# Few Matches or Almost Periodicity: Faster Pattern Matching with Mismatches in Compressed Texts

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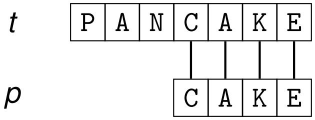
# Pattern Matching with Mismatches

## Pattern Matching

Given a text $t$ and a pattern $p$, is $p$ a substring of $t$?




Finding CAKE

# Pattern Matching with Mismatches

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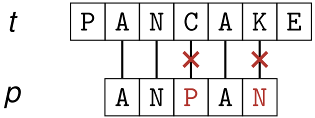
Given a text $t$, a pattern $p$, and an integer $k$, does $t$ have a length-$|p|$ substring with Hamming-distance at most $k$ to $p$?




Finding ANPAN, $k = 2$

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Pattern matching with $k$ mismatches on a text of length $n$ and a pattern of length $m$ can be solved in time $\widetilde{O}((m + k\sqrt{m}) \cdot n/m)$.

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Matching (conditional) lower bound [GU'18]

What if the text is much larger than the pattern?

SESWEETROLLMOSTCOMMONLYFILLEDWITHREDBEANPASTEANPANCANALSOBEPREPAREDWITHOTHERFILL

ANPAN

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What if the text is much larger than the pattern and given in a compressed representation?

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# Grammar Compression

**Straight-Line Program (SLP)**

A Straight-Line Program or SLP $\mathcal{T}$ is a context-free grammar that generates exactly one string eval($\mathcal{T}$).

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An SLP $\mathcal{T}$ is a set of non-terminals $\{T_1, \dots, T_n\}$ and productions of the form $T_i \to \sigma$ or $T_i \to T_\ell T_r$, where $\ell, r < i$.
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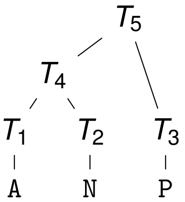

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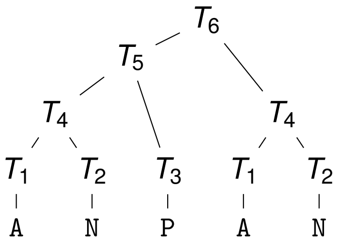

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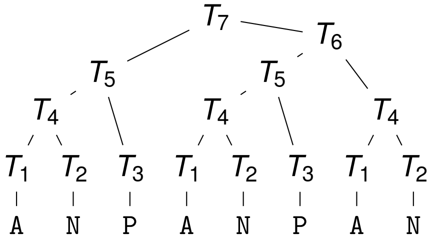

# Known Results

| Problem | uncompressed | LZW/LZ78 text $n = \Omega(\sqrt{N})$ | SLP text $n = \Omega(\log N)$ |
|---|---|---|---|
| Pattern Matching | $O(N + m)$ [KMP'77] | $O(n + m)$ ※ [G'12] | $\widetilde{O}(n + m)$ ※ [J'15] |
| PM with $k$ Mismatches | $\widetilde{O}(\frac{N}{m}(m + k\sqrt{m}))$ [GU'18] | $O(n\sqrt{m}k^2)$ [GS'13] | $\widetilde{O}(nm\,\mathrm{poly}(k))$ [T'14,BLRS'15] |

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Improvement obtained via new structural insight in solution structure

# Solution Structure of Pattern Matching

**Fact (Folklore)**

Let text $t$ and pattern $p$, $|t| \leq \frac{3}{2}|p|$, be given such that there are $\geq 2$ matches of $p$ in $t$ that together match $t$ completely. Then, both $p$ and $t$ are periodic with some period $x$ and every match of $p$ in $t$ starts at a position $1 + i \cdot |x|$.

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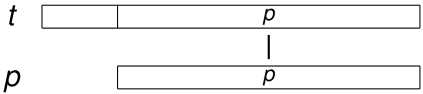

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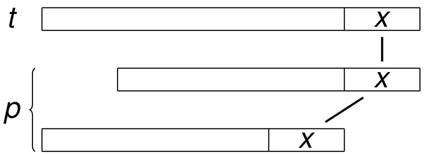

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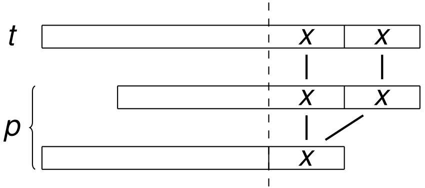

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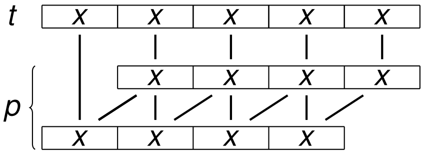

What is the solution structure of
Pattern Matching with Mismatches?

# Solution Structure of Pattern Matching with Mismatches

If there are at least 2 $k$-matches of $p$ in $t$, then $p$ and $t$ are periodic and every $k$-match of $p$ starts at a position $1 + i|x|$?

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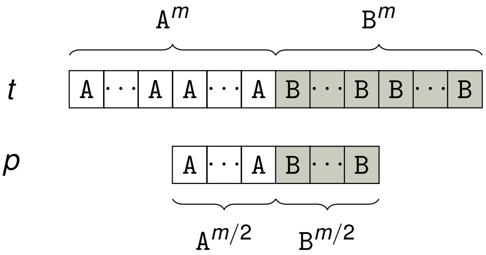

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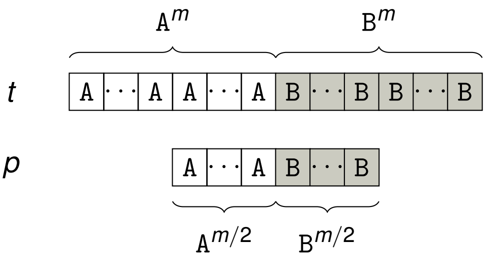
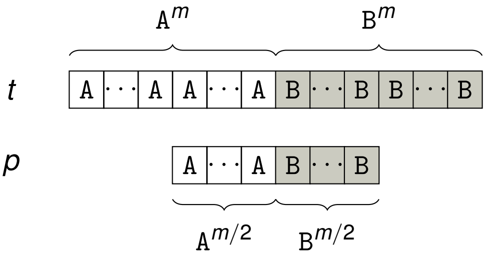



- $p$ and $t$ not periodic, but $2k$ $k$-matches of $p$ in $t$

# Solution Structure of Pattern Matching with Mismatches

If there are at least ~~two~~ $\Omega(\text{poly}(k))$ $k$-matches of $p$ in $t$, then $p$ and $t$ are periodic and every $k$-match of $p$ starts at a position $1 + i|x|$?




