Advances in Quantum Computer Vision

Workshop on Quantum Information (08.12.2022)

Vladislav Golyanik
4D and Quantum Vision (4DQV) Group

- Independent research group integrated in the VCAI Department of MPI for Informatics
- Research agenda at the intersection of CV, CG and ML
- We collaborate in Germany and internationally on these topics
4D and Quantum Vision (4DQV) Group

Two Core Research Areas

- Non-rigid 3D (=4D) Perception/Reasoning
- Quantum Computer Vision

1) explicit and 2) implicit 3D representations

1) What are the relevant problems?
2) Real quantum hardware is in the foreground.
Non-rigid 3D (=4D) Perception/Reasoning

General Objects  Hands  3D Human MoCap (different settings)

Volumetric 3D Representations  Video/Action Manipulation

Quantum Computer Vision

Problems solved by synchronisation

(Iterative) Matching methods
Quantum Computer Vision

Problems solved by synchronisation

(Iterative) Matching methods

qubit properties
Constrained optimisation

Methodology of mapping problems to quantum hardware

AQC + ML
3D Human MoCap with Physics Priors

Shimada et al., ACM SIGGRAPH 2021.
3D Human-Object Motion Capture

\[ x_i = f \frac{X_i}{Z_i} + c_x, \quad y_i = f \frac{Y_i}{Z_i} + c_y, \quad \forall i, \]
\[ \text{s.t.} \quad \sqrt{g_x^2 + g_y^2 + g_z^2} = 9.81 \text{ m/s}^2, \]

where
\[ \begin{cases} 
X_i = X_0 + u_xt + \frac{1}{2}g_xt^2, \\
Y_i = Y_0 + u_yt + \frac{1}{2}g_yt^2, \quad \text{and} \\
Z_i = Z_0 + u_zt + \frac{1}{2}g_zt^2.
\end{cases} \]

Dabral et al., ICCV 2021.
3D Human MoCap with a Scene Prior

HULC (Ours) (Top View)

POSA
[Hassan et al., CVPR 2021]

PROX (RGB)
[Hassan et al., ICCV 2019]
3D Human MoCap with a Scene Prior

Contact Label Annotations on GTA-IM dataset

Scale and Depth Ambiguity

GTA-IM dataset [Cao et al.]

Our Contact Annotations
Playable Environments

The Approach to Construct Playable Environments (PE)

Menapace et al., CVPR 2022.
Playable Environments

The Approach to Construct Playable Environments (PE)

Menapace et al., CVPR 2022.
Playable Environments: Players Control

Tennis

Minecraft

Menapace et al., CVPR 2022.
Playable Environments: Style Manipulation
Playable Environments: Style Manipulation
From a Classical to a Quantum Perspective
From a Classical to a Quantum Perspective

- QC are steadily improving
- How can computer vision benefit from QC?
From a Classical to a Quantum Perspective

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- QC are steadily improving
- How can computer vision benefit from QC?
### From a Classical to a Quantum Perspective

**QC are steadily improving**

**How can computer vision benefit from QC?**

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Copyright: Forschungszentrum Jülich / JUNIQ
Quantum annealing in the transverse Ising model
Tadashi Kadowaki and Hidetoshi Nishimori

We introduce quantum fluctuations into the simulated annealing process of optimization problems, aiming at faster convergence to the optimal state. Quantum fluctuations cause transitions between states and thus play the same role as thermal fluctuations in the conventional approach. The idea is tested by the transverse Ising model, in which the transverse field is a function of time similar to the temperature in the conventional method. The goal is to find the ground state of the diagonal part of the Hamiltonian with high accuracy as quickly as possible. We have solved the time-dependent Schrödinger equation numerically for small size systems with various exchange interactions. Comparison with the results of the corresponding classical (thermal) method reveals that the quantum annealing leads to the ground state with much larger probability in almost all cases if we use the same annealing schedule. [S1063-651X(98)02910-9]

Kadowaki and Nishimori, 1998

A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem
Edward Farhi,1,2* Jeffrey Goldstone,1 Sam Gutmann,2
Joshua Lapan,2 Andrew Lundgren,2 Daniel Prode2

A quantum system will stay near its instantaneous ground state if the Hamiltonian that governs its evolution varies slowly enough. This quantum adiabatic behavior is the basis of a new class of algorithms for quantum computing. We tested one such algorithm by applying it to randomly generated hard instances of an NP-complete problem. For the small examples that we could simulate, the quantum adiabatic algorithm worked well, providing evidence that quantum computers (if large ones can be built) may be able to outperform ordinary computers on hard sets of instances of NP-complete problems.

