Algorithms and Data Structures
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Exercises 34 and 35

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morning and afternoon

Motivation

We fill in the missing details of the min-cut algorithm and develop our first geometric algorithms.

The discussion of the min-cut algorithm was based on Section 10.2 in the book by Motwani and Raghavan: Randomized Algorithms. The treatment of geometric algorithms follows the books by de Berg, van Kreveld, Overmars and Schwarzkopf: Computational Geometry, by Mehlhorn and Näher: LEDA and the article Four Results on Randomized Incremental Constructions by K. Clarkson, K. Mehlhorn and R. Seidel (CGTA, 1993, 185–212). You can download the article from `www.mpi-sb.mpg.de/~mehlhorn/ftp/CMS-FourResults.ps`. Only the first 14 pages are relevant for class.

1. In class, we analyzed a randomized algorithm for finding a minimum cut in a multi-graph. We left an efficient implementation to the exercises.

   Let $G = (V, E)$ be a multi-graph with n nodes and m edges; m might be much larger than n^2. We have a procedure *random* to our avail that on input N, an integer, produces a random integer in $[1 \ldots N]$

   In each iteration of the min-cut algorithm, one chooses an edge uniformly at random and contracts it.

   Design a representation for multi-graphs so that an iteration can be carried out in time $O(n)$.

2. Consider the following variant of the min-cut algorithm.

   (a) we reduce the number of nodes from n to $\lceil 1 + \sqrt{n}/2 \rceil$ nodes by random contractions. Let H be the resulting graph.

   (b) We make two copies H_1 and H_2 of H and reply the algorithm recursively to H_1 and H_2.

   In class, we obtained H_1 and H_2 by independent sequences of contractions. Now we obtain them by the same sequence.

   (a) Derive a recurrence for the success probability.

(b) Does the analysis of the success probability given in class stay valid?

3. Let p, q, and r be points in the plane. Proof that the determinant of the matrix below is twice the signed area of the triangle formed by the three points.

$$\begin{pmatrix} 1 & 1 & 1 \\ p_x & q_x & r_x \\ p_y & q_y & r_y \end{pmatrix}$$

4. The diameter of a point set is the width of a minimum width slab containing the point set. Design an algorithm for computing the diameter of a finite point set. What is the running time of your algorithm.

   A slab is the region between two parallel lines. The width of a slab is the distance of the lines.

   Hint: compute the convex hull first.

5. Consider the following point set. It consists of the points $(0,-1)$ and $(0,+1)$ and the points $(i,0)$, $1 \leq i \leq n$. What is the running time of the incremental algorithms, if the points are inserted in the following order?

   (a) First the points $(0,-1)$ and $(0,+1)$ and then the points $(i,0)$, $1 \leq i \leq n$, in this order.

   (b) First the points $(0,-1)$ and $(0,+1)$ and then the points $(i,0)$, $1 \leq i \leq n$, in reversed order.

   (c) First the points $(0,-1)$ and $(0,+1)$ and then the points $(i,0)$, $1 \leq i \leq n$, in random order.

Have fun with the solutions.