Exercises for Non-linear Computational Geometry

Assignment 1

Due 31.10.2008

Exercise 1 (ε Points)
Write an email to mkerber[at]mpi-sb.mpg.de with subject “Non-linear Computational Geometry” containing your name and Matrikelnummer.

Exercise 2 (5 Points)
a) Consider the circles

\[ C_1 = V((x - 1)^2 + y^2 - 2) \]
\[ C_2 = V(x^2 + (y + 2)^2 - 3) \]
\[ C_3 = V(x^2 + y^2 - 4.1599) \]

How many different pairwise intersection points exist? Is \( C_1 \cap C_2 \cap C_3 = \emptyset \)? (proof!)

b) For which values of \( \lambda \) is \( C_1 \cap C_2 \cap C^{(\lambda)} \neq \emptyset \), where \( C^{(\lambda)} = V(x^2 + y^2 - \lambda) \)?

Exercise 3 (5 Points)
For this exercise, we recommend to use the web demo at http://exacus.mpi-sb.mpg.de.
Consider the parameterized curve

\[ C^{(\lambda_1, \ldots, \lambda_6)} := V(y^4 - y^3 + 2x^2y^2 + 3x^2y + x^4 + \lambda_1x^2y + \lambda_2xy + \lambda_3x + \lambda_4) \]

a) Start with \( \lambda_1 = \ldots = \lambda_4 = 0 \). Where does \( C^{(0, \ldots, 0)} \) have a singular point (self-intersection)?

b) Set \( \lambda_1 = 0.001 \), is there a significant change?

c) Set \( \lambda_2 = 0.001 \) as well. What is the number of singular points? Is there a significant change at all?

d) What happens if we set \( \lambda_3 = 0.001 \) as well?

e) Set \( \lambda_4 = 0.001 \). Is there a self intersection?

PLEASE TURN OVER
Exercise 4 (5 Points)
We consider the orientation predicate for three planar points:

$$\text{orient}(p, q, r) = \text{sgn} \left( (q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x) \right)$$

a) Consider the points

$$p^{(x,y)} := \left( \frac{0.5 + 2^{-64}x}{0.5 + 2^{-64}y} \right) \quad q := \left( \frac{12}{12} \right) \quad r := \left( \frac{24}{24} \right)$$

Write a C++ program using long double arithmetic that produces a 256 x 256 pixel image. For $0 \leq i, j < 256$, the pixel $(i, j)$ is drawn red if $\text{orient}(p^{(i,j)}, q, r) = 1$, it is drawn yellow if $\text{orient}(p^{(i,j)}, q, r) = 0$, and it is drawn blue if $\text{orient}(p^{(i,j)}, q, r) < 0$.

We suggest that the picture is stored in the very simple ppm-format (Portable Pixmap). The format is supported by, e.g., gimp (GNU Image Manipulation Program). A short documentation of the format can be found at: [http://de.wikipedia.org/wiki/Portable_Pixmap](http://de.wikipedia.org/wiki/Portable_Pixmap).

Send the source code and the resulting picture to mkerber[at]mpi-sb.mpg.de.

b) Consider three points $p, q, r$ in the plane. The lines $pq$ and $pr$ subdivide the plane into 4 regions $F_1, \ldots, F_4$.

Define a predicate to test whether a point $s := (s_x, s_y)$ lies inside $F_1$. 

\[\begin{array}{c}
F_1 \\
F_2 \\
F_3 \\
F_4 \\
p \\
q \\
r
\end{array}\]