MAX PLANCK INSTITUTE
FOR INFORMATICS

SIC Saarland Informatics Campus

Optimal Algorithms for Bounded Weighted Edit Distance

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Key Messages

- Bounded Weighted Edit Distance is harder than Bounded Unweighted Edit Distance
- (Weighted) Edit Distance is Shortest Paths on Planar Graphs
- APSP-based Lower Bounds via Grid Construction and Dynamic Intermediate Problem

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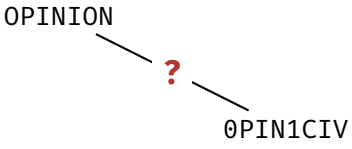

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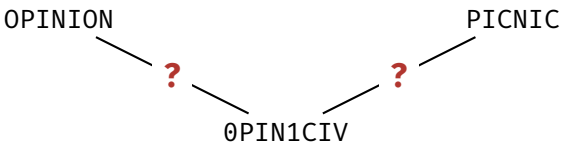

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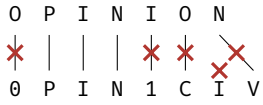

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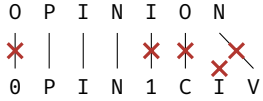

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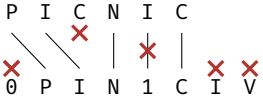

$\mathrm{ed}(\texttt{0PIN1CIV}, \texttt{PICNIC}) = 5$

Weighted Edit Distance

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Min cost of transforming X to Y using character edits, where:

- inserting y costs $w(\varepsilon, y)$;
- deleting x costs $w(x, \varepsilon)$;
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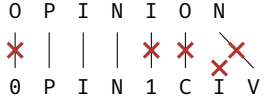
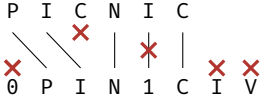
$w(0, \mathrm{O}) := 1 \quad w(1, \mathrm{I}) := 1 \quad w(\mathrm{C}, 0) := 1 \qquad w(*, *) := 2 \quad w(*, \varepsilon) := 1 \quad w(\varepsilon, *) := 10$





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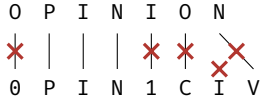

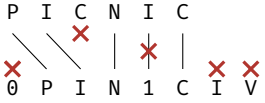

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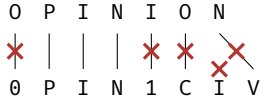

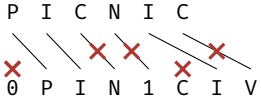

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$\rightsquigarrow O(n^2)$ (you know this!)

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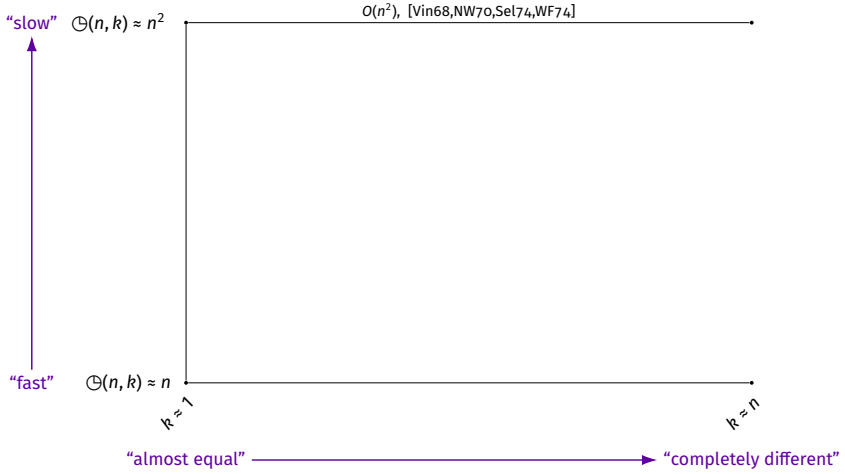
$\rightsquigarrow$ Bounded (Weighted) Edit Distance

