Dr. Ulrich Meyer **WS 2003/4**

Exercises for Parallel and Distributed Algorithms

7. Assignment Due 15. Dec. 2003

Exercise 1 (2 *Points*)
Let $G = (V, E)$ be a graph with constant max. node degree d. A node set $I \subseteq V$ is called *independent* if $\{u, v\} \notin E$ for all $u, v \in I$. Show that there is a constant c_d which only depends on d such that on a CRCW PRAM one can choose an independent set $I \subseteq V$ that has size $|I| \geq c_d|V|$ with high probability in constant time. You may assume that the graph is given in a suited representation.

Exercise 2 (2 *Points*)
Show how to obtain an adjacency list representation (see JaJa P.112 Fig 3.10. if you do not know what an adjacency list resp. looks like) of an undirected graph that is given by an unordered sequence of edges. Your algorithm should run in $O(\log n)$ time using $O(n \log n)$ work.

Exercise 3 ($1 + 1$ *Points*)
A directed cycle of length n (given by a successor function $S(\cdot)$) can be colored with n PUs (and n colors) by initially assigning the unique PU-number to each respective node.

(a) Show how to reduce the number of colors to $O(\log n)$ in constant time.

(b) How many colors remain if you apply (a) several times?

Hint: The idea was already sketched in both the lecture and the discussion group. A look at the JaJa book may also be helpful.