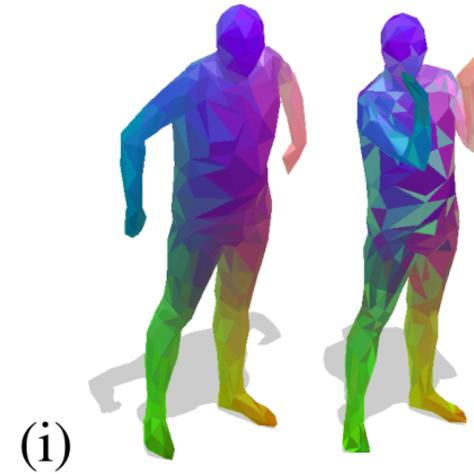
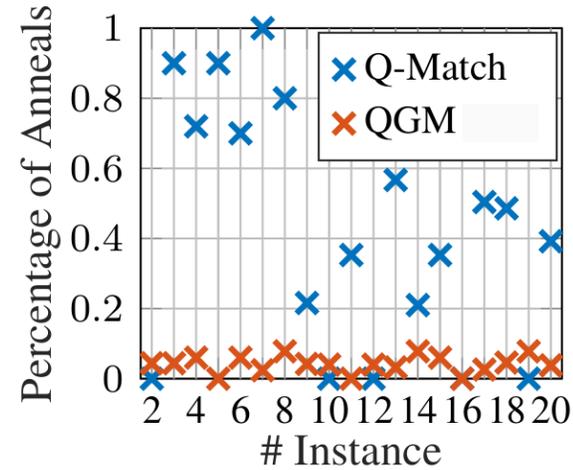
NP-hard; decides to apply C_i or not



- Entries of QAP are highly correlated (isometry)
 - Target explicitly points with high energy scores based on detection of point mapping inconsistencies

Experimental Evaluation

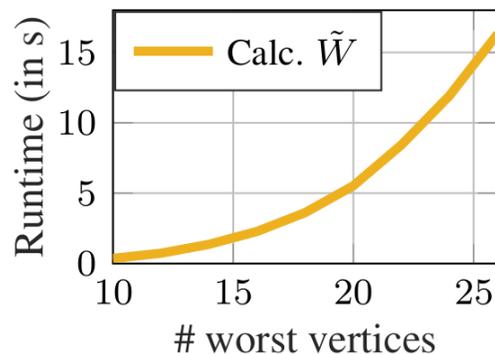
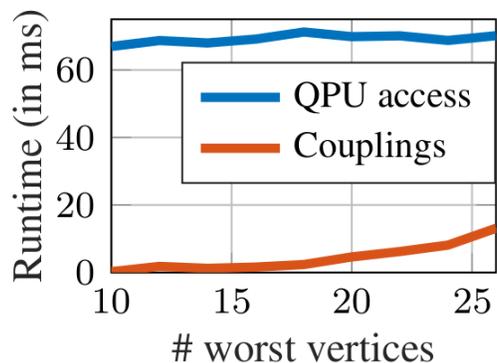
Success rates on 20 random problems (\rightarrow).



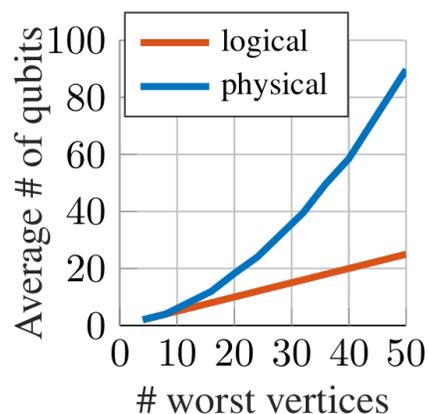
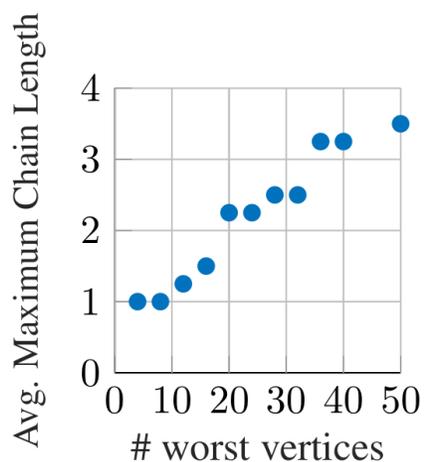
Cumulative error (left) and convergence (right) on FAUST.