Although a large quantum computer has yet to be built, the rules for programming such a device, which are derived from the laws of quantum mechanics, are well established. It is already known that quantum computers could solve problems believed to be intractable on classical (i.e., nonquantum) computers. An intractable problem is one that necessarily takes too long to solve when the input gets too big. More precisely, a classically intractable problem is one that cannot be solved using any classical algorithm whose running time grows only polynomially as a function of the length of the input. For example, all known classical factoring algorithms require a time that grows faster than any polynomial as a function of the number of digits in the integer to be factored. Shor’s quantum algorithm for the factoring problem (7) can factor an integer in a time that grows (roughly) as the square of the number of digits. This raises the question of whether quantum computers could solve other classically difficult prob-

Farhi et al., 2001

Foundations of AQC (D-Wave)

Initial state:

$$|\psi(t = 0)\rangle = \bigotimes_{i=1}^{n} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

Transition (simplified):

$$H(t) = \left(1 - \frac{t}{\tau}\right) H_I + \frac{t}{\tau} H_P$$

Final state encoding the problem and the data:

$$\min_{s \in \{-1,1\}^n} s^\top J s + b^\top s$$

Exemplary Q (QUBO, 21 qubits):

Quadratic Unconstrained Binary Optimisation (QUBO) problem:

$$\arg\min_{x \in \mathbb{B}^n} x^\top Q x + s^\top x$$

Annealing functions (schedules)


Five Steps of Every AQC Algorithm

1. **Formulation**
2. **QUBO**
3. **Logical Problem**
4. **Minor Embedding**
5. **Solution**

Image: Birdal and Golyanik et al., CVPR 2021.
Five Steps of Every AQC Algorithm

1. Formulation
2. QUBO
3. Logical Problem
4. Quantum Annealing
5. Unembedding

Image: Birdal and Golyanik et al., CVPR 2021.
D-Wave Quantum Annealers

2000Q
- 2048 qubits (16x16x8)
- Nominal length 4 (internal couplers)
- Degree 6 (+2 external qubits)
- Internal and external couplers

Advantage
- 5640 qubits (~16x16x24)
- Nominal length 12 (internal couplers)
- Degree 15 (+3 external qubits)
- Internal, external and odd couplers

Advantage 2
- 7440 qubits (~15x15x32)
- Nominal length 16 (internal couplers)
- Degree 20 (+4 external qubits)
- Internal, external and odd couplers

Images: Silva et al., QIP, 2021; Dwave. D-Wave QPU Architecture: Topologies.
Permutation Synchronisation

Birdal and Golyanik et al., CVPR 2021.
Permutation Synchronisation

Birdal and Golyanik et al., CVPR 2021.
Permutation Synchronisation

\[
\arg\min_{\{x_i \in \mathcal{P}_n\}} \sum_{(i,j) \in \mathcal{E}} \|P_{ij} - X_i X_j^\top\|_F^2 = \arg\min_{\{x_i \in \mathcal{P}_n\}} x^\top Q'x,
\]

\[
Q' = -\begin{bmatrix}
I \otimes P_{11} & I \otimes P_{12} & \cdots & I \otimes P_{1m} \\
I \otimes P_{21} & I \otimes P_{22} & \cdots & I \otimes P_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
I \otimes P_{m1} & I \otimes P_{m2} & \cdots & I \otimes P_{mm}
\end{bmatrix}.
\]
Permutation Synchronisation

