Counting and Finding Homomorphisms is Universal for Parameterized Complexity Theory

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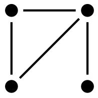
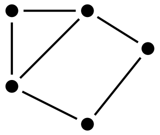
Finding Graph Homomorphisms

Graph Homomorphism

Mapping from graph H to G that preserves edges;
Write $\mathrm{Hom}(H \to G)$ for the set of all graph hom's from H to G.



H



G

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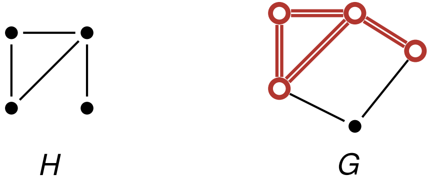

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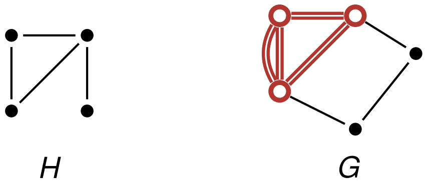

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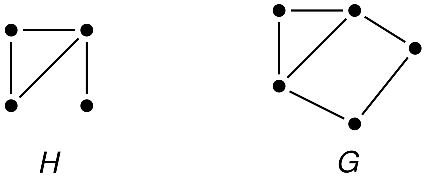

$$\#\mathrm{Hom}(H \to G) = 14$$

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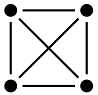

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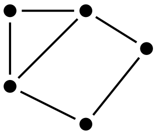

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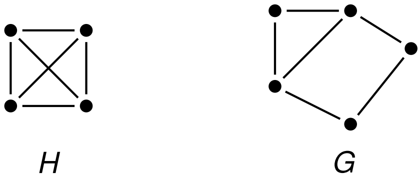

No homomorphisms from H to G.

Finding Graph Homomorphisms

HOM($\mathcal{H} \to \mathcal{G}$)

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, check if there is a graph hom from H to G.

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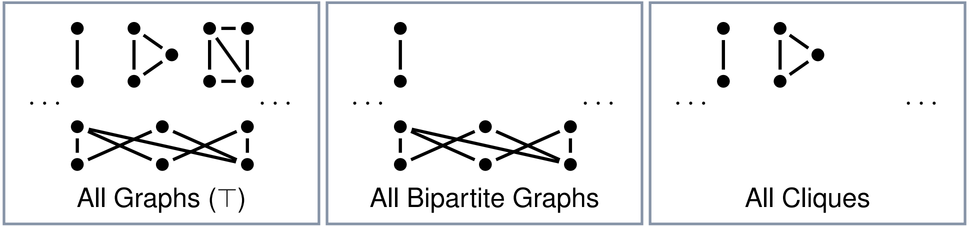

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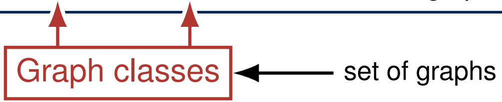

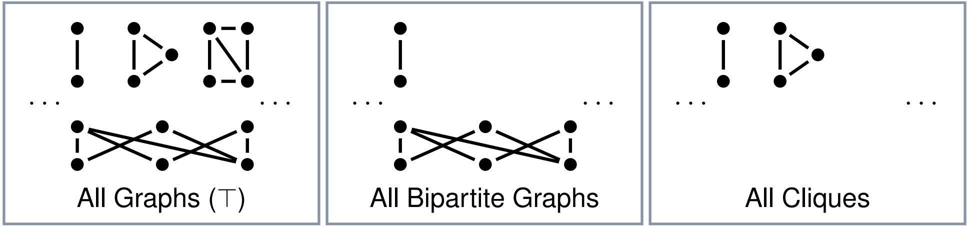

Known Results

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NP-complete

$\textsc{Hom}(\top \to \top)$

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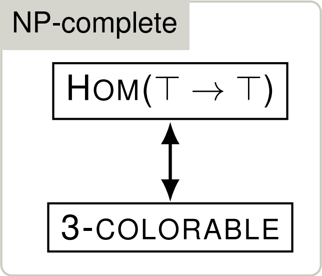

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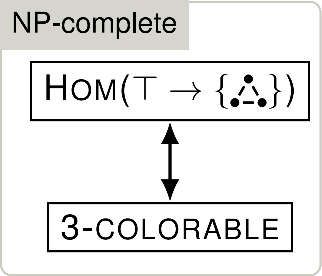

Known Results

$\textsc{Hom}(\mathcal{H} \to \mathcal{G})$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, check if there is a graph hom from H to G.