## Insight 1

Periodicity only if number of $k$-matches of $p$ in $t$ is $\Omega(\text{poly}(k))$

# Solution Structure of Pattern Matching with Mismatches

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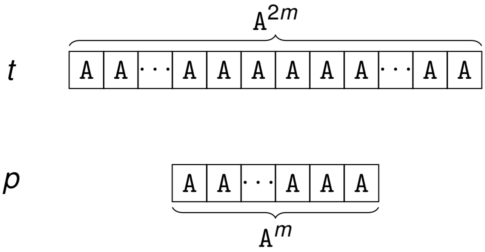

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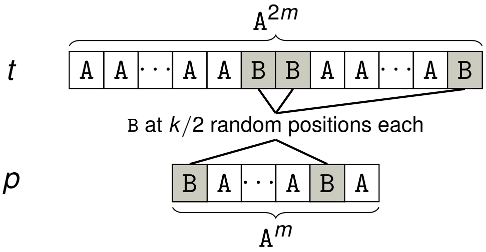

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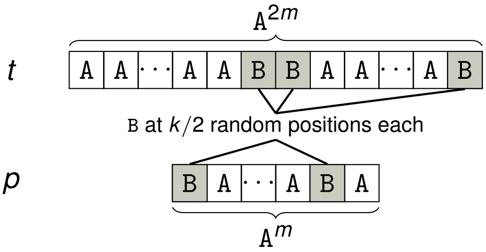
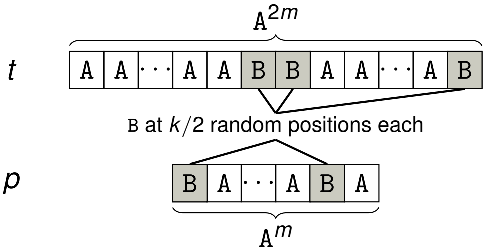



- $O(m)$ $k$-matches of $p$ in $t$, but $p$ and $t$ not perfectly periodic

# Solution Structure of Pattern Matching with Mismatches

If there are at least $\Omega(\text{poly}(k))$ $k$-matches of $p$ in $t$, then $p$ and $t$ are ~~periodic~~ periodic up to $O(k)$ mismatches and every $k$-match of $p$ starts at a position $1 + i|x|$?




## Insight 2

Periodicity only up to $O(k)$ mismatches

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# Main Result

**Theorem (Structural Insight)**

For pattern $p$ and text $t$, $|t| \leq 2|p|$, at least one of the following holds:

- The number of $k$-matches of $p$ in $t$ is at most $O(k^2)$, or
- $t'$: shortest substring of $t$ such that any $k$-match of $p$ in $t$ is also a $k$-match in $t'$ Both $t'$ and $p$ have HD $O(k)$ to the same periodic string $x$ and all $k$-matches of $p$ in $t'$ start at a position $1 + i \cdot |x|$.

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For pattern $p$ and text $t$, $|t| \leq 2|p|$, at least one of the following holds:

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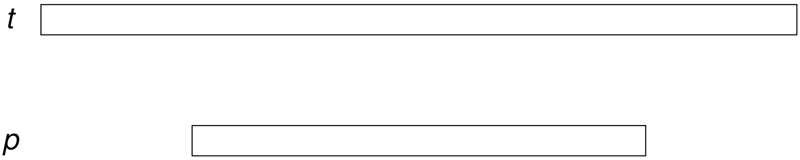

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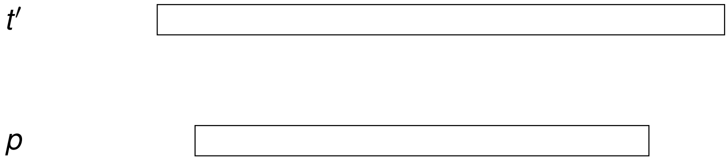


- Consider $t'$: shortest substring of $t$ that contains all $k$-matches

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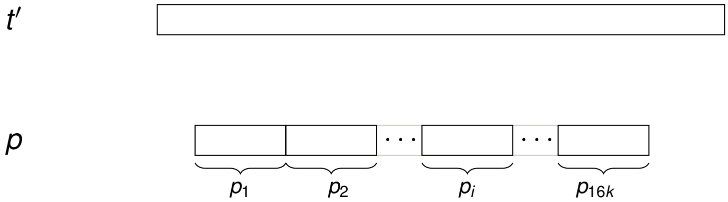


- Split $p$ into $16k$ parts $p_i$ of equal length

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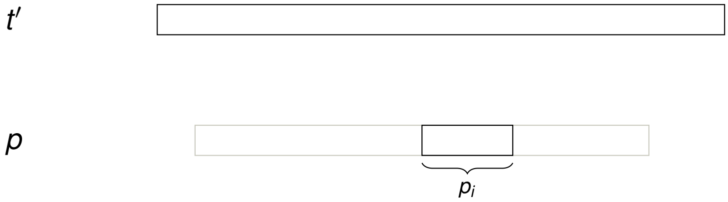


- Fix a $p_i$

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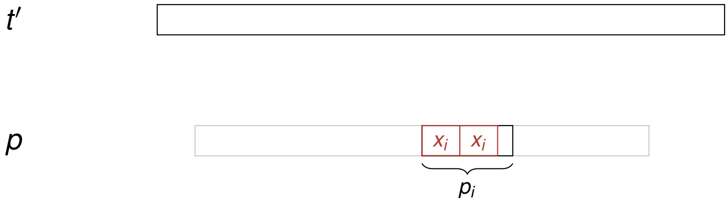


- Consider prefix $x_i$ of $p_i$ that is also a period of $p_i$

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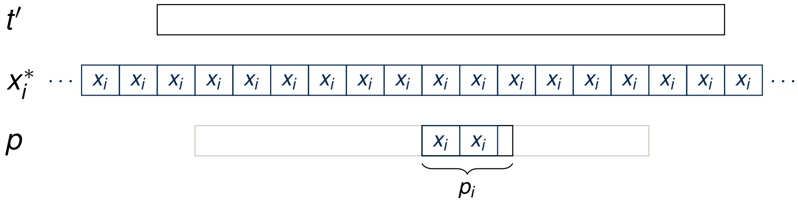


