Dynamically Reparameterized Light Fields & Fourier Slice Photography

Oliver Barth, 2009
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Background

What we are talking about?
Background

What we are talking about?

- We want to reconstruct new pictures potentially from arbitrary viewpoints
What we are talking about?

- We want to reconstruct new pictures potentially from arbitrary viewpoints

- We want to adjust the depth-of-field (the things to be in focus) after a real scene was taken
Example
Part I

- Dynamical Reparameterization of Light Fields
  - Focal Surface Parameterization
  - Variable Aperture
  - Variable Focus
  - Analysis
  - Further Application
Content
Part II

• Prerequisites
  • *Simple Fourier Slice Theorem in 2D Space*

• Photographic Imaging in Fourier Space
  • Generalization of Fourier Slice Theorem
  • Fourier Slice Photography
Light Field
Conventional Camera

Ray carrying $L_F(s, t, u, v)$
What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database
What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database
- Aliasing effects in high frequency regions
Light Field
Conventional Ray Reconstruction

What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database
- Aliasing effects in high frequency regions
- Only suitable for constant depth scenes
Light Field
Conventional Ray Reconstruction

What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database
- Aliasing effects in high frequency regions
- Only suitable for constant depth scenes
- Lumigraph uses depth correction
Light Field
Conventional Ray Reconstruction

\[ \begin{align*}
uv_1 &= \square + \square \\
uv_2 &= \square + \square \\
uv_3 &= \square + \square
\end{align*} \]
Avoiding aliasing effects by low pass filtering the ray database
Avoiding aliasing effects by low pass filtering the ray database.

Aperture filtering has to be done before reconstruction process.
Light Field
Conventional Ray Reconstruction

- Avoiding aliasing effects by low pass filtering the ray database
- Aperture filtering has to be done before reconstruction process
- Therefore static and fixed xy uv planes
Avoiding aliasing effects by low pass filtering the ray database

Aperture filtering has to be done before reconstruction process

Therefore static and fixed xy uv planes

Aperture filtering results in a blurred reconstruction image
Avoiding aliasing effects by low pass filtering the ray database

Aperture filtering has to be done before reconstruction process

Therefore static and fixed xy uv planes

Aperture filtering results in a blurred reconstruction image

Unpractical high sampling rate would be needed
Dynamical Reparameterization of Light Fields

Idea

- Scene
- Front aperture
- Back aperture
- Lens
- Sensor plane
- Point (s, t)
Dynamical Reparameterization of Light Fields

Idea
Dynamical Reparameterization of Light Fields

Idea

\[ E_F(s, t) = \frac{1}{F^2} \int \int L_F(s, t, u, v) \cos^4 \phi \, du \, dv \]
Dynamical Reparameterization of Light Fields
Focal Surface Parameterization

\[ r = (s, t, u, v) = (s, t, f, g)_F \]

- \( D_{s,t} \)
- \((u,v)\)
- \((f,g)_F\)
- data cameras
- camera surface \( C \)
- focal surface \( F \)
Dynamical Reparameterization of Light Fields
Focal Surface Parameterization
Dynamical Reparameterization of Light Fields
Focal Surface Parameterization
Dynamical Reparameterization of Light Fields
Ray Reconstruction

\begin{equation}
(s', t', u', v') = (s', t', f, g)_F
\end{equation}

\begin{equation}
(s'', t'', u'', v'') = (s'', t'', f, g)_F
\end{equation}

\begin{equation}
(u', v') = (s', t', f, g)_F
\end{equation}

\begin{equation}
(u'', v'') = (s'', t'', f, g)_F
\end{equation}
Dynamical Reparameterization of Light Fields
Variable Aperture
Dynamical Reparameterization of Light Fields Using a Weighting Function
Dynamical Reparameterization of Light Fields
Big Aperture Example
Dynamical Reparameterization of Light Fields
Big Aperture Example
Dynamical Reparameterization of Light Fields
Big Aperture Example
Dynamical Reparameterization of Light Fields

Big Aperture Example
Dynamical Reparameterization of Light Fields
Variable Focus
Dynamical Reparameterization of Light Fields
Variable Focus Example
What about arbitrary selected points to be in focus?
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- Real camera has only one continuous plane in focus
What about arbitrary selected points to be in focus?