Example correspondences from the FAUST registrations.

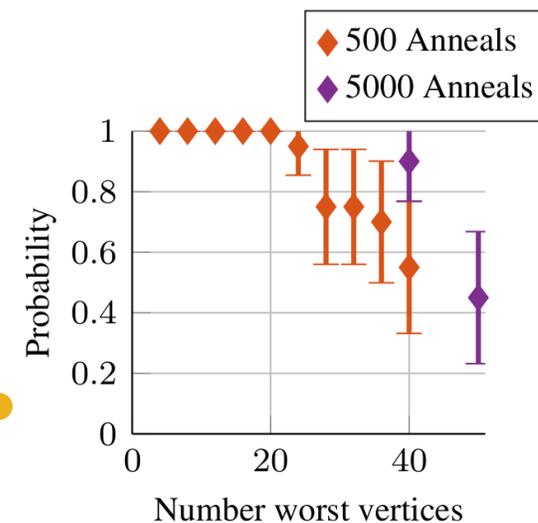
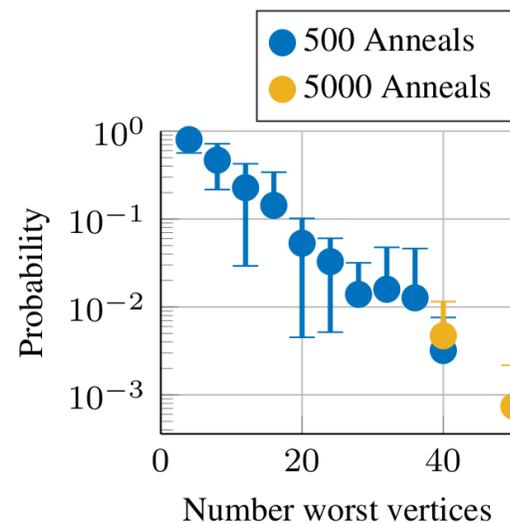
Experimental Evaluation



Influence of the problem size on the runtime.

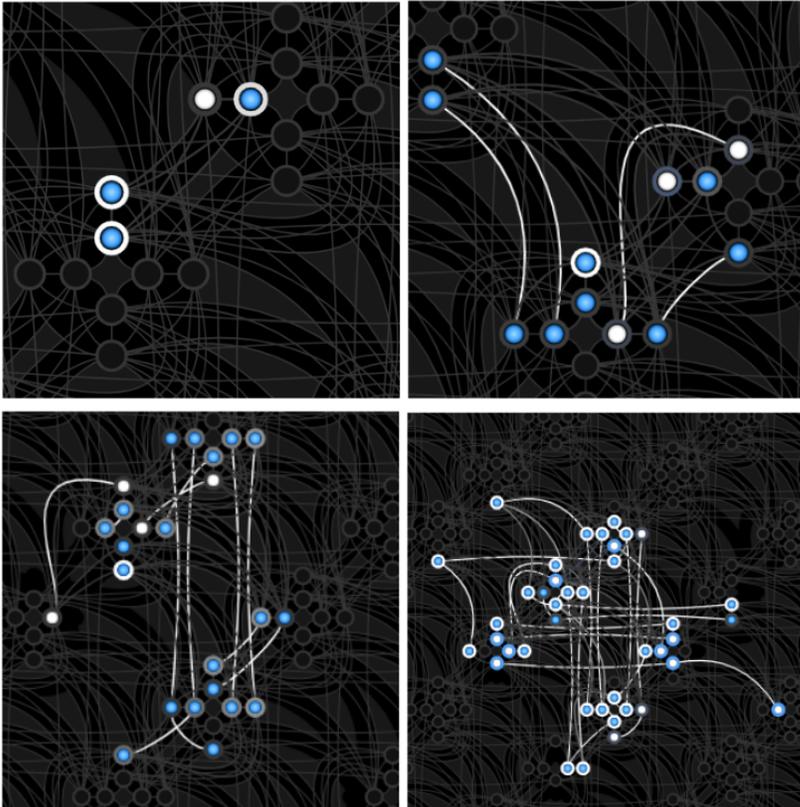


Minor embedding characteristics.