Algorithms for (Bounded) Edit Distance



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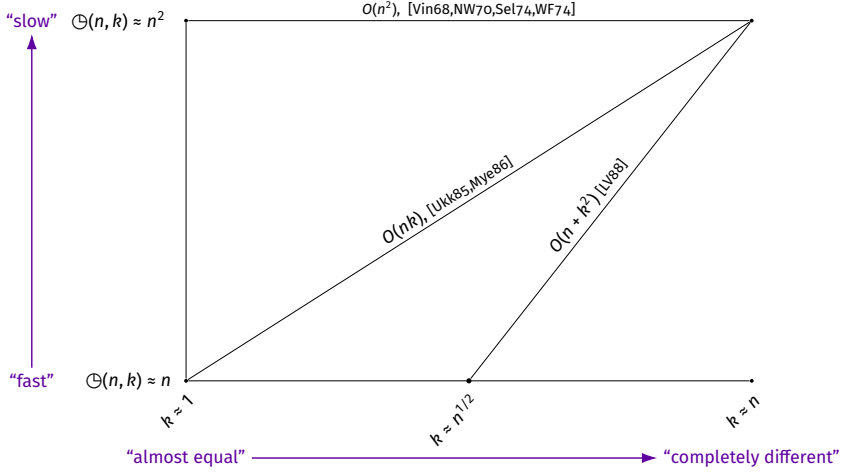

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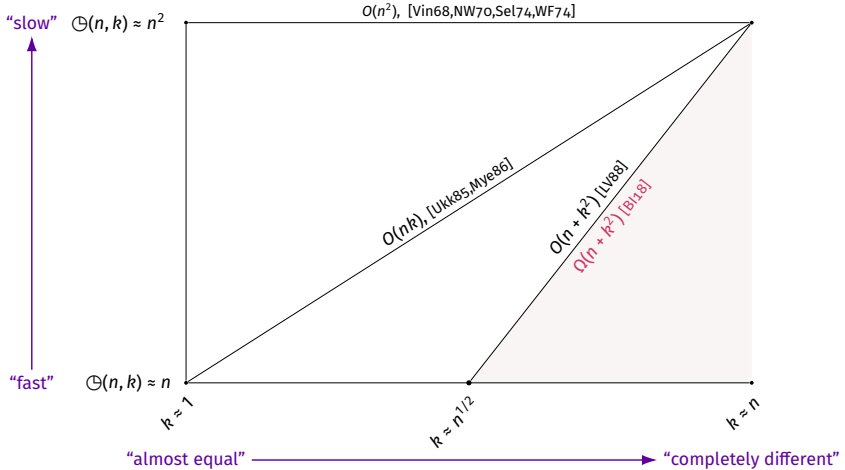

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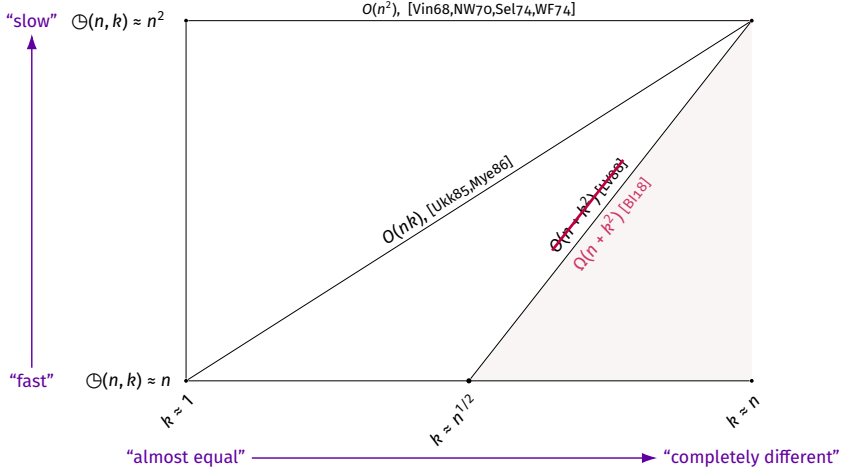
What about Weighted Edit Distance?