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- Multiple focal planes can highlight several regions of different depth
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- No constraints of physical optics
- Multiple focal planes can highlight several regions of different depth
- Multiple apertures can reduce vignette effects near edges
Dynamical Reparameterization of Light Fields
Multiple Regions in Focus
Dynamical Reparameterization of Light Fields
Multiple Apertures and Vignette Effects
Dynamical Reparameterization of Light Fields
Ray Space Analysis
Dynamical Reparameterization of Light Fields
Ray Space Analysis

(e) (f) (g)
Dynamical Reparameterization of Light Fields
Frequency Domain Analysis
Dynamical Reparameterization of Light Fields
Frequency Domain Analysis

(e)

(f)
Dynamical Reparameterization of Light Fields
Frequency Domain Analysis

(a)  

(b)  

(c)  

(d)
Dynamical Reparameterization of Light Fields
Frequency Domain Analysis
Dynamical Reparameterization of Light Fields
Special Lens For Capturing Light Fields
Dynamical Reparameterization of Light Fields
Special Lens For Capturing Light Fields
Dynamical Reparameterization of Light Fields

Autostereoscopic Light Fields

image
principal point

lenslet
Dynamical Reparameterization of Light Fields
Autostereoscopic Light Fields
Dynamical Reparameterization of Light Fields
Autostereoscopic Light Fields
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized

- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays
- Algorithm is in $O(n^4)$
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays
- Algorithm is in $O\left(n^4\right)$
- Many different application approaches (refocusing, view through objects, 3D displays)
Variable apertures could be synthesized

For every pixel in \((s, t)\) direction one has to integrate over the neighborhood \((u, v)\) rays

Algorithm is in \(O(n^4)\)

Many different application approaches (refocusing, view through objects, 3D displays)

A photograph is a integral over a shear of the ray space
Photographic Imaging in Fourier Space

Part II

- Goal: Speed Up by Working in Frequency Domain
- Prerequisites
- Generalization of Fourier Slice Theorem
- Fourier Slice Photography
Prerequisites
Projection
Prerequisites
Reconstructions
Prerequisites
Radon Transform
Prerequisites
Radon Transform
Prerequisites
Radon Transform
Prerequisites
Simple Fourier Slice Theorem in 2D Space
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
- Slicing
Photographic Imaging in Fourier Space

Operator Definition

- Integral Projection
- Slicing
- Change of Basis
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
- Slicing
- Change of Basis
- Fourier Transform
Photographic Imaging in Fourier Space
Fourier Slice Theorem in 2D
Photographic Imaging in Fourier Space

Main Idea

- simple theorem exists: shearing a space is equivalent to rotating and dilating the space
Photographic Imaging in Fourier Space

Idea

Main Idea

• simple theorem exists: shearing a space is equivalent to rotating and dilating the space

• slicing and dilating the 4D Fourier transform of a light field and back transform
Photographic Imaging in Fourier Space

Idea

Main Idea

- simple theorem exists: shearing a space is equivalent to rotating and dilating the space
- slicing and dilating the 4D Fourier transform of a light field and back transform
- should be equivalent to an integral over a sheared light field
Main Idea

- simple theorem exists: shearing a space is equivalent to rotating and dilating the space

- slicing and dilating the 4D Fourier transform of a light field and back transform