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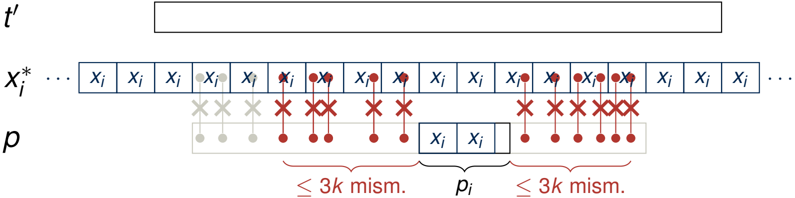


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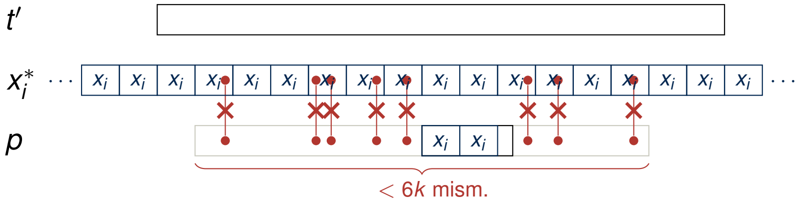

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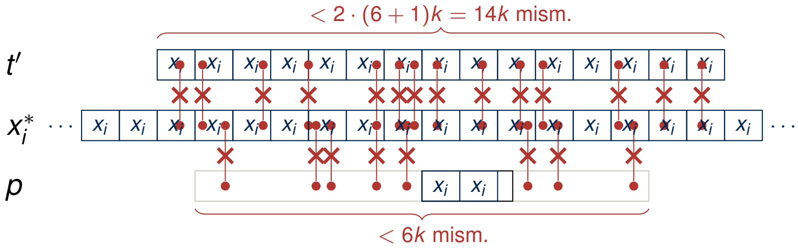

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**Theorem (Structural Insight)**

For pattern $p$ and text $t$, $|t| \leq 2|p|$, at least one of the following holds:

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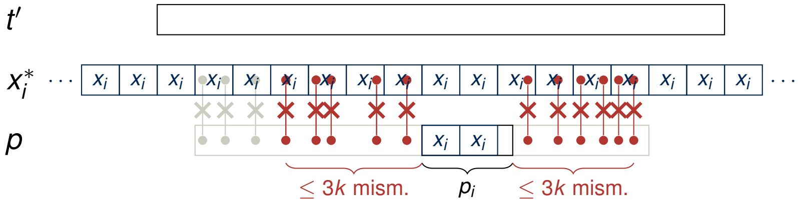

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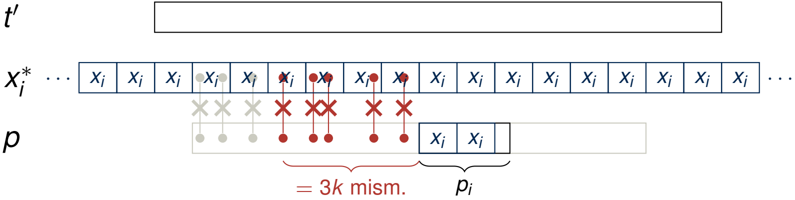

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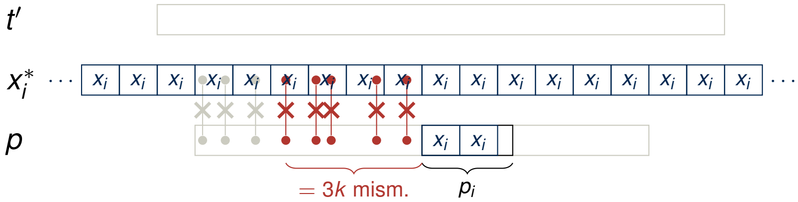


## Insight

Any $k$-match of $p$ in $t'$ must match at least one $p_i$'s **exactly**.

# Main Result, Proof Overview

**Theorem (Structural Insight)**

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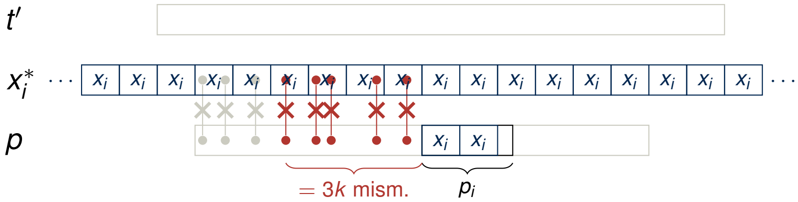


- Fix a $p_i$

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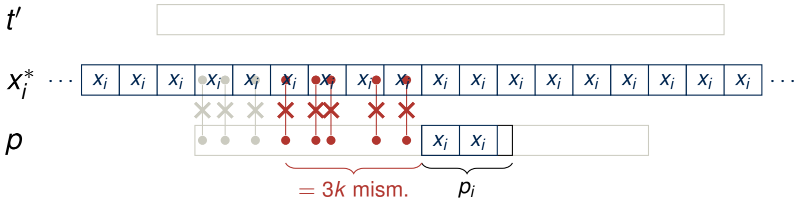


- Fix a $p_i$; count $k$-matches where $p_i$ is matched exactly

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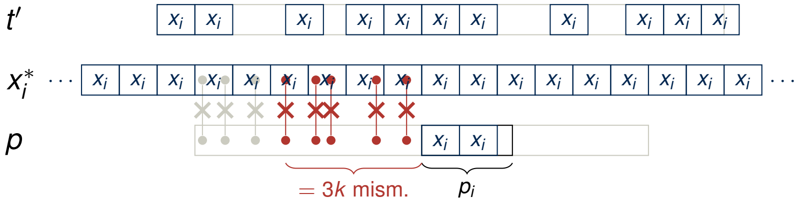


- Consider occurrences of $x_i$ in $t'$

# Main Result, Proof Overview

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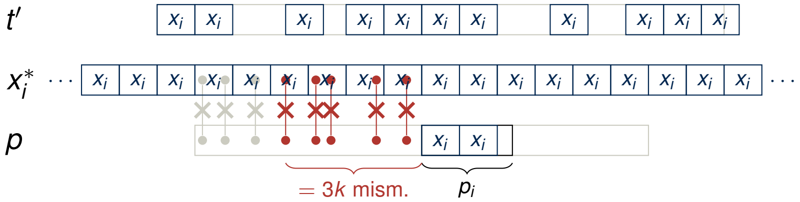


## Problem

Up to $O(m)$ exact matches of $x_i$ in $t'$.