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\vdots & \vdots & \ddots & \vdots \\
I \otimes P_{m1} & I \otimes P_{m2} & \cdots & I \otimes P_{mm}
\end{bmatrix}
\]

is turned into

\[
\arg\min_{x \in \mathcal{B}} x^\top Q' x \quad s.t. \quad Ax = b
\]

where \(Q = Q' + \lambda A^\top A\) and \(s = -2\lambda A^\top b\).
Permutation Synchronisation

\[
\arg\min_{\{X_i \in \mathcal{P}_n\}} \sum_{(i,j) \in \mathcal{E}} \left\| P_{ij} - X_i X_j^T \right\|_F^2 = \arg\min_{\{X_i \in \mathcal{P}_n\}} x^T Q' x,
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\[
\arg\min_{x \in \mathcal{B}} x^T Q' x \quad s.t. \quad A x = b
\]

where

\[
Q = Q' + \lambda A^T A \quad \text{and} \quad s = -2\lambda A^T b.
\]

\[
\mathcal{P}_n := \{ P \in \{0,1\}^{n \times n} : P1_n = 1_n, \ 1_n^T P = 1_n^T \}
\]

Binary
Rows sum to 1
Cols sum to 1

\[
A_i = \begin{bmatrix}
I \otimes 1^T \\
1^T \otimes I
\end{bmatrix}
\]

\[
b_i = 1
\]

\[
A = \begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_n
\end{bmatrix}
\]

\[
b = 1
\]

\[
\begin{array}{c}
\text{unconstrained} \\
\text{constrained}
\end{array}
\]

Birdal and Golyanik et al., CVPR 2021.
## Permutation Synchronisation

### Evaluations on the synthetic dataset (4 views and 4 points)

<table>
<thead>
<tr>
<th></th>
<th>Car</th>
<th>Duck</th>
<th>Motorbike</th>
<th>Winebottle</th>
<th>Average</th>
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<td>Exhaustive</td>
<td>0.84 ± 0.104</td>
<td>0.91 ± 0.115</td>
<td>0.82 ± 0.10</td>
<td>0.95 ± 0.096</td>
<td>0.88 ± 0.104</td>
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<td>EIG</td>
<td>0.81 ± 0.083</td>
<td>0.86 ± 0.102</td>
<td>0.77 ± 0.059</td>
<td>0.87 ± 0.107</td>
<td>0.83 ± 0.088</td>
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<td>ALS</td>
<td>0.84 ± 0.095</td>
<td>0.90 ± 0.102</td>
<td>0.81 ± 0.078</td>
<td>0.94 ± 0.092</td>
<td>0.87 ± 0.092</td>
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<td>LIFT</td>
<td>0.84 ± 0.102</td>
<td>0.90 ± 0.103</td>
<td>0.81 ± 0.078</td>
<td>0.94 ± 0.092</td>
<td>0.87 ± 0.094</td>
</tr>
<tr>
<td>Birkhoff</td>
<td>0.84 ± 0.094</td>
<td>0.90 ± 0.107</td>
<td>0.81 ± 0.079</td>
<td>0.94 ± 0.093</td>
<td>0.87 ± 0.093</td>
</tr>
<tr>
<td>D-Wave(Ours)</td>
<td>0.84 ± 0.104</td>
<td>0.90 ± 0.104</td>
<td>0.81 ± 0.080</td>
<td>0.93 ± 0.095</td>
<td>0.87 ± 0.096</td>
</tr>
</tbody>
</table>

### Detailed evaluations over all the subsets

Birdal and Golyanik et al., CVPR 2021.
Permutation Synchronisation

Bit corrections using multiple measurements of different energies

Evaluations on the synthetic dataset (4 views and 4 points)

Detailed evaluations over all the subsets

Birdal and Golyanik et al., CVPR 2021.
Exemplary minor embeddings in the experiments with

- $n = 3, m = 3$ (A, $N = 49$),
- $n = 4, m = 4$ (B, $N = 341$), and
- $n = 3, m = 8$ (C, $N = 550$).
Permutation Synchronisation

Exemplary minor embeddings in the experiments with

- $n = 3, m = 3$ (A, $N = 49$),
- $n = 4, m = 4$ (B, $N = 341$), and
- $n = 3, m = 8$ (C, $N = 550$).