- should be equivalent to an integral over a sheared light field

- what we know is a simple photograph of the light field
Photographic Imaging in Fourier Space
Generalization of Fourier Slice Theorem

\[ \mathcal{F}^M \circ \mathcal{I}^N_M \circ \mathcal{B} \equiv S^N_M \circ \frac{\mathcal{B}^{-T}}{|\mathcal{B}^{-T}|} \circ \mathcal{F}^N \]
Photographic Imaging in Fourier Space
Generalization of Fourier Slice Theorem

integral

projection

$I_M^N \circ B$

$[O(n^N)]$

$\mathcal{F}^N$

$[O(n^N \log n)]$

$\mathcal{S}_M^N \circ \frac{\mathcal{B}^{-T}}{|\mathcal{B}^{-T}|}$

$[O(n^M)]$

$\mathcal{F}^M$

$[O(n^M \log n)]$

$G_N$

$G_M$

$G_N$

$G_M$

$S_M^N$

slicing

$\mathcal{S}_M^N$

$\mathcal{G}_N$

$\mathcal{G}_M$

$\mathcal{G}_N$

$\mathcal{G}_M$
Photographic Imaging in Fourier Space
Fourier Slice Photography

\[ L_F \xrightarrow{4D \text{ Fourier Transform}} \mathcal{F}^4 [O(n^4 \log n)] \xrightarrow{2D \text{ Fourier Transform}} E_{\alpha \cdot F} \]

\[ \mathcal{P}_\alpha [O(n^4)] \xrightarrow{\mathcal{F}^2} \mathcal{F}_\alpha [O(n^2)] \xrightarrow{\mathcal{F}} \mathcal{P}_\alpha [O(n^4)] \]

\[ \mathcal{E}_{\alpha \cdot F} \]

Photograph Synthesis

Fourier-space Photograph Synthesis
Photographic Imaging in Fourier Space
Filtering the Light Field

\[
\begin{align*}
\mathcal{P}_\alpha & \quad \mathcal{C}_k^4 \quad 4D \text{ Convolution} \quad \overline{L}_F \\
L_F & \quad 4D \text{ kernel} \quad \mathcal{P}_\alpha \\
E_{\alpha \cdot F} & \quad \mathcal{C}_{\mathcal{P}_\alpha[k]}^2 \quad 2D \text{ Convolution} \quad \overline{E}_{\alpha \cdot F}
\end{align*}
\]
Photographic Imaging in Fourier Space

Result

- Algorithm is in $O(n^2)$
Photographic Imaging in Fourier Space

Result

- Algorithm is in $O(n^2)$
- Only one focal plane can be sliced
Photographic Imaging in Fourier Space
Result

- Algorithm is in $O\left(n^2\right)$
- Only one focal plane can be sliced
- The plane is always perpendicular to the camera plane
Photographic Imaging in Fourier Space
Result

Fourier Slice

Conventional
Discussion

• Questions?
Discussion

• **Question:**
  What about non planar slices in Fourier Space?
Photographic Imaging in Fourier Space
Fourier Slice Photography

$$\mathcal{P}_\alpha [L_F] \equiv \frac{1}{\alpha^2 F^2} \mathcal{I}_2^4 \circ \mathcal{B}_\alpha [L_F]$$

$$\mathcal{B}_\alpha = \begin{bmatrix} \alpha & 0 & 1 - \alpha & 0 \\ 0 & \alpha & 0 & 1 - \alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathcal{B}_\alpha^{-1} = \begin{bmatrix} \frac{1}{\alpha} & 0 & 1 - \frac{1}{\alpha} & 0 \\ 0 & \frac{1}{\alpha} & 0 & 1 - \frac{1}{\alpha} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{\alpha^2 F^2} \int \int L_F \left( u(1-1/\alpha) + x/\alpha, \ v(1-1/\alpha) + y/\alpha, \ u, \ v \right) \, du \, dv$$
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**What we are talking about?**

- We want to reconstruct new pictures potentially from arbitrary viewpoints
- We want to adjust the depth-of-field (the things to be in focus) after a real scene was taken

- for synthetic scenes that means 3D scenes with meshes and textures and all that virtual stuff this is quite simple, all information is available