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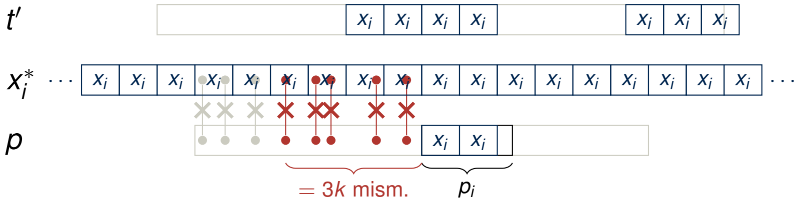


- Consider **power stretches** of $x_i$ in $t'$ of length $\geq |p_i|$

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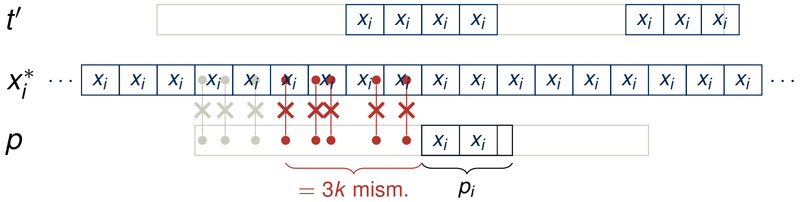


- Consider **power stretches** of $x_i$ in $t'$ of length $\geq |p_i|$
  $\rightsquigarrow$ at most $150k$ different power stretches

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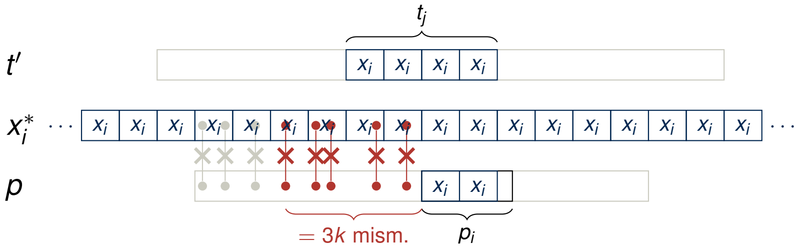


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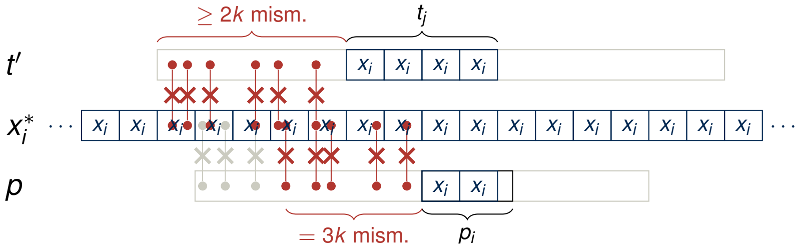


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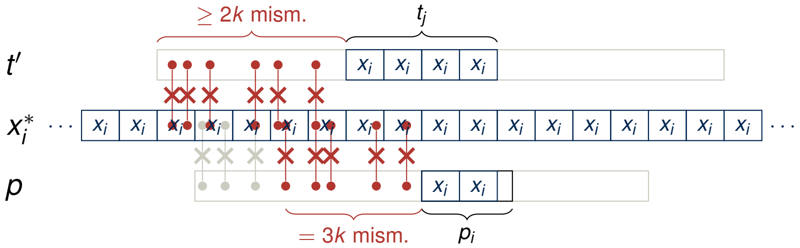


## Insight

Must align at least one mismatch.

# Main Result, Proof Overview

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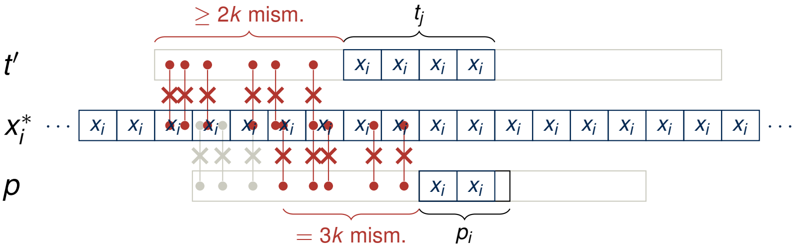


## Insight

At most $O(k^4)$ matches: $O(k)$ parts in $p$, $O(k)$ stretches, $O(k^2)$ matches per combination.

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# Faster Algorithm

**Theorem (Algorithm)**

Pattern matching with $k$ mismatches on a text $t$ given by an SLP of size $n$ and a pattern $p$ of length $m$ can be solved in time $O(n\,k^3\,(k \log k + \log m) + k\,m)$.

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**Pattern-Compressed String [GS'13]**

Let $p$ be a string of length $m$. We call a string $f = v_1 \dots v_q, \sum_{i=1}^{q} |v_i| \leq 2m$ a $p$-pattern-compressed string (pc-string) if every $v_i$ is a substring of $p$.
We call the $v_i$'s factors of $f$.

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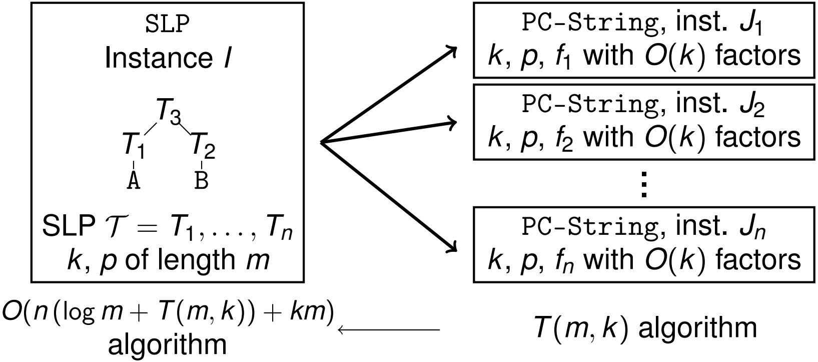

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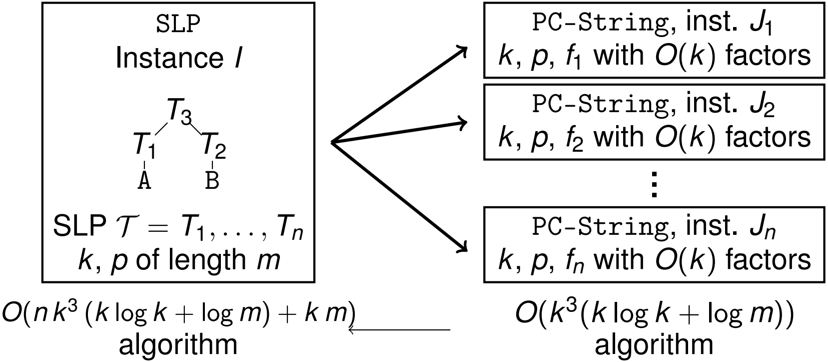