Algorithms for (Bounded) Weighted Edit Distance



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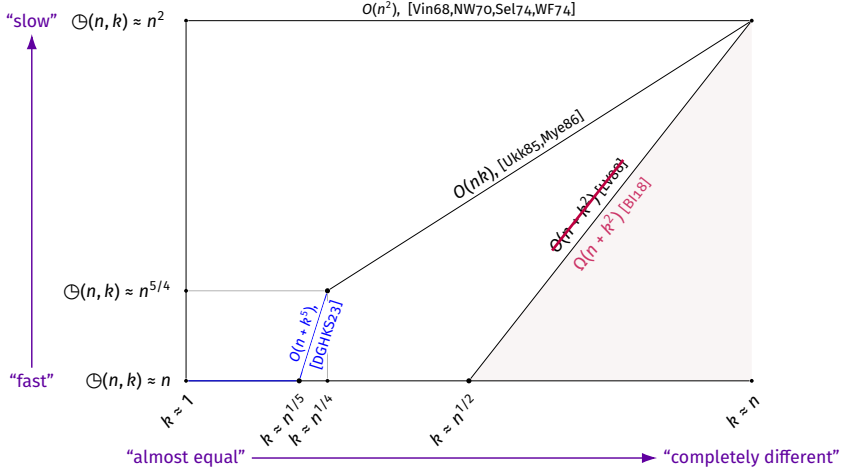

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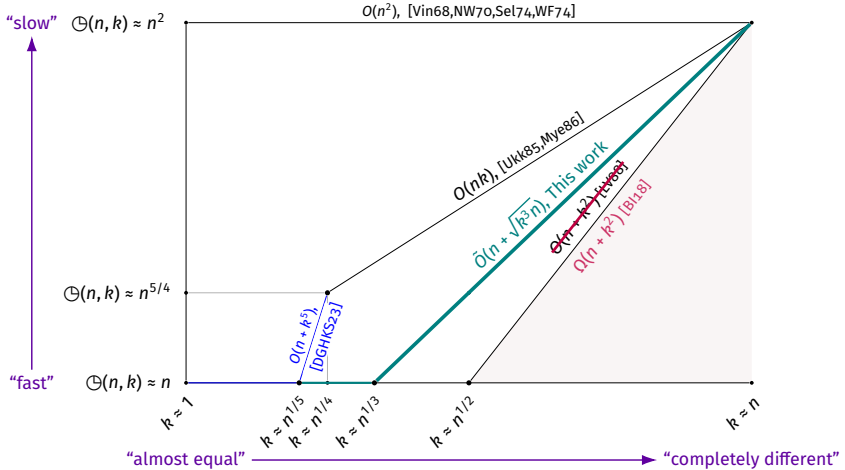

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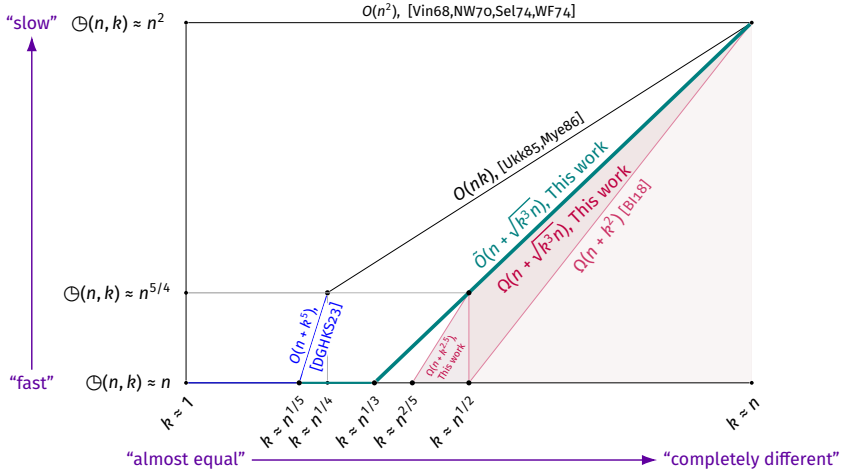