- with the standard lightfield or lumigrah parametrization this is not possible or only under some special restrictions

- adjustment of depth-of-field as post-processing
- left image: sharp regions in foreground

- right image: same scene, sharp regions in background

- focus varying in the same scene

- goal is to adjust this as a post-process

- one application could be a specialized tool for image designers
• Dynamical Reparameterization of Light Fields
  • Focal Surface Parameterization
  • Variable Aperture
  • Variable Focus
  • Analysis
  • Further Application
Content
Part II

• Prerequisites
  • *Simple Fourier Slice Theorem in 2D Space*

• Photographic Imaging in Fourier Space
  • Generalization of Fourier Slice Theorem
  • Fourier Slice Photography
- already known and very popular st uv parametrization, known from the very first talk

- highly sampled uv

- low sampled st

- sensor chip is discretized

- the lens is continuous (respectively some distortions)
What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database

- Ray database is a 4 dimensional function \((s, t, u, v)\) that returns a color value of the radiance along that ray

- Commonly the conventional reconstruction gives only one ray

- And the st uv planes are fixed
What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database
- Aliasing effects in high frequency regions

- in the previous talks we have seen how high frequency regions behave under the reconstruction process
- aliasing effects occur
- high frequency means very sharp edges, very rapidly change of color in a relatively small region, big gradient in the color map
- we also have seen how to avoid this by aperture pre-filtering, low-pass filtering the scene
- this results in blurring the scene
Light Field
Conventional Ray Reconstruction

**What is the problem with a conventional reconstruction?**

- Reconstruction by querying a ray database
- Aliasing effects in high frequency regions
- Only suitable for constant depth scenes

- not that deep scenes

- light field has many aliasing effects on reconstruction process if too much depth in scene
Light Field
Conventional Ray Reconstruction

What is the problem with a conventional reconstruction?

- Reconstruction by querying a ray database
- Aliasing effects in high frequency regions
- Only suitable for constant depth scenes
- Lumigraph uses depth correction

- depth map is needed, hardly to obtain
- so depth correction is possible
- but everything is in focus then
- process dependent on unwanted information of the scene
- left side: entry plane, right side: exit plane

- the standard light field parametrization uses a fixed uv exit plane

- 3 scenarios

- best reconstruction with uv_2, the plane approximates the scene geometry

- highly sampled uv plane, low sampled st plane

- moving ray r switches between colors => aperture filtering
• Avoiding aliasing effects by low pass filtering the ray database
• Avoiding aliasing effects by low pass filtering the ray database
• Aperture filtering has to be done before reconstruction process
Avoiding aliasing effects by low pass filtering the ray database

Aperture filtering has to be done before reconstruction process

Therefore static and fixed xy uv planes
• Avoiding aliasing effects by low pass filtering the ray database
• Aperture filtering has to be done before reconstruction process
• Therefore static and fixed xy uv planes
• Aperture filtering results in a blurred reconstruction image
- to avoid these artifacts

- Avoiding aliasing effects by low pass filtering the ray database
- Aperture filtering has to be done before reconstruction process
- Therefore static and fixed xy uv planes
- Aperture filtering results in a blurred reconstruction image
- Unpractical high sampling rate would be needed
- how does a conventional camera lens system work

- a point (s,t) is an integral, a sum up of the light rays entering at that point

- a lens will provide a lot of rays to sum up

- if the point P was in focus (s,t) will only sum up rays coming from P
- if point P is not in focus (s,t) will sum up rays from the neighborhood, resulting in a blurring of point P

- this is what a camera will do

- very intuitive
Dynamical Reparameterization of Light Fields

Idea

\[ E_F(s, t) = \frac{1}{F^2} \int \int \overline{L}_F(s, t, u, v) \cos^4 \phi \, du \, dv \]
the new parametrization like a camera array

-D_st is a single camera, (u,v) is a pixel on the image of D_st

-(s,t,u,v) will intersect the focal surface at certain point (f,g).