Are there fast algorithms for special cases of $\textsc{Hom}(\top \to \top)$?

Known Results

$\textsc{Hom}(\mathcal{H} \to \mathcal{G})$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, check if there is a graph hom from H to G.

What makes $\textsc{Hom}(\top \to \top)$ hard?

Known Results

$\textsc{Hom}(\mathcal{H} \to \mathcal{G})$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, check if there is a graph hom from H to G.

	poly-time solvable	NP-complete
$\textsc{Hom}(\top \to \mathcal{G})$	$\mathcal{G}$ contains only bipartite graphs [Hell, Nešetřil ’90]	$\mathcal{G}$ contains a non-bipartite graph [Hell, Nešetřil ’90]

Known Results

#Hom($\mathcal{H} \to \mathcal{G}$)

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, count all graph homomorphisms from H to G.

	poly-time solvable	#P-complete
#Hom($\top \to \mathcal{G}$)	(explicit criterion exists) [Dyer, Greenhill '00]	(explicit criterion exists) [Dyer, Greenhill '00]

Known Results

$\textsc{Hom}(\mathcal{H} \to \mathcal{G})$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, check if there is a graph hom from H to G.

What about *the other side*, $\textsc{Hom}(\mathcal{H} \to \top)$?

Known Results

HOM($\mathcal{H} \to \mathcal{G}$) **Parameter: $|V(H)|$**

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What about *the other side*, HOM($\mathcal{H} \to \top$)?

Known Results

Hom($\mathcal{H} \to \mathcal{G}$) — Parameter: $|V(H)|$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, check if there is a graph hom from H to G.

	FPT ($f(\|V(H)\|) \cdot \text{poly}(\|V(G)\|)$ time)	W[1]-hard (not faster than k-Clique)
Hom($\mathcal{H} \to \top$)	"$\mathcal{H}$ contains only graphs with small treewidth" [Grohe '03]	"$\mathcal{H}$ contains graphs with arbitrary large tw" [Grohe '03]

Known Results

#HOM($\mathcal{H} \to \mathcal{G}$) Parameter: $|V(H)|$

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#HOM($\mathcal{H} \to \top$)	"$\mathcal{H}$ contains only graphs with small treewidth" [Dalmau, Jonsson '04]	"$\mathcal{H}$ contains a graph with large treewidth" [Dalmau, Jonsson '04]

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Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, count all graph homomorphisms from H to G.

Complexity dichotomies when restricting either $\mathcal{G}$ or $\mathcal{H}$.

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What if we restrict *both sides*?

Known Results

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What if we restrict *both sides*?

This talk.

Main Result

$\#\textsc{Hom}(\mathcal{H} \to \mathcal{G})$ Parameter: $|V(H)|$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, count all graph homomorphisms from H to G.

Theorem

For any problem P in #W[1] (or W[1]), there are graph classes $\mathcal{H}_P$ and $\mathcal{G}_P$ such that P is equivalent to $\#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P)$ (or $\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P)$).

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- Cannot hope for clear categorization into FPT/W[1]-hard for all pairs $(\mathcal{H}, \mathcal{G})$ (think of Ladner's Theorem)

Proof Ideas

#HOM($\mathcal{H} \to \mathcal{G}$) — Parameter: $|V(H)|$

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Recall: #Hom($\mathcal{H} \to \top$) is #W[1]-hard if $\mathcal{H}$ has "unbounded treewidth" [DalJon'04]

Proof Ideas

#HOM($\mathcal{H} \to \mathcal{G}$) Parameter: $|V(H)|$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, count the number of graph hom's from H to G.