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Strings X, Y each of length at most n

Oracle access to (normalized) weight function w

Can compute $k = \mathrm{ed}^w(X, Y)$ in time $O(n + \sqrt{nk^3} \log^3 n)$

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Main Theorem 2 (Lower Bound)

Assuming the APSP Hypothesis and for $\sqrt{n} \leq k \leq n$,
Main Theorem 1 is tight (up to $n^{o(1)}$-factors)

Tool 0: Unweighted ED and [DGHKS23]-Kernel

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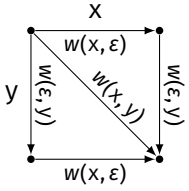

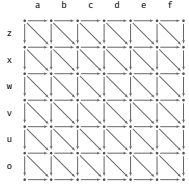

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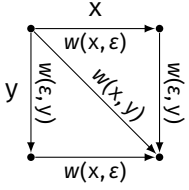

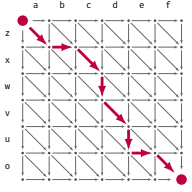

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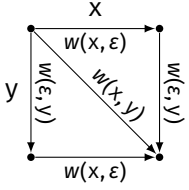

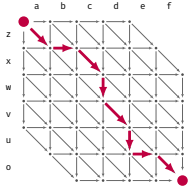

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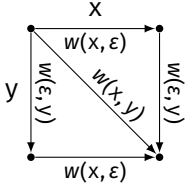
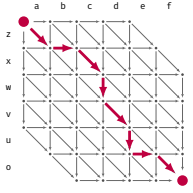
  $\rightsquigarrow$ AG has $O(k^5)$ vertices $\rightsquigarrow$ Dijkstra yields $\tilde{O}(k^5)$ algo





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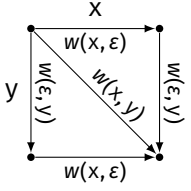
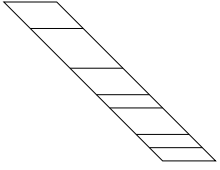
  Compute all b-to-b dist [Kleino5] + stitch together results ((min, +)-product [SMAWK87])





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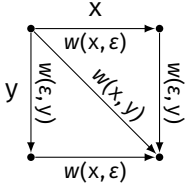
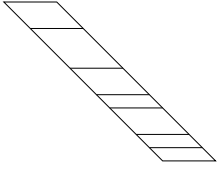
  $\rightsquigarrow k^2 \cdot \tilde{O}(k^3)$ for periodic pieces + $k^2 \cdot \tilde{O}(k^2)$ for stitching





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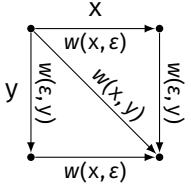
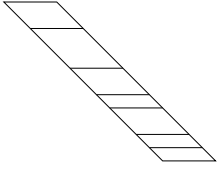
  Compute all b-to-b dist [Klein05] + stitch together results ((min, +)-product [SMAWK87]) for periodic pieces: fast exponentiation;
  $\rightsquigarrow$ $k^2 \cdot \tilde{O}(k^2)$ for periodic pieces + $k^2 \cdot \tilde{O}(k^2)$ for stitching