-focal surface is not static, could be moved, a certain ray intersects at different positions if one moves the fs toward or away from the cs

-st poor, low resolution
-uv high density – high sampling rate
- example for such a camera setup

- for each camera intrinsic an extrinsic parameters have to be estimated
Dynamical Reparameterization of Light Fields
Focal Surface Parameterization

- notice: not aligned accurately
- how to reconstruct a ray $r$ with such a setup

- estimate intersecting point with $F$, then look for the rays in the neighborhood

- notice the rotation of each camera

- different thing $(f,g)$ vs $(u,v)$ (dynamic plane)

- one could take some more cameras into account
- a reconstruction of $r'$ considers certain rays of the the $D_{st}$ cameras in the neighborhood

- the number of cameras give a synthetic aperture

- for each point (single reconstruction) it's possible to adjust an arbitrary aperture size

- $r''$ is intersecting a region in scene not approximated by the focal plane, ray integral will sum up to a blurring effect

- behaves like a lens

- very natural and intuitive setup
- r is the ray we want to reconstruct

- it's possible to use a weighting function

- this could be used for each ray separately

- in w_1 six rays are considered

- in w_3 only 2 rays are considered

- it is important that the values sum up to 1
  Otherwise brightness will not be correct
Dynamical Reparameterization of Light Fields
Big Aperture Example

- with big apertures it is possible to view through objects
- view from above

- rays are surrounding the tree
- with a big aperture it is possible to view through bushes and shrubberies
- big apertures can produce vignette effects on the boundaries of the image

- this is because the weighting will not sum up to 1 anymore
- different planes are possible, different shapes, especially non planar ones
- by moving the plane towards and away from the camera plane one can adjust the things to be in focus
Dynamical Reparameterization of Light Fields
Multiple Apertures and Focal Surfaces

What about arbitrary selected points to be in focus?
Dynamical Reparameterization of Light Fields
Multiple Apertures and Focal Surfaces

What about arbitrary selected points to be in focus?

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What about arbitrary selected points to be in focus?

- Real camera has only one continuous plane in focus
- Simulation with a set of pictures and post-processing
- No constraints of physical optics
- Multiple focal planes can highlight several regions of different depth

- with a focal plane approximating the geometry of the scene everything will be in focus

- this could also be done by moving the focal plane away from the camera surface estimate what is in focus and what is not (sigma function)
What about arbitrary selected points to be in focus?

- Real camera has only one continuous plane in focus
- Simulation with a set of pictures and post-processing
- No constraints of physical optics
- Multiple focal planes can highlight several regions of different depth
- Multiple apertures can reduce vignette effects near edges

- by reducing the aperture at the boundaries the weighting sums up to 1
Dynamical Reparameterization of Light Fields
Multiple Regions in Focus
Dynamical Reparameterization of Light Fields
Multiple Apertures and Vignette Effects

- circle is the area of considered rays => aperture
- sf slice, top view

- 4 feature points

- think of a line intersection the feature point and moving along the s axis

- shear along the dotted line

- if surface remains perpendicular to cs a position change results in linear shear of ray space

- non linear for non orthogonal

- this is called a epi polar image
- epi with 3 different apertures

- the red feature is in focus

- the same apertures with a different c) shear

- orange and green feature is in focus
Dynamical Reparameterization of Light Fields
Frequency Domain Analysis

- epi of one feature
- ideal fourier transform of a continuus light field
- repetitions from sampling rate
- not intersection because of proper sampling rate
- artefacts from improper reconstruction filter
- blue box is an aperture prefilter
Dynamical Reparameterization of Light Fields
Frequency Domain Analysis

- result of the improper reconstruction

- with dynamical reparametrization one could get reconstruction filters
- two features