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SLP
Instance $I$

$T_3$
$T_1$ $T_2$
A B

SLP $\mathcal{T} = T_1, \dots, T_n$
$k$, $p$ of length $m$

PC-String, inst. $J_1$
$k$, $p$, $f_1$ with $O(k)$ factors

PC-String, inst. $J_2$
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⋮

PC-String, inst. $J_n$
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$O(n\,k^3\,(k \log k + \log m) + k\,m)$ algorithm

$O(k^3(k \log k + \log m))$ algorithm



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SIC Saarland Informatics Campus

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**Theorem (Algorithm for pc-strings)**

Pattern matching with $k$ mismatches on a pattern $p$ of length $m$ and a $p$-pc-string $f$ of size $O(k)$ representing at most $2m$ characters, can be solved in time $O(k^3(k \log k + \log m))$.
(With $O(km)$ preprocessing on $p$.)

- Implementation of structural insight
- Need e.g. tools for finding first $O(k)$ mismatches to a periodic string or finding all power stretches of a given string in a pc-string

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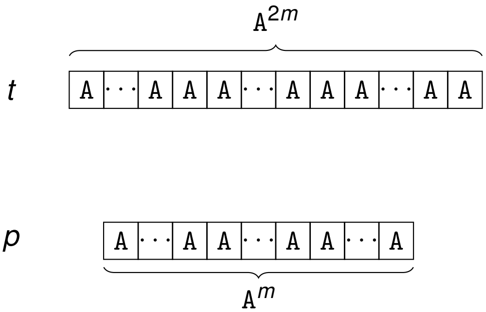
# Open Problems

- ~~Improve insight to $O(k)$ mismatches in the aperiodic case~~
- ~~Improve dependence on $k$ in the algorithm~~
- ~~Fully-compressed setting ($p$ also given as an SLP)~~
- ~~Pattern Matching with Errors (Edit distance instead of Hamming distance)~~

# Solution Structure of Pattern Matching with Mismatches



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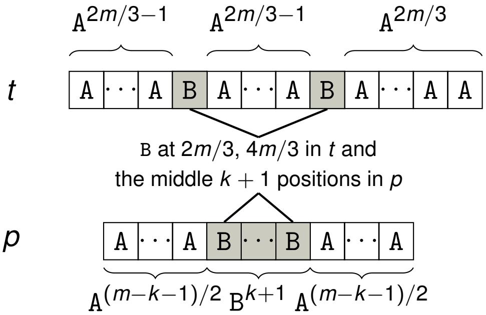

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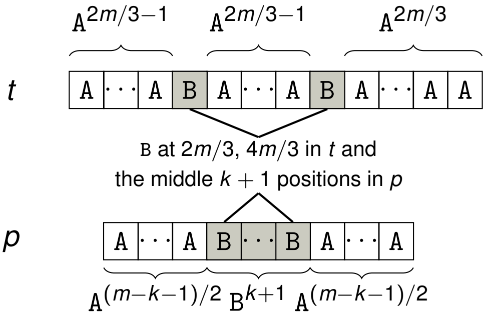
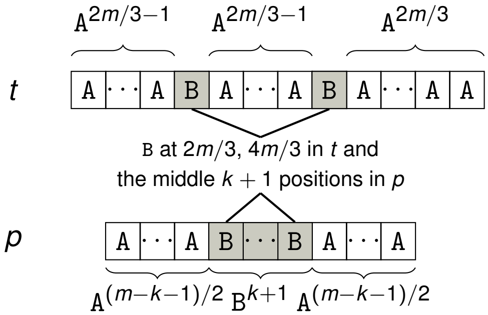



- All matches start at the union of two intervals.

# Solution Structure of Pattern Matching with Mismatches




**Insight 3**

Arithmetic progression only approximates all matches

# Main Result

**Theorem (Structural Insight)**

Given strings $p$ of length $m$ and $t$ of length at most $2m$, at least one of the following holds:

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| $t$ | P | A | N | C | A | K | E | P | A | N |
|---|---|---|---|---|---|---|---|---|---|---|

| $t$ | P | U | N | R | A | N | P | A | M | P | A | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|

Finding ANPAN, $k = 2$
non-periodic case

Finding PANPAN, $k = 2$
periodic case

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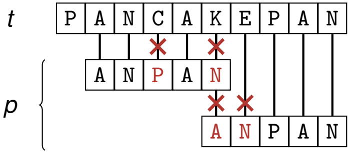


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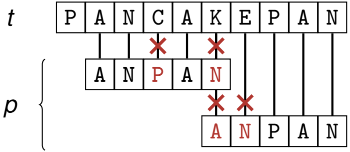


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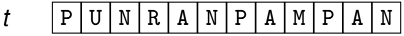


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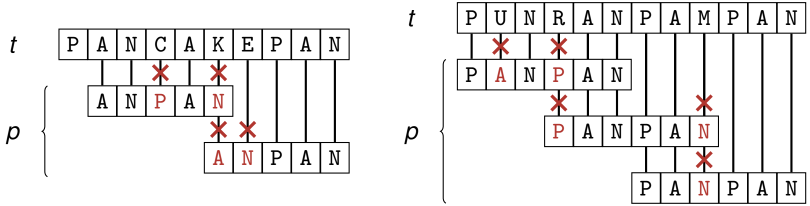


Finding `ANPAN`, $k = 2$
non-periodic case

Finding `PANPAN`, $k = 2$
periodic case

## Main Result, Proof Overview

**Theorem (Structural Insight)**

For pattern $p$ and text $t$, $|t| \leq 2|p|$, it holds at least one of:

- The number of $k$-matches of $p$ in $t$ is at most $O(k^2)$, and
- Both $t$ and $p$ have HD $O(k)$ to the same periodic string.

# Main Result, Proof Overview

**Theorem (Structural Insight)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, then both $t$ and $p$ have a HD $< 20k$ to the same periodic string.

# Main Result, Proof Overview

**Theorem (Structural Insight)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, then both $t$ and $p$ have a HD $< 20k$ to the same periodic string.