Tool 2: Divide and Conquer

- X and Y consist in $O(k^2)$ periodic pieces of length $O(k^2)$ and with period $O(k)$ each
- Idea: Use AG, trimmed to $O(k)$ diags $\rightsquigarrow \mathrm{ed}^{w}_{\leq k}(X, Y)$ is distance $(0, 0) \rightsquigarrow (|X|, |Y|)$
- Idea[3]: Compute all b-to-b dist for periodic pieces [Klein05] + fast exponentiation;
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Tool 3: Compressibility instead of Periodicity

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- Idea[5]: Use tailor-made compressibility measure instead of periodicity + (w)ED algorithms for compressed strings

  $\rightsquigarrow \tilde{O}(n + \sqrt{k^3 n})$ time in total

Main Theorem 1 (Upper Bound) ✓

Strings X, Y each of length at most n
Oracle access to (normalized) weight function w
Can compute $k = \mathrm{ed}^w(X, Y)$ in time $O(n + \sqrt{nk^3} \log^3 n)$

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Assuming the APSP Hypothesis and for $\sqrt{n} \leq k \leq n$,
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Tool 0: APSP Hypothesis and Negative Triangle

All-Pairs Shortest Paths Hypothesis [VWW18]

There is no algorithm for APSP on n vertex graphs with running time $O(n^{3-\varepsilon})$.

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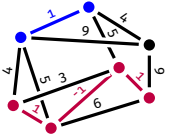
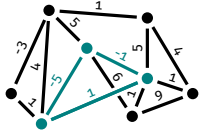
Negative Triangle

Check if n vertex graph has a negative triangle (3 vertices u, v, x with $w(u,v) + w(v,x) + w(u,x) < 0$)

APSP-H $\overset{\text{[VWW18]}}{\Longleftrightarrow}$ Negative Triangle





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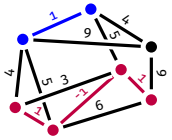
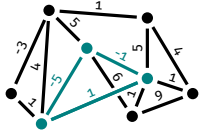
Given 3 matrices A, B, C, check if min diag entry of (min, +)-product $A \oplus B \oplus C$ is negative

APSP-H $\overset{\text{[VWW18]}}{\Longleftrightarrow}$ Negative Triangle $\overset{\text{[VWW18]}}{\Longleftrightarrow}$ MPP-Minimum





$$\min\left(\boxed{A} \oplus \boxed{B} \oplus \boxed{C}\right) < 0$$

Tool 1: Matrix-Vector Min-Plus Multiplication via wED

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Given 3 matrices A, B, C, check if min diag entry of $(\min,+)$-product $A \oplus B \oplus C$ is negative

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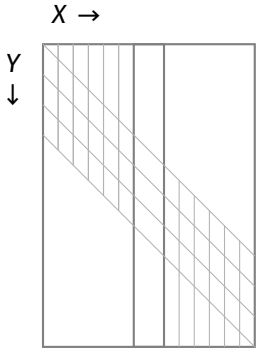
  $\rightsquigarrow$ encode matrix/vector as substitution costs



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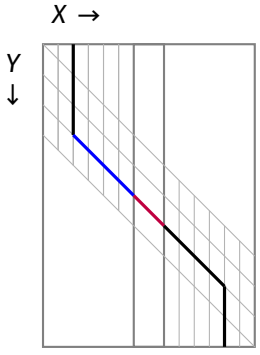

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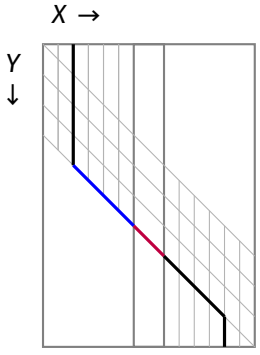

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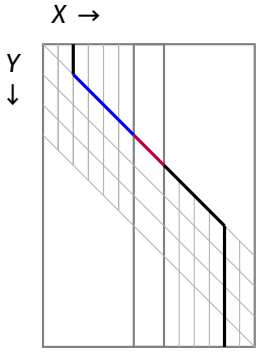