- continuous signal and the sampled version

- bigger apertures will result in smaller reconstruction filters
- first with a small aperture
- second with a big aperture
- artefacts will get unperceptable
- an other method for capturing light fields
- not camera array but lens array
- could be used with conventional cameras
- 16megapixel cameras get acceptable results
- each circle is a D_st and has contains all information about the entering light from all directions covering the view angle for this single lens

- one circle will be used to reconstruct one pixel of an arbitrary view point image, respectively averaging over more pixels for aperture synthesis

-
- gives possibility to construct real 3d displays with different perspectives for each viewer

- each lens-let in the lens array acts as a view dependent pixel
- a light field can be re-parametrized into a integral photograph

- integration is done by the retina in the eye
- an auto-stereoscopic image that can be viewed with a hexagonal lens array
Dynamical Reparameterization of Light Fields

**Result**

- Variable apertures could be synthesized
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays

- better to say: every new pixel (s',t') and the integration not over the own neighborhood but over the neighborhood of different cams
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays
- Algorithm is in $O\left(n^4\right)$
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays
- Algorithm is in $O(n^4)$
- Many different application approaches (refocusing, view through objects, 3D displays)
Dynamical Reparameterization of Light Fields

Result

- Variable apertures could be synthesized
- For every pixel in (s, t) direction one has to integrate over the neighborhood (u, v) rays
- Algorithm is in $O\left(n^4\right)$
- Many different application approaches (refocusing, view through objects, 3D displays)
- A photograph is a integral over a shear of the ray space
Photographic Imaging in Fourier Space

Part II

- Goal: Speed Up by Working in Frequency Domain
- Prerequisites
- Generalization of Fourier Slice Theorem
- Fourier Slice Photography
- a projection is a sum up of all values

- a discrete version sums up the values with a comb (dirac function)

- the distance between the teeth of the comb is our sampling rate

- the steps of theta is also a sampling rate
- first was the original image

- these are reconstructions

- reconstruction with 1, 2, 3, 4 projections and 45 degree

- reconstruction with over 40 projections and around 6 degree
- the radon transform does the same thing

- every slice of the right is a sum up of all values in one direction

- used for ct scanners
Prerequisites
Radon Transform

θ
0° 180°
Prerequisites
Radon Transform
- $P(\theta, t)$ is the sum up of all values in direction theta

- Fourier slice state that a slice indirection theta of the whole 2d transform is a 1d transform of the sum up in direction theta in the original space

- We could reconstruct the sum up by slicing the 2d fourier spectrum and backtransform

- And we remember and keep in mind that a sum up of a 4d space of a light field is a fotograf

- Somehow a fourier transform is a rotational respresentation of the original space
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
- Slicing
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
- Slicing
- Change of Basis
Photographic Imaging in Fourier Space
Operator Definition

- Integral Projection
- Slicing
- Change of Basis
- Fourier Transform
Photographic Imaging in Fourier Space
Fourier Slice Theorem in 2D

\[ G(x, y) \xrightarrow{\mathcal{F}^2} \mathcal{G}(u, v) \]
\[ \mathcal{I}_1^2 \circ \mathcal{R}_\theta [O(n^2)] \]
\[ S_1^2 \circ \mathcal{R}_\theta [O(n)] \]
\[ \mathcal{F}^1 [O(n \log n)] \]
\[ g_\theta(x') \rightarrow \mathcal{G}_\theta(u') \]
- a shear operation could be expressed as rotation rotation the space and dilating it

- dilation means expanding the size in one dimension, along one axis

- so a shear is composition of rotations and resize operations along the dimensional axes
Photographic Imaging in Fourier Space

Idea

Main Idea

- simple theorem exists: shearing a space is equivalent to rotating and dilating the space
- slicing and dilating the 4D Fourier transform of a light field and back transform
Photographic Imaging in Fourier Space

Idea

Main Idea

- simple theorem exists: shearing a space is equivalent to rotating and dilating the space

- slicing and dilating the 4D Fourier transform of a light field and back transform