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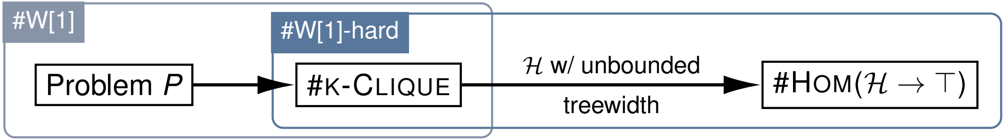
For any P in #W[1], there are $\mathcal{H}_P$, $\mathcal{G}_P$ such that P is equivalent to #HOM($\mathcal{H}_P \to \mathcal{G}_P$).



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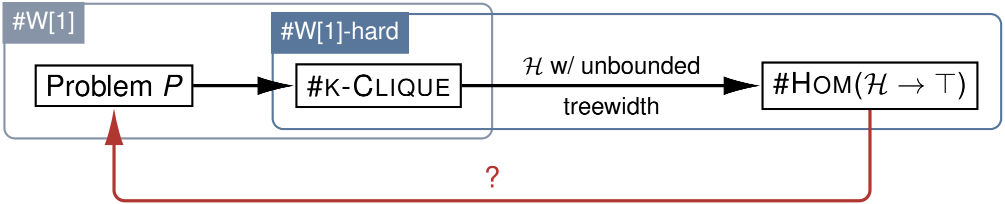

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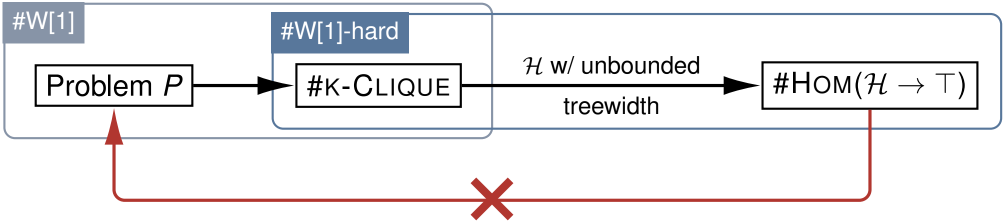

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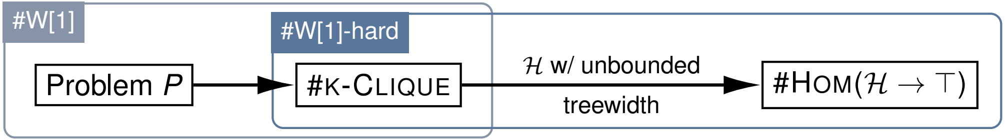

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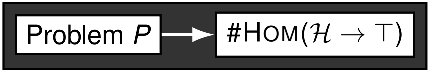

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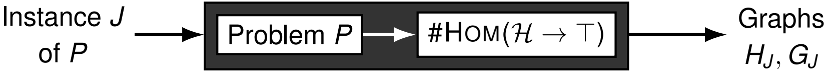

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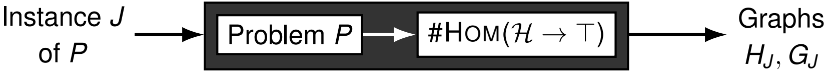

Approach:

$$\mathcal{H}_P := \{H_J \mid \text{instance } J \text{ of } P\}$$
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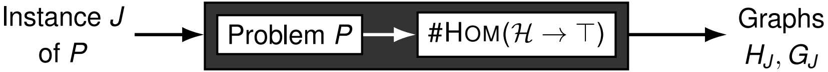

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$\mathrm{P} \preccurlyeq \#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P)$ ✓

Proof Ideas

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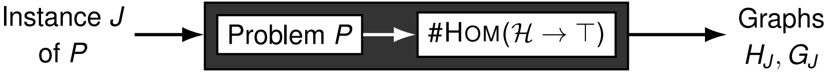

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$\#\text{Hom}(\mathcal{H}_P \to \mathcal{G}_P) \overset{?}{\preccurlyeq} \mathrm{P}$