Average number of measured optimal solutions

Minor embedding functions

Birdal and Golyanik et al., CVPR 2021.
Motion Segmentation

Goal: Classify points in multiple images into different motions

two-frame matches (known)

$Z_{ij} = X_i X_j^T$

absolute segmentations (unknown)

Arrigoni et al., ECCV 2022.
Motion Segmentation

Goal: Classify points in multiple images into different motions

\[
\min_{X_1, \ldots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,
\]

s.t. \(\text{vec}(X_i) \in \mathcal{B}^{p_i}, \quad X_i 1_d = 1_{p_i} \quad \forall i = 1, \ldots, n\)

\[
X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & 0 & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & 0 \end{bmatrix}
\]

Arrigoni et al., ECCV 2022.
Motion Segmentation

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\]

\[
Z_{ij} = X_i X_j^T
\]

\[
\begin{align*}
\text{two-frame matches} & \quad \text{(known)} \\
\text{absolute segmentations} & \quad \text{(unknown)}
\end{align*}
\]

\[
\min_{\mathbf{y} \in \mathcal{B}^k} \mathbf{y}^T Q \mathbf{y} + s^T \mathbf{y} + \sum_{i} \lambda_i \| A_i \mathbf{y} - \mathbf{b}_i \|^2
\]

Arrigoni et al., ECCV 2022.
Permutation Synchronisation

\[
\min_{X_1, \ldots, X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,
\]

s.t. \( \text{vec}(X_i) \in \mathcal{B}^{p_i} \), \( X_i 1_d = 1_{p_i} \) \( \forall i = 1, \ldots, n \)

binary variables each point belongs to one motion

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X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & 0 & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & 0 \end{bmatrix}
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Arrigoni et al., ECCV 2022.
Permutation Synchronisation

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\min_{X_1,\ldots,X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,
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s.t. \( \text{vec}(X_i) \in \mathcal{B}^{p_i} \), \( X_i 1_{d_i} = 1_{p_i} \) \( \forall i = 1, \ldots, n \)

binary variables \quad each point belongs to one motion

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\]

\[
\text{maximize}_{X_1,\ldots,X_n} \sum_{(i,j) \in \mathcal{E}} \text{trace}(X_i^T (2Z_{ij} - 1_{p_i \times p_j}) X_j)
\]

s.t. \( \text{vec}(X_i) \in \mathcal{B}^{p_i} \), \( X_i 1_{d_i} = 1_{p_i} \) \( \forall i = 1, \ldots, n \)

\[
\text{max}_{X} \text{vec}(X)^T (I_{d \times d} \otimes (2Z - 1_{p \times p})) \text{vec}(X),
\]

s.t. \( \text{vec}(X) \in \mathcal{B}^{dp} \), \( X 1_{d} = 1_{p} \).

QuMoSeg-v1, dense Q

Arrigoni et al., ECCV 2022.
Permutation Synchronisation

$$\min_{X_1,\ldots,X_n} \sum_{(i,j) \in \mathcal{E}} \|Z_{ij} - X_i X_j^T\|_F^2,$$

s.t. $\text{vec}(X_i) \in B^{p_i}$, $X_i 1_d = 1_{p_i}$ $\forall i = 1, \ldots, n$

binary variables each point belongs to one motion

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & Z_{12} & \ldots & Z_{1n} \\ Z_{21} & 0 & \ldots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \ldots & 0 \end{bmatrix}$$

QuMoSeg-v1, dense Q

$$\max_{X_1,\ldots,X_n} \sum_{(i,j) \in \mathcal{E}} \text{trace}(X_i^T (2 Z_{ij} - 1_{p_i \times p_j}) X_j),$$

s.t. $\text{vec}(X_i) \in B^{p_i}$, $X_i 1_d = 1_{p_i}$ $\forall i = 1, \ldots, n$

$$\max X \text{vec}(X)^T (I_{d \times d} \otimes (2Z - 1_{p \times p})) \text{vec}(X),$$

s.t. $\text{vec}(X) \in B^{dp}$, $X 1_d = 1_p$.