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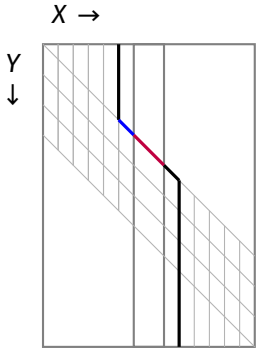

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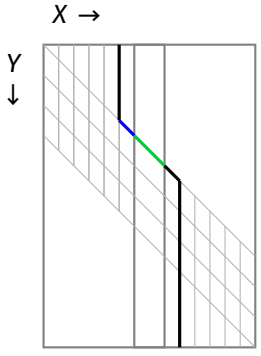

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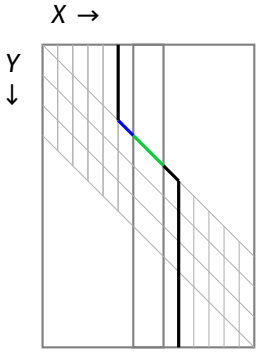
  - $\rightsquigarrow$ Allows to compute $A \oplus B$; encode C similarly to B
  - $\rightsquigarrow$ Use selection gadget for $\mathrm{ed}^w(X, Y) \approx \min(A \oplus B \oplus C)$
  - $\rightsquigarrow$ Lower bound for dynamic problem



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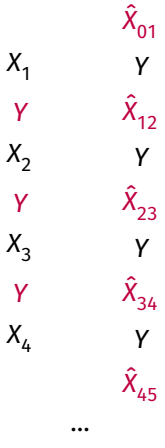

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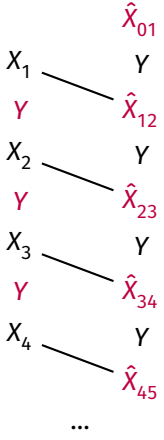

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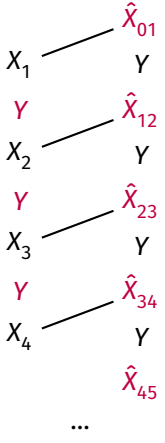

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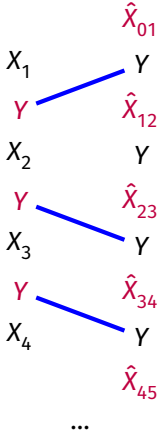

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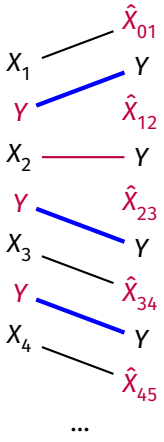

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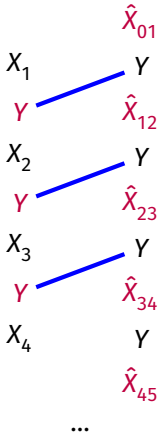

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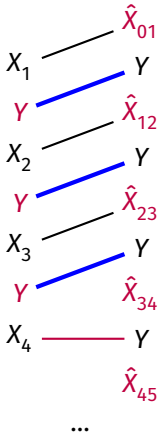

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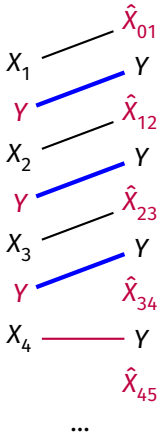
  ⇝ Minimum gadget: $\mathrm{ed}^w(\hat{X}, \hat{Y}) \approx \min \mathrm{ed}^w(X_i, Y)$
  ⇝ Static LB



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Open Questions

- What is the true complexity of $n^{1/3} \leq k \leq n^{1/2}$?
  Must be between $n + k^{2.5}$ and $\sqrt{nk^3}$.
- Is the problem easier for small (constant-sized) alphabets?
- Is there any easy class of weight functions?

Start

End