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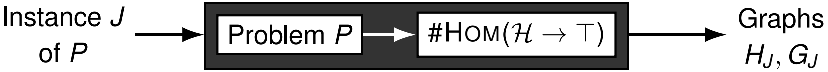

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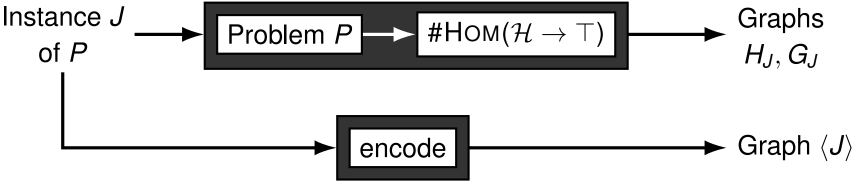
$\#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P) \not\preccurlyeq \mathrm{P}$

How do we obtain instance J from (H_J, G_J)?

Proof Ideas

Theorem

For any P in #W[1], there are $\mathcal{H}_P, \mathcal{G}_P$ such that P is equivalent to $\#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P)$.



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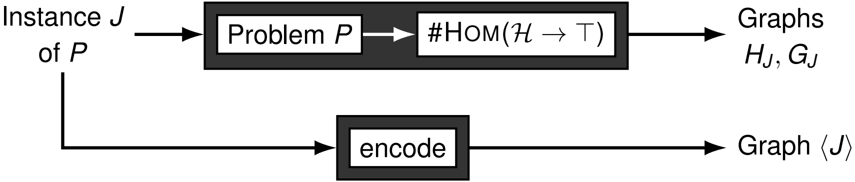

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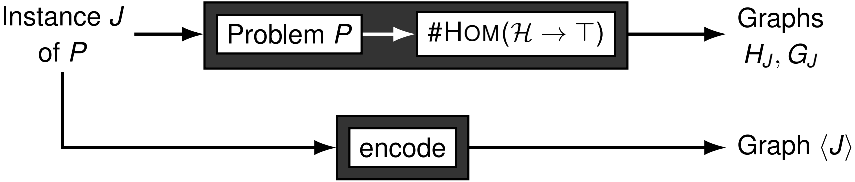

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$P \preccurlyeq \#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P)$ ✓

(ensure $\#\mathrm{Hom}(H_J \to \langle J \rangle) = 0$)

Proof Ideas

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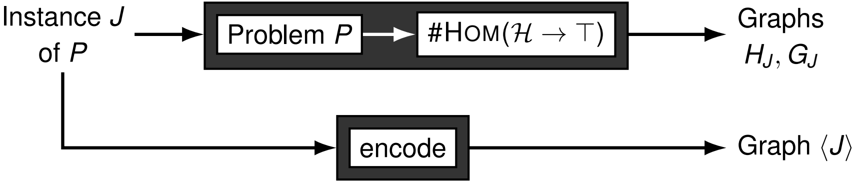

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$\#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P) \overset{?}{\preccurlyeq} P$

Proof Ideas

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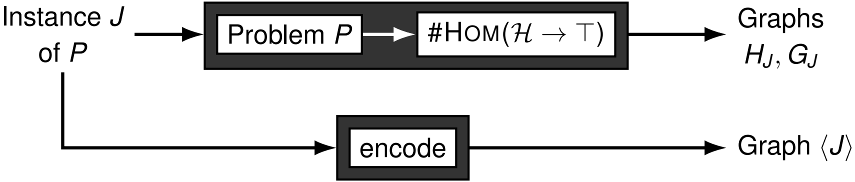

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$\#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P) \overset{?}{\preccurlyeq}$ P
How do we handle malformed input (H_J, G_L)?