- should be equivalent to an integral over a sheared light field
Photographic Imaging in Fourier Space

Idea

Main Idea

- simple theorem exists: shearing a space is equivalent to rotating and dilating the space
- slicing and dilating the 4D Fourier transform of a light field and back transform
- should be equivalent to an integral over a sheared light field
- what we know is a simple photograph of the light field
Photographic Imaging in Fourier Space
Generalization of Fourier Slice Theorem

\[ \mathcal{F}^M \circ \mathcal{I}^N_M \circ B = S^N_M \circ \frac{B^{-T}}{|B^{-T}|} \circ \mathcal{F}^N \]

- (tafel)
Photographic Imaging in Fourier Space
Generalization of Fourier Slice Theorem

$G_N \xrightarrow{\mathcal{F}^N} [O(n^N \log n)] \rightarrow \mathcal{S}_N$

Integral Projection

$I_M \circ B [O(n^M)]$

$G_M \xrightarrow{\mathcal{F}^M} [O(n^M \log n)] \rightarrow \mathcal{S}_M$

Slicing

$S_M \circ \frac{B^{-T}}{|B^{-T}|} [O(n^M)]$
Photographic Imaging in Fourier Space
Fourier Slice Photography

\[ L_F \xrightarrow{\mathcal{F}^4} 4D \text{ Fourier Transform} \]
\[ \mathcal{F}^4 \left[ O(n^4 \log n) \right] \]

Photograph Synthesis

\[ \mathcal{P}_\alpha \left[ O(n^4) \right] \]

\[ \mathcal{F}^2 \left[ O(n^2 \log n) \right] \]

\[ E_{\alpha,F} \xrightarrow{2D \text{ Fourier Transform}} \]

Fourier-space Photograph Synthesis

\[ \mathcal{E}_{\alpha,F} \]

\[ \mathcal{P}_\alpha \left[ O(n^2) \right] \]
Photographic Imaging in Fourier Space
Filtering the Light Field

\[ L_F \xrightarrow{\mathcal{C}_k^4} \xrightarrow{4D \text{ Convolution}} \bar{L}_F \]

\[ P_\alpha \]

\[ E_{\alpha \cdot F} \]

\[ P_\alpha \]

\[ E_{\alpha \cdot F} \]

\[ C_{P_\alpha[k]}^2 \xrightarrow{2D \text{ Convolution}} \]
Photographic Imaging in Fourier Space

Result

- Algorithm is in $O(n^2)$
Photographic Imaging in Fourier Space

Result

- Algorithm is in $O\left(n^2\right)$
- Only one focal plane can be sliced
Photographic Imaging in Fourier Space

Result

- Algorithm is in $O\left(n^2\right)$
- Only one focal plane can be sliced
- The plane is always perpendicular to the camera plane
Photographic Imaging in Fourier Space

Result

Fourier Slice

Conventional
Discussion

• Questions?
Discussion

• Question: What about non planar slices in Fourier Space?
Photographic Imaging in Fourier Space
Fourier Slice Photography

- (tafel)
Photographic Imaging in Fourier Space

Fourier Slice Photography

\[
P_\alpha [L_F] \equiv \frac{1}{\alpha^2 F^2} T_2^1 \circ B_\alpha [L_F]
\]

\[
B_\alpha = \begin{bmatrix}
\alpha & 0 & 1 - \alpha & 0 \\
0 & \alpha & 0 & 1 - \alpha \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
B_\alpha^{-1} = \begin{bmatrix}
\frac{1}{\alpha} & 0 & 1 - \frac{1}{\alpha} & 0 \\
0 & \frac{1}{\alpha} & 0 & 1 - \frac{1}{\alpha} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\frac{1}{\alpha^2 F^2} \iint L_F (u(1-1/\alpha)+x/\alpha) \cdot v(1-1/\alpha)+y/\alpha, \ u, \ v) \ du \ dv
\]

- (tafel)