QuMoSeg-v2, sparse Q (additional assumptions)

$$\max_{X_1,\ldots,X_n} \sum_{(i,j) \in \mathcal{E}} \text{trace}(X_i^T Z_{ij} X_j),$$

s.t. $\text{vec}(X_i) \in B^{p_i}$, $X_i 1_d = 1_{p_i}$, $1_{p_i}^T X_i = m_i^T$ $\forall i = 1, \ldots, n$.

extra constraints

$$\max X \text{vec}(X)^T (I_{d \times d} \otimes Z) \text{vec}(X),$$

s.t. $\text{vec}(X) \in B^{dp}$, $X 1_d = 1_p$, $KX = M$.

Arrigoni et al., ECCV 2022.
Motion Segmentation

μ = 0.97

ground truth

QuMoSeg

Mode, ICCV’19

Synch, ICCVW’19

Xu et al., TPAMI’19

Arrigoni et al., ECCV 2022.
## Motion Segmentation

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<td>0.89</td>
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</table>
Motion Segmentation

Accuracy under noisy inputs

Optimal solution probabilities

Chain strengths

Minor embedding functions

Arrigoni et al., ECCV 2022.
Iterative AQC Algorithms
Iterative AQC Algorithms

Initial Logical Problem

Precomputed Minor Embedding

Updated Problem

QPU

Quantum Annealing

Intermediate Solution

Iterate (CPU/GPU)

Final Solution
Shape Matching with AQC

\[
\begin{align*}
\min_{X \in \mathbb{P}_n} E(X) := x^T W x \\
x = \text{vec}(X) \quad W \in \mathbb{R}^{n^2 \times n^2} \\
\mathbb{P} \subset \{0, 1\}^{n \times n} \text{ (permutation matrix)} \\
\mathbb{P}_n = \{ X \in \{0, 1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \ \forall i, j \}. 
\end{align*}
\]
Shape Matching with AQC

\[ \min_{X \in \mathcal{P}_n} E(X) := x^T W x \]
\[ x = \text{vec}(X) \quad W \in \mathbb{R}^{n^2 \times n^2} \]
\[ \mathcal{P} \subset \{0, 1\}^{n \times n} \text{ (permutation matrix)} \]
\[ \mathcal{P}_n = \{ X \in \{0, 1\}^{n \times n} | \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j \} \]

k-cycles:

Disjoint permutations commute:

1 → 4 → 6 → 2 → 1 → 2 → 6
7 ← 3 ← 8 ← 5 ← 3 ← 4 ← 7

six-cycle fixed points four-cycle two-cycle

Seelbach Benkner et al., ICCV 2021.
Shape Matching with AQC

\[ \min_{X \in \mathbb{P}_n} E(X) := x^T W x \]
\[ x = \text{vec}(X) \quad W \in \mathbb{R}^{n^2 \times n^2} \]
\[ \mathbb{P} \subset \{0,1\}^{n \times n} \text{ (permutation matrix)} \]
\[ \mathbb{P}_n = \{X \in \{0,1\}^{n \times n} \mid \sum_i X_{ij} = 1, \sum_j X_{ij} = 1 \forall i, j\}. \]

Given two cycles \( C_1, C_2 \) we parametrize all combinations with two binary variables \( \alpha_1, \alpha_2 \):

\[
\begin{pmatrix}
1-\alpha_1 & 0 & \alpha_1 & 0 & 0 \\
\alpha_1 & 1-\alpha_1 & 0 & 0 & 0 \\
0 & \alpha_1 & 1-\alpha_1 & 0 & 0 \\
0 & 0 & 0 & 1-\alpha_2 & \alpha_2 \\
0 & 0 & 0 & \alpha_2 & 1-\alpha_2
\end{pmatrix}
\]

Possible permutations for all choices of \( \alpha_1, \alpha_2 \):
Shape Matching with AQC

Given: 3D shapes $M$ and $N$, both discretised with $n$ vertices.

$$W_{i \cdot n + k \cdot j \cdot n + l} = |d^g_M(i, j) - d^g_N(k, l)|$$

Find: optimal $P$
Shape Matching with AQC

Given: 3D shapes $M$ and $N$, both discretised with $n$ vertices.

