Selected Topics in Algorithms
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Exercise 3

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We will discuss this exercise sheet on May 22^{th} **and May** 25^{th}

Motivation

We continue our study of cycle bases.

NP-Completeness

Three-dimensional Matching Let X, Y, Z be disjoint sets of equal cardinality and let $T \subset X \times Y \times Z$, i.e., each element of T is a triple whose first component is in X, second component is in Y, and third component belongs to Z. Is there a subset $M \subseteq T$ such that no two elements of M agree in any coordinate and such that $|M| = |X|$.

Establish NP-completeness.

Hint: Reduce 3-SAT to three dimensional matching.

Exact Set Cover Let t be an integer and let $S = \{S_1, \ldots, S_k\}$ be a family of 3-element subsets of $T = [1, \ldots, t]$. Is there a subfamily $S' \subseteq S$ of pairwise disjoint sets such that any element of T belongs to exactly one member of the subfamily.

Establish NP-completeness.

Addition and Multiplication in 2-Complement

Let k be an integer. For digits $d_\ell \in \{0, 1\}$, $0 \le \ell \le k$, let

$$D = \sum_{0 \le \ell < k} d_\ell 2^\ell - d_k 2^k \ .$$

- Which numbers D can be represented in this way.
- Show that the following algorithm can be used to add two such numbers.
  - Add the numbers as usual binary numbers, i.e., ignore the fact that d_k contributes $-d_k 2^k$ to D.
  - If there is a carry into position $k+1$, then declare overflow, i.e., the result cannot be represented. Otherwise, return the result.
- Show that the following algorithm can be used to multiply two such numbers.

- Multiply the numbers as usual binary numbers, i.e, ignore the special interpretation of position k.
- Return the result, if ??? (this is for you to fill in).

Remark: You probably learned the solution to this exercise in your introductory course on machine organization and computer architecture.

Have fun with the solution!