Main Steps:

- At least $1000k^2$ $k$-matches of $p$ in $t$ and
  $p$ has a HD $< 6k$ to a specific periodic string $x \in x(p)$
  $\implies t$ has a Hamming Distance $< 20k$ to $x$
- $p$ has HD $\geq 6k$ to any specific periodic string $x \in x(p)$
  $\implies$ Less than $1000k^2$ $k$-matches of $p$ in $t$

# Main Result, Proof Overview

**Theorem (Structural Insight)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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Main Steps:

- At least $1000k^2$ $k$-matches of $p$ in $t$ and
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  $\implies$ $t$ has a Hamming Distance $< 20k$ to $x$

- $p$ has HD $\geq 6k$ to any specific periodic string $x \in x(p)$
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# Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, and $p$ has HD $< 6k$ to a periodic string $x \in x(p)$, then $t$ has HD $< 20k$ to $x$.

# Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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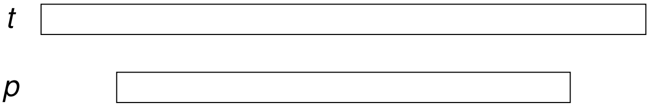

# Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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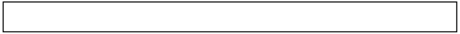
$p$



- Split $p$ into $16k$ parts $p_i$ of equal length

## Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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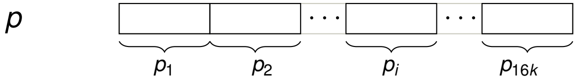


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# Main Result, Proof Overview

## Lemma (Step 1)

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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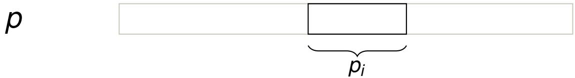


- Fix a $p_i$

# Main Result, Proof Overview

## Lemma (Step 1)

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, and $p$ has HD $< 6k$ to a periodic string $x \in x(p)$, then $t$ has HD $< 20k$ to $x$.


- Consider prefix $x_i$ of $p_i$ that is also a period of $p_i$

# Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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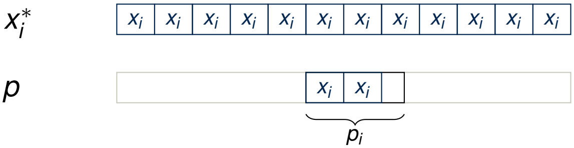


- Find first $3k$ mismatches between $p$ and $x_i^*$ before and after $p_i$

# Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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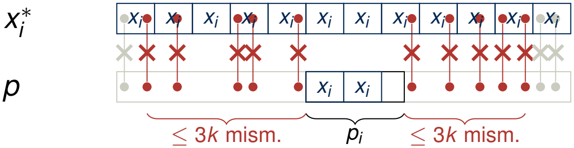


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# Main Result, Proof Overview

## Lemma (Step 1)

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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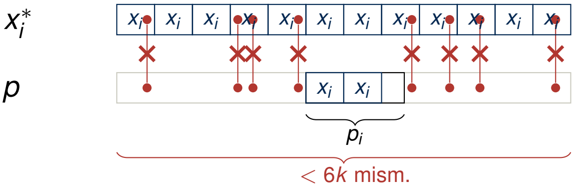

## Main Result, Proof Overview

**Lemma (Step 1)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, and $p$ has HD $< 6k$ to some $x_i^*, 1 \leq i \leq 16k$, then $t$ has HD $< 20k$ to $x_i^*$.

**Claim (Proof omitted)**

If there are at least $2 + 16k$ $k$-matches of $p$ in $t$, all starting positions of $k$-matches differ by (integer) multiples of $|x_i|$.

# Main Result, Proof Overview

## Lemma (Step 1)

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, and $p$ has HD $< 6k$ to some $x_i^*, 1 \leq i \leq 16k$, then all starting positions of $k$-matches differ by multiples of $|x_i|$ and $t$ has HD $< 20k$ to $x_i^*$.

# Main Result, Proof Overview

## Lemma (Step 1)

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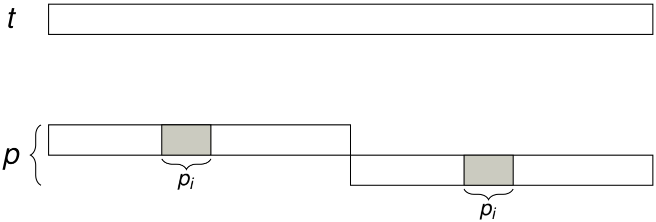

# Main Result, Proof Overview

## Lemma (Step 1)

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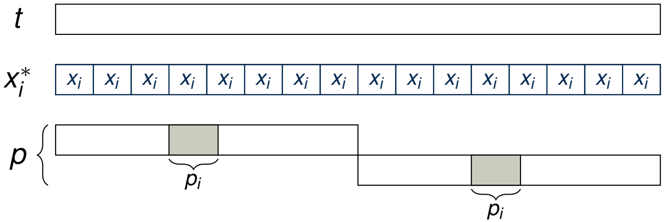

# Main Result, Proof Overview

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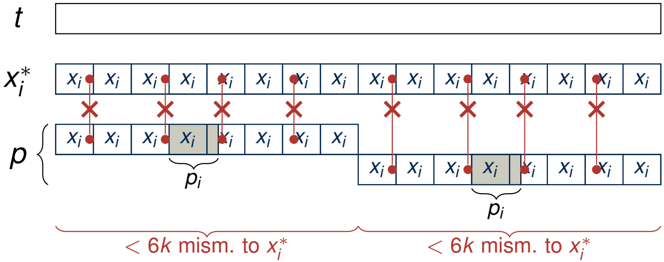

# Main Result, Proof Overview

## Lemma (Step 1)

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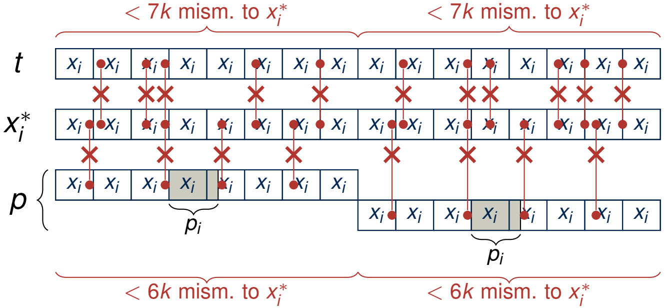

# Main Result, Proof Overview

## Lemma (Step 1) ✓

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, and $p$ has HD $< 6k$ to some $x_i^*, 1 \leq i \leq 16k$, then all starting positions of $k$-matches differ by multiples of $|x_i|$ and $t$ has HD $< 20k$ to $x_i^*$.