$$W_{i\cdot n+k, j\cdot n+l} = |d^g_M(i, j) - d^g_N(k, l)|$$

Find: optimal $P$

---

Want to solve but cannot:

$$\min_{X \in \mathbb{R}^n} E(X) := x^T W x$$

$$W_{i\cdot n+k, j\cdot n+l} = |d^g_M(i, j) - d^g_N(k, l)|$$

Instead solve

$$\arg\min_{\{P \in \mathbb{R}^m \mid \exists \alpha \in \{0,1\}^m : P = (\Pi_i c^\alpha_i) P_0\}} E(P)$$

$C = \{c_1, ..., c_m\}$

---

... leading to

$$\min_{\alpha \in \{0,1\}^m} \alpha^T \tilde{W} \alpha$$

$$\tilde{W}_{ij} = \begin{cases} E(C_i, C_j) & \text{if } i \neq j, \\ E(C_i, C_i) + E(C_i, P_0) + E(P_0, C_j) & \text{otherwise.} \end{cases}$$

---

$$E(Q, R) = \text{vec}(Q)^T W \text{vec}(R)$$

$$P(\alpha) = P_0 + \sum_{i=1}^{m} \alpha_i (c_i - I) P_0$$

Seelbach Benkner et al., ICCV 2021.
Shape Matching with AQC

**Initialise** $P_0$ via descriptor-based similarity

**repeat until converged**

obtain $I_M$ and $I_N$ and choose from them a set of $k$ random and disjoint 2-cycles

construct a submatrix of worst matches $W_S$

**repeat until every 2-cycle occurred**

choose a random set of 2-cycles

calculate $\tilde{W}$ and solve $\min_{\alpha \in \{0,1\}^m} \alpha^T \tilde{W} \alpha$ on QPU

$$P_i = \left( \prod_j c_{j}^{\alpha_j} \right) P_{i-1}$$

**apply the obtained permutation to worst matches**
**Shape Matching with AQC**

**Initialise** $P_0$ via descriptor-based similarity

**repeat until converged**
- obtain $I_M$ and $I_N$ and choose from them a set of $k$ random and disjoint 2-cycles
- construct a submatrix of worst matches $\tilde{W}_s$

**repeat until every 2-cycle occurred**
- choose a random set of 2-cycles
- calculate $\tilde{W}$ and solve $\min_{\alpha \in \{0,1\}^m} \alpha^T \tilde{W} \alpha$ on QPU
- apply the obtained permutation to worst matches

$P_i = \left( \prod_j c_j^{\alpha_j} \right) P_{i-1}$

$\text{NP-hard; decides to apply } c_i \text{ or not}$
Shape Matching with AQC

Accuracy and convergence (FAUST)

Runtime

Seelbach Benkner et al., ICCV 2021.
Shape Matching with AQC

Accuracy and convergence (FAUST)

Runtime

optimal solution probabilities

minor embedding statistics

Minor embeddings (40 and 50 vertices)

Seelbach Benkner et al., ICCV 2021.
WIP: Matching Multiple Shapes with AQC

cycle consistency

evolution of the matches
WIP: Quantum Annealing with Learnt Couplings

Seelbach Benkner et al., arXiv, 2022.
WIP: Quantum Annealing with Learnt Couplings

Encode Problem Instance

Regress Couplings with MLP

Set up QUBO

Initialise Quantum Annealer

Anneal

QUBO Solution

$\mathbf{p} : 
\begin{bmatrix} 1.4 \\ 3.1 \\ \vdots \end{bmatrix} \quad \mathbf{A} : 
\begin{bmatrix} -0.3 & 2.3 \\ 1.0 & -1.2 & \cdots \end{bmatrix} \quad \text{arg min} \ x^\top \mathbf{A} x \quad x \in \{0,1\}^n$

$\mathbf{x} : 
\begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix}$

Backpropagate & Update MLP

Compute Losses

Hamming distance evolution over training epochs

Seelbach Benkner et al., arXiv, 2022.
• QCV gains momentum
• QPUs as accelerators for CV, CG and ML
• Our mission: To enhance the visibility of QCV in the communities
References


Thank You!