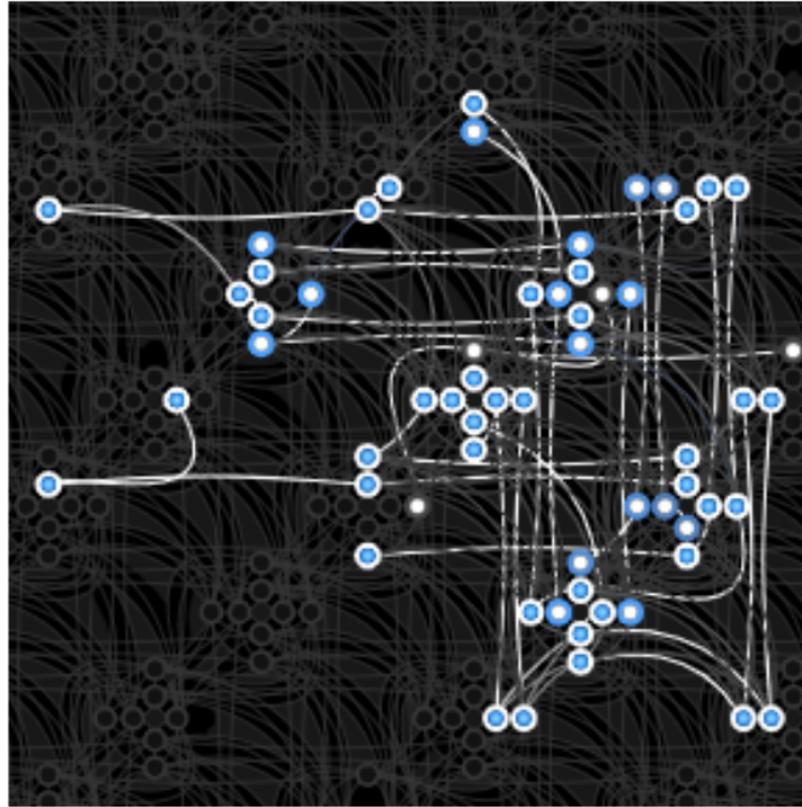


Success probability (left) and the fraction of executions where the best solution is the optimum (right).

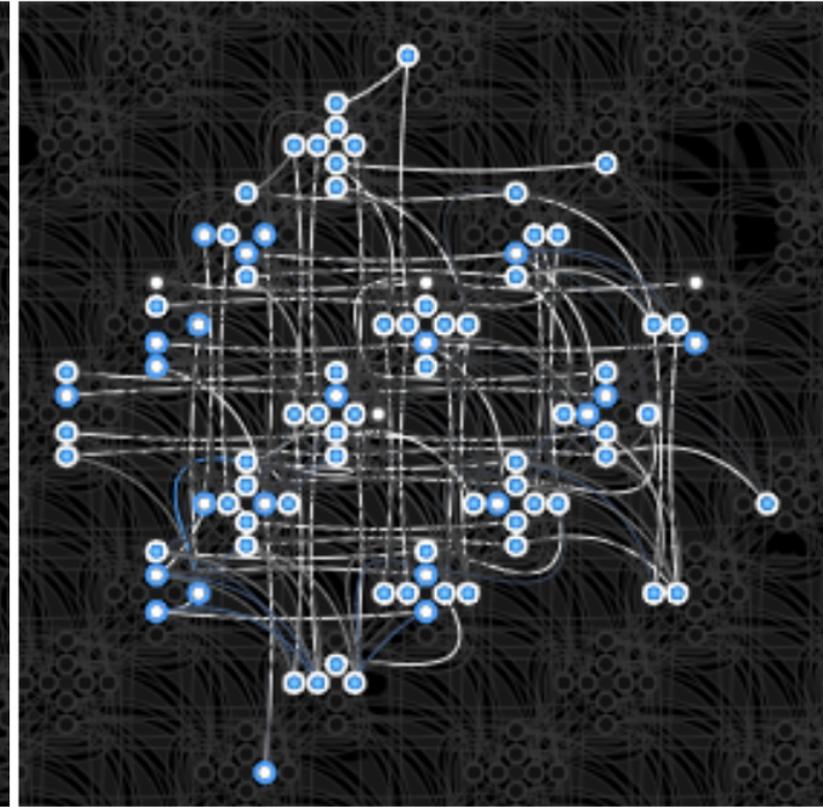
Examples of Minor Embeddings



8, 16, 24, 32 worst vertices



40 worst vertices



50 worst vertices

4D and Quantum Vision (4DQV)

4D and Quantum Computer Vision Group

Visual Computing and Artificial Intelligence Department



Quantum Visual Computing

This page summarises the activities of 4DQV related to quantum visual computing (QVC).