Proof Ideas

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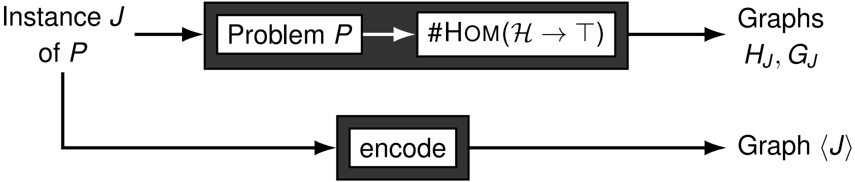

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(ensure $\#\mathrm{Hom}(H_J \to \langle J \rangle) = 0$)

$\#\textsc{Hom}(\mathcal{H}_P \to \mathcal{G}_P) \overset{?}{\preccurlyeq} P$

How do we ensure $\#\mathrm{Hom}(H_J \to G_L \cup \langle L \rangle) = 0$?

Theorem

For any P in #W[1], there are $\mathcal{H}_P$, $\mathcal{G}_P$ such that P is equivalent to #HOM($\mathcal{H}_P \to \mathcal{G}_P$).

P $\preccurlyeq$ #HOM($\mathcal{H}_P \to \mathcal{G}_P$)	#HOM($\mathcal{H}_P \to \mathcal{G}_P$) $\preccurlyeq$ P
Can solve instance J with $(H_J, G_J \cup \langle J \rangle)$ by computing #Hom($H_J \to G_J \cup \langle J \rangle$)	Can extract instance J from pair $(H_J, G_J \cup \langle J \rangle)$
(ensuring #Hom($H_J \to \langle J \rangle$) = 0)	How do we ensure #Hom($H_J \to G_L \cup \langle L \rangle$) = 0?

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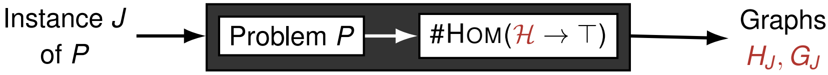

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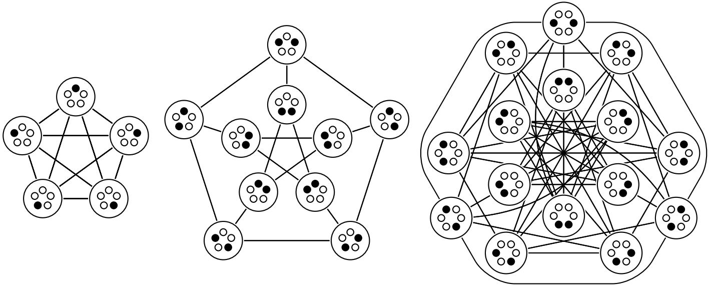

Theorem

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$P \preccurlyeq \#\text{Hom}(\mathcal{H}_P \to \mathcal{G}_P)$

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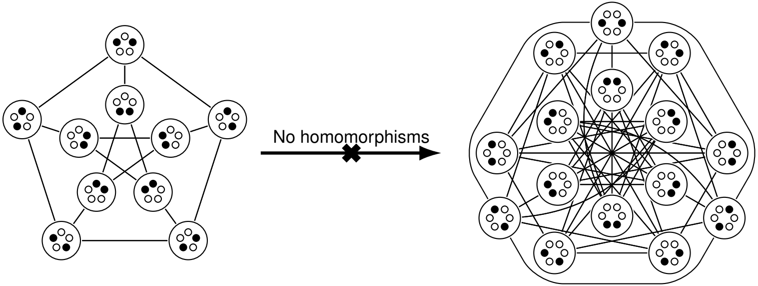
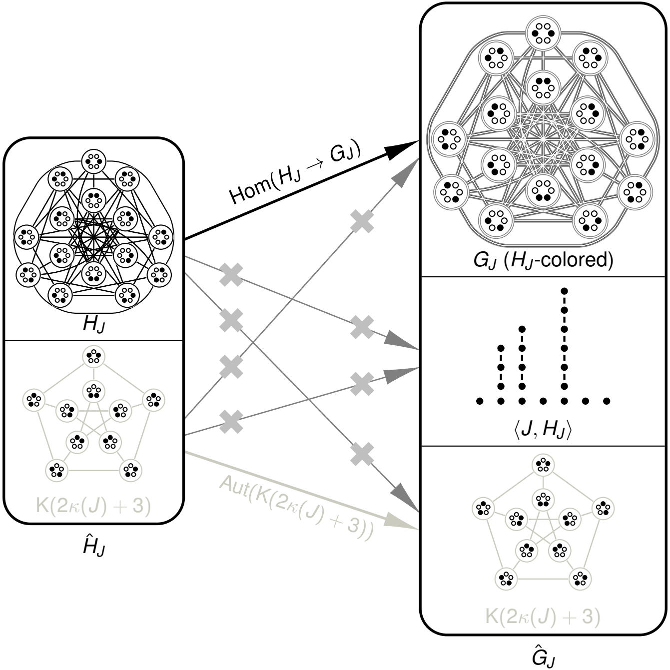
(ensuring $\#\text{Hom}(H_J \to \langle J \rangle) = 0$)