## Lemma (Step 2)

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the pattern $p$ has a HD $\geq 6k$ to all strings $x_i^*, 1 \leq i \leq 16k$, then there are less than $1000k^2$ $k$-matches of $p$ in $t$.

## Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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# Main Result, Proof Overview

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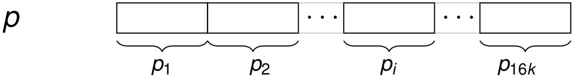


- Recall: Split $p$ into $16k$ parts $p_i$ of equal length

# Main Result, Proof Overview

## Lemma (Step 2)

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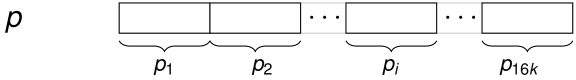


## Insight

Any $k$-match of $p$ in $t$ must match at least $15k$ $p_i$'s **exactly**.

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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- Fix a $p_i$

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the pattern $p$ has a HD $\geq 6k$ to all strings $x_i^*, 1 \leq i \leq 16k$, then there are less than $1000k^2$ $k$-matches of $p$ in $t$.


- Fix a $p_i$; count $k$-matches where $p_i$ is matched exactly

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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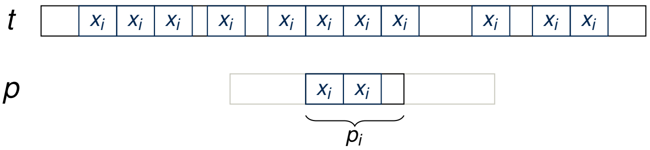


- Search for $x_i$ in $t$

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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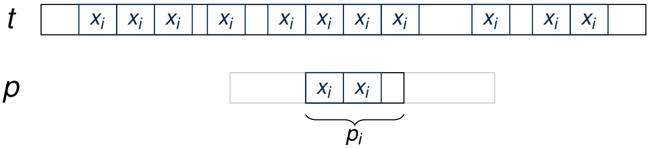


**Problem**

Up to $O(m)$ exact matches of $x_i$ in $t$.

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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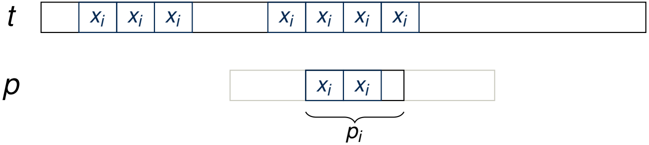


- Search for **power stretches** of $x_i$ in $t$ of length $\geq |p_i|$

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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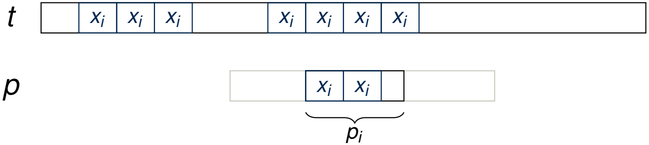


**Insight**

Only $\leq 150k$ different power stretches of $x_i$ in $t$.

# Main Result, Proof Overview

**Lemma (Step 2)**

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
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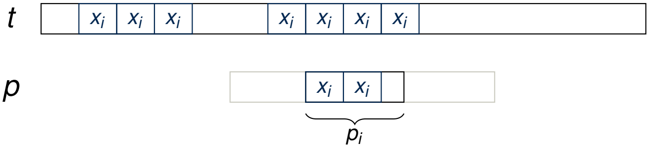


- Fix a power stretch $t_j$ of $x_i$ in $t$.

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# Main Result, Proof Overview

**Lemma (Step 2)**

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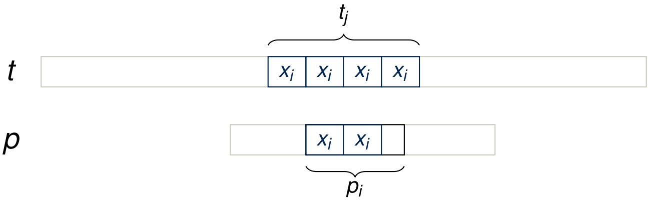


- Fix a power stretch $t_j$ of $x_i$ in $t$.

# Main Result, Proof Overview

**Lemma (Step 2)**

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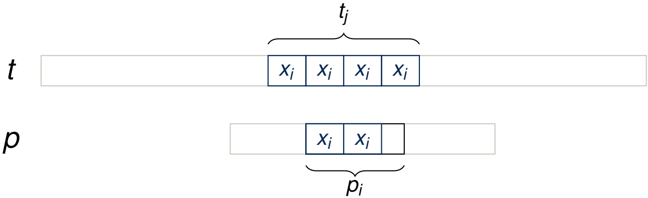

# Main Result, Proof Overview

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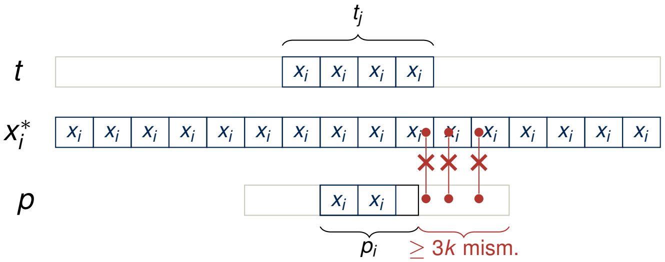
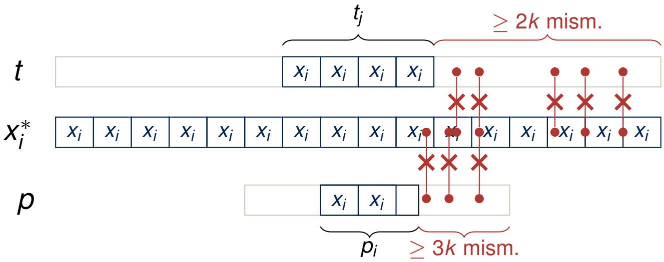
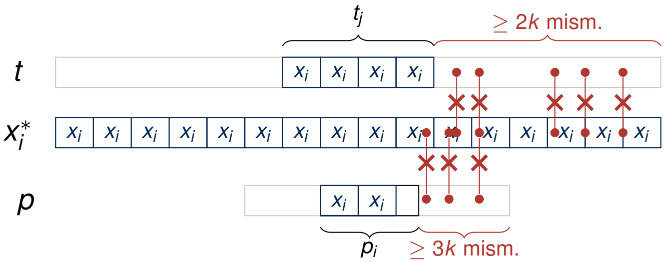


# Main Result, Proof Overview



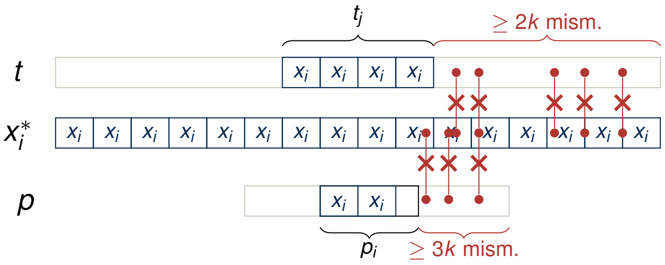
# Main Result, Proof Overview




**Insight**

Must align at least $k$ mismatches.