Our long-term goal is to demonstrate the practical advantages of using quantum hardware in visual computing. This is one of the differentiation factors compared to research groups predominantly investigating the theoretical aspects of the quantum computational paradigm. Like many like-minded colleagues across different research fields, we believe there is no alternative to quantum computing in the long term. The question is not whether the quantum era will arrive but how well we as a community are prepared for the new disruptive technology.

4DQV and Collaborators



Speaker



Christian



Shuteng



Marcel



Natacha



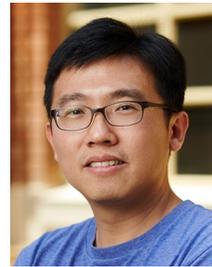
Tolga



Michael



Zorah



Tat-Jun



Federica



Luca



Willi



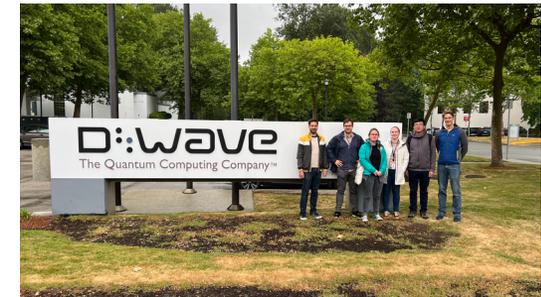
Elisa



MPI for Informatics, Saarbrücken, Germany



4D and Quantum
Vision Group



Visiting D-Wave Headquarters,
Burnaby, BC (2023)

Research Interests:

- 3D/4D Reconstruction and Neural Rendering
- 4D Generative Models
- Quantum-enhanced Computer Vision/Visual Computing

Overview of Activities Related to QeCV

Introduction to Quantum Visual Computing

07.02.2024

4D and Quantum
Vision Group $\langle \mathcal{A} | \psi \rangle$



Dr. Vladislav Golyanik - Advanced Topics in Neural Rendering and Reconstruction



Call for Papers: Special Issue on Quantum Visual Computing

IEEE CG&A seeks submissions for this upcoming special issue.

Important Dates

- Submissions due: 22 February 2024
- Publication: September/October 2024

In the near future, quantum computing is expected to provide superpolynomial speedups—and/or reduction in energy consumption—compared to conventional (super-)computers in a multitude of use cases. However, to benefit from these speedups, a radically new approach to computing is required, relying on the quantum-mechanical phenomena of qubit superposition, entanglement, interference, and resulting exponential quantum parallelism. However, qubits cannot be copied and collapse to binary states on measurement. The significant potential benefits when full-fledged quantum computers become available, coupled with the challenges of a novel, often unintuitive approach.



Lecture at Saarland University

Special Issue of CG&A

2nd QCVML Workshop / ECCV 2024

- **Invited Lecture at the European Summer School on Quantum AI (EQAI) 2024**
- **Guest Editor** at IEEE CG&A (Special Issue on Quantum Visual Computing, 2024)
- **Teaching at Saarland University:**
 - QCV within the lecture **Advanced Topics in Neural Rendering and Reconstruction** in WS 23/24
 - Seminar *Quantum Computer Vision and Machine Learning* (QCVML) in WS 23/24
- **The QCVML Workshop** at international Computer Vision conferences (CVPR 2023, ECCV 2024)
- **Presentations at QTML 2023 and 2024 (tentatively)**