$\#\text{Hom}(\mathcal{H}_P \to \mathcal{G}_P) \preccurlyeq P$

Can extract instance J from pair $(H_J, G_J \cup \langle J \rangle)$

How do we ensure $\#\text{Hom}(H_J \to G_L \cup \langle L \rangle) = 0$?





Hom($H_J \to G_J$)
H_J
K($2\kappa(J)+3$)
$\hat{H}_J$
G_J (H_J-colored)
$\langle J, H_J \rangle$
K($2\kappa(J)+3$)
$\hat{G}_J$
Aut(K($2\kappa(J)+3$))

Main Result

#Hom($\mathcal{H} \to \mathcal{G}$) Parameter: $|V(H)|$

Given graphs $H \in \mathcal{H}$ and $G \in \mathcal{G}$, count all graph homomorphisms from H to G.

Theorem ✓

For any problem P in #W[1] (or W[1]), there are graph classes $\mathcal{H}_P$ and $\mathcal{G}_P$ such that P is equivalent to #Hom($\mathcal{H}_P \to \mathcal{G}_P$) (or Hom($\mathcal{H}_P \to \mathcal{G}_P$)).

- Cannot hope for clear categorization into FPT/W[1]-hard for all pairs $(\mathcal{H}, \mathcal{G})$

$\rightsquigarrow$ Need to look at specific pairs of graph classes

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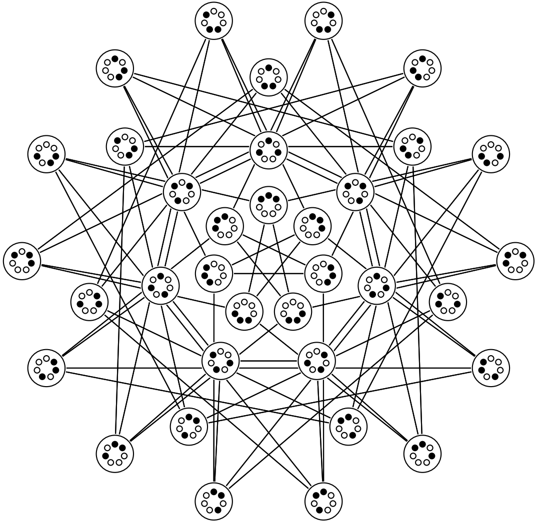
- Cannot hope for clear categorization into FPT/W[1]-hard for all pairs $(\mathcal{H}, \mathcal{G})$

$\rightsquigarrow$ Need to look at specific pairs of graph classes

Open Problems

- Can we find a "hierarchy" of homomorphism problems?
  $\#\text{HOM}(\mathcal{H}_1 \to \mathcal{G}_1)$ "$\leq$" $\#\text{HOM}(\mathcal{H}_2 \to \mathcal{G}_2)$ "$\leq$" $\cdots$ "$\leq$" $\#\text{HOM}(\top \to \top)$
- (Grunt work?) Obtain algorithms/hardness for specific pairs of graph classes $\mathcal{H}$, $\mathcal{G}$
  (Done for $\mathcal{G} = F$-colorable graphs, line graphs, claw-free graphs, ...)



Thank you!

TikZ code for Kneser graphs available on GitHub

`github.com/PH111P/tikz-kneser`

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