# Main Result, Proof Overview




**Insight**

At most $O(k^4)$ matches: $O(k)$ parts in $p$, $O(k)$ streches, $O(k^2)$ matches per combination.

# Main Result, Proof Overview

**Lemma (Step 1)** ✓

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the number of $k$-matches of $p$ in $t$ is at least $1000k^2$, and $p$ has HD $< 6k$ to some $x_i^*, 1 \leq i \leq 16k$, then all starting positions of $k$-matches differ by multiples of $|x_i|$ and $t$ has HD $< 20k$ to $x_i^*$.

**Lemma (Step 2)** ✓

Fix a pattern $p$ of length $m$ and a text $t$ of length at most $2m$.
If the pattern $p$ has a HD $\geq 6k$ to all strings $x_i^*, 1 \leq i \leq 16k$, then there are less than $1000k^2$ $k$-matches of $p$ in $t$.

## Main Result, Proof Overview

**Theorem (Structural Insight) ✓**

Given strings $p$ of length $m$ and $t$ of length at most $2m$, at least one of the following holds:

- The number of $k$-matches of $p$ in $t$ is at most $O(k^2)$.
- $t'$: shortest substring of $t$ such that any $k$-match of $p$ in $t$ is also a $k$-match in $t'$
  There is a substring $x$ of $p$, with $|x| = O(m/k)$, such that $\delta_H(p, x^*[1, m]) \leq O(k)$ and $\delta_H(t', x^*[1, |t'|]) \leq O(k)$.
  Moreover, any $k$-match of $p$ in $t'$ starts at a position of the form $1 + i \cdot |x|$ with $0 \leq i \leq (|t'| - |p|)/|x|$ (but not every starting position $1 + i \cdot |x|$ necessarily yields a $k$-match).

# Faster Algorithm for Pattern-Compressed Strings

**Pattern-Compressed String [GS'13]**

Let $p$ be a string of length $m$. We call a string $f = v_1 \dots v_q, \sum_{i=1}^{q} |v_i| \leq 2m$ a $p$-pattern-compressed string (pc-string) if every $v_i$ is a substring of $p$. We call the $v_i$'s factors of $f$.

# Faster Algorithm for Pattern-Compressed Strings

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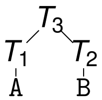
`SLP` Instance $I$




SLP $\mathcal{T} = T_1, \dots, T_n$
$k$, $p$ of length $m$

`PC-String`, inst. $J_1$
$k$, $p$, $f_1$ with $O(k)$ factors

`PC-String`, inst. $J_2$
$k$, $p$, $f_2$ with $O(k)$ factors

⋮

`PC-String`, inst. $J_n$
$k$, $p$, $f_n$ with $O(k)$ factors

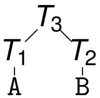
$O(n(\log m + T(m,k)) + km)$ algorithm ⟵ $T(m,k)$ algorithm

# Faster Algorithm for Pattern-Compressed Strings

**Pattern-Compressed String [GS'13]**

Let $p$ be a string of length $m$. We call a string $f = v_1 \dots v_q, \sum_{i=1}^{q} |v_i| \leq 2m$ a $p$-pattern-compressed string (pc-string) if every $v_i$ is a substring of $p$. We call the $v_i$'s factors of $f$.

`SLP` Instance $I$




SLP $\mathcal{T} = T_1, \dots, T_n$
$k$, $p$ of length $m$

→

`PC-String`, inst. $J_1$
$k$, $p$, $f_1$ with $O(k)$ factors

`PC-String`, inst. $J_2$
$k$, $p$, $f_2$ with $O(k)$ factors

⋮

`PC-String`, inst. $J_n$
$k$, $p$, $f_n$ with $O(k)$ factors

$O(n\,k^3\,(k \log k + \log m) + k\,m)$ algorithm ← $O(k^3(k \log k + \log m))$ algorithm

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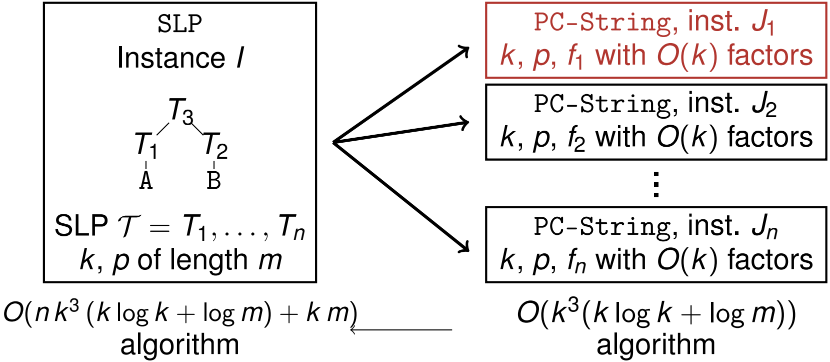

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**Theorem (Algorithm for pc-strings)**

Pattern matching with $k$ mismatches on a pattern $p$ of length $m$ and a $p$-pc-string $f$ of size $O(k)$ representing at most $2m$ characters, can be solved in time $O(k^3(k \log k + \log m))$.
(With $O(km)$ preprocessing on $p$.)

- Implementation of structural insight
- Need e.g. tools for finding first $O(k)$ mismatches to a periodic string or finding all power stretches of a given string in a pc-string

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# Navigation

Start

Definition SLP | Known Results

Insight Ex. 1 | Insight Ex. 2 | Insight Ex. 3

Insight Theorem | Insight Step 1 Overview | Insight Step 1 Details | Insight Step 2 Overview

Algorithm General Idea | Algorithm Overview

Open Problems

End