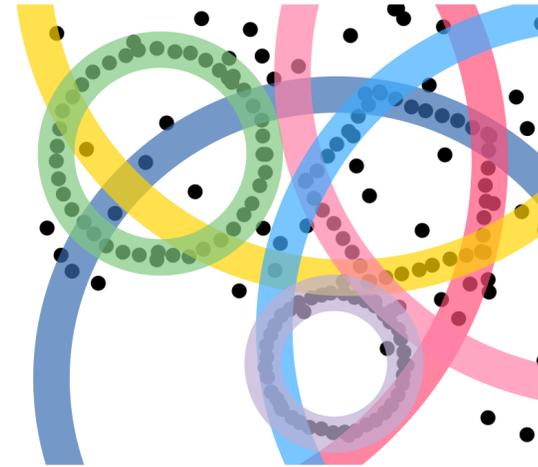
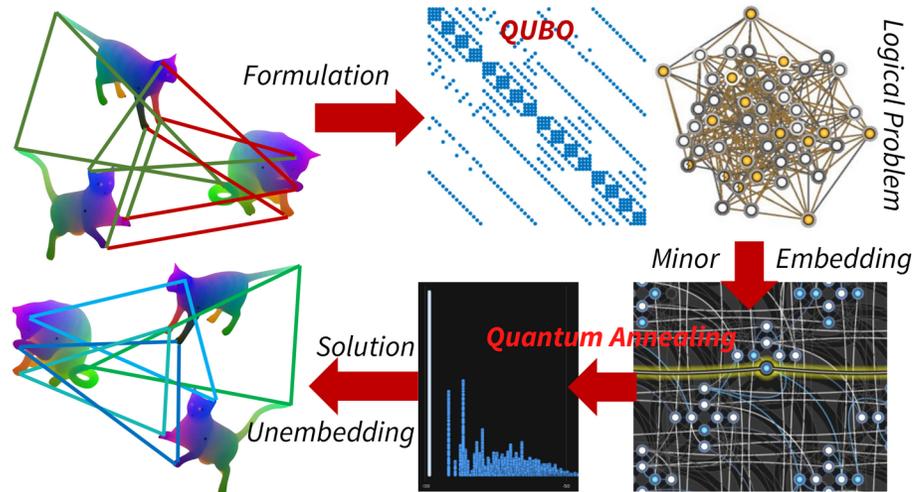
Conclusions



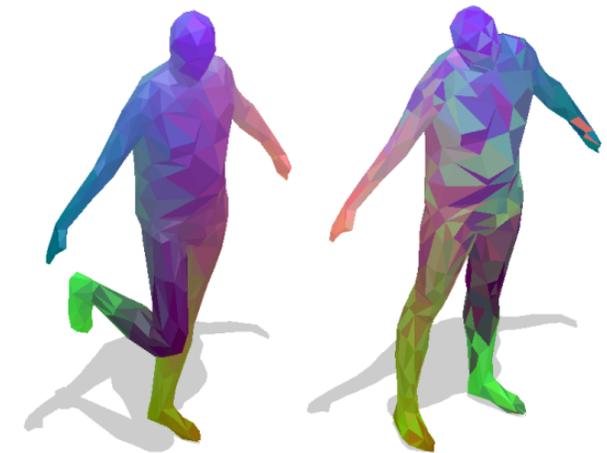
“It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.”

— Richard P. Feynman

tags: experimentation, science, theory



Magri and Fusiello, CVPR 2019.



- **Ising encodings** for MMF and mesh alignment **bring advantages w.r.t. previous methods**
- **Optimising over swaps** is more efficient than over permutation matrices (cyclic alpha expansion)
- **QA works for moderate problem instances** but is not in step with SA yet
 - Iterative nature of algorithms relying on QA compensates for hardware imperfections
- QeCV approaches relying on simulation **can have higher accuracy** than classical algorithms
- CV and QeCV are experimental research fields (mostly driven by experimental results):
 - Understand the problem: Does it have a **combinatorial optimisation** part?
 - Can the combinatorial part be mapped to **the Ising form**?
 - What has to be and can be pre-processed on the CPU?

Thanks! Questions?

The Corresponding Unitary Transformation

$$H(t) = A(t)H_{\text{init}} + B(t)H_{\text{final}}$$

Time-dependent Hamiltonian in QA

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H(t) |\psi(t)\rangle$$

Time-dependent Schrödinger Equation

$$U(T) = \tau \exp \left(\frac{-i}{\hbar} \int_0^T H(t) dt \right)$$

Unitary transformation that evolves the state from $t = 0$ to $t=T$

$$H_{\text{final}} = \sum_i h_i \sigma_i^z + \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

Final Ising Hamiltonian (representing the Ising problem)

$$U(t) = \exp \frac{-iH(t)t}{\hbar}$$

Solution to time-dependent Schrödinger Equation (instantaneous unitary transformation for time t)

$$|\psi(T)\rangle = U(T) |\psi(0)\rangle$$

Final wavefunction